Bilinear Model of Behavior Reinforced Concrete Column under High-intensity Lateral Loads after Fire

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Abstract. The experience of destructive earthquakes shows that the problem of determining the response of RC frames under seismic loads after a fire is relevant. Calculation models for individual elements and buildings as a whole should reflect nonlinear behavior in this case. A brief review of models for describing reinforced concrete elements under low-cycle loads is given. It is noted that such models for elements damaged by fire do not exist at present. A model based on a bilinear diagram for calculating an eccentrically compressed RC column damaged by fire is proposed. Only three parameters are required to describe the model: the ultimate moment, the ultimate curvature, and the effective initial stiffness. The ultimate moment and curvature are determined by analyzing the distribution of stresses and strains in the column cross-section during the fracture stage. The column cross section is represented by a set of separate layers heated to different temperatures. The effective stiffness is defined by a straight line passing through the point in the curvilinear diagram where the initial yielding in the reinforcement or concrete occurs. The model takes into account different levels of axial loading, indirect reinforcement by transverse hoops, second-order effects and non-uniform distribution of stresses in the compressed zone of concrete. On the basis of the proposed model, bilinear deformation diagrams of RC columns, which are subjected to standard fire of different duration, are compared. The calculation results have shown a significant decrease in bearing capacity and stiffness of the damaged columns and an increase in their plasticity. The resulting model is simple enough to be used and suitable for most engineering calculations. This model can be used as a basis for constructing a hysteresis diagram for low-cycle impacts after a fire, which is necessary for seismic analysis of structures in the time domain.

1 Introduction

Reinforced concrete buildings are subjected to a whole complex of force and environmental influences in the process of operation. Such combinations include, for example, problems of survivability against progressive collapse of buildings damaged by fire [1] and corrosion

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[2], fire impacts in combination with seismic [3] loads, combinations of emergency impacts [4], the problem of RC column stability under horizontal impact [5].

Fig. 1. Bilinear approximation of the "moment-curvature" diagram for a reinforced concrete column.

The authors' attention to the issue of combination of seismic and fire impacts is mainly represented by experimental [6] and numerical studies [7].

For the development of engineering calculation methods, it is also important to develop theoretical models of the performance of individual elements (beams and columns) [8] and of the load-bearing frame as a whole. At the same time, such models should reflect the nonlinear behavior of structures and be simple enough in practical application.

A simplified approach in describing the nonlinear operation of columns and beams as part of the frame is the introduction of plasticity hinges into the model. In this case, the nonlinear properties are concentrated in the sections of length $L_p$, which are adjacent to the joints of columns and beams. The rest of the structure is considered elastic.


From a computational point of view, the simplest model for describing hysteresis behavior is the model based on a bilinear diagram (Fig. 1).

In [12], a bilinear model is proposed, which has a linear-elastic first section with an equivalent stiffness $K_e$, after reaching the bearing capacity the stiffness becomes zero.

When bending reinforced concrete columns and beams, as a result of redistribution of stresses, some sections of the section will be involved gradually, which is caused by the non-linear behavior of reinforcement and concrete. Thus, the bearing capacity resource of the elements will be realized. In the deformation diagram, this can be accounted for by introducing a nonzero stiffness after the ultimate moment is reached. This approach is realized in the bilinear model proposed in [13].

The above models are quite suitable for describing the performance of reinforced concrete elements without initial damage and have the engineering simplicity. However, there is no theoretical description of such a model for the elements pre-exposed to fire.

This paper develops a model of nonlinear operation of a reinforced concrete column after a fire based on a bilinear model. This model can be further generalized to the case of low-cycle loading - hysteresis diagram.
An important feature of the proposed model is the consideration of transverse reinforcement, which, in addition to ensuring the strength of inclined sections, perform the role of indirect reinforcement. Transverse reinforcement makes it possible to increase the carrying capacity of the elements and the capacity of plastic deformation, which contributes to the redistribution of forces in the system and a fuller use of the reserves of carrying capacity. In addition, after the ultimate moment are reached, when the stresses in the hoops reach the yield strength, there is no sudden failure of the element, but a gradual decrease in the carrying capacity with increasing plastic deformations follows [14]. Unstrengthening can take place taking into account such phenomena as loss of stability of compressed reinforcement, geometric nonlinearity, spalling of the protective layer of concrete.

2 Methods

Consider a reinforced concrete column of rectangular cross-section, which is under the influence of vertical $N$ and horizontal $H$ loads applied at the level of the upper embedment.

After exposure to a standard fire in the cross-section of the column, there will be temperature fields with a maximum value of temperature at the edges, which will decrease toward the center of the section. In this paper, we assume that the fire affects the column uniformly from four sides.

Assume the following assumptions:
- the cross sections are flat before deformation and remain so after;
- the following deformation diagrams are adopted for the materials: bilinear for compressed concrete (Fig. 2a) and reinforcement (Fig. 2b); three-linear for confined concrete (Fig. 2c) [15];
- second-order effect (axial bending of a reinforced concrete column), taken into account by the coefficient axial bending $\eta$;
- the work of tensile concrete is not taken into account;
- stresses in concrete and reinforcement are found by making and solving equations of equilibrium.

As noted, to describe the response of a reinforced concrete column, it is necessary to consider the section where the greatest bending moment occurs - the plastic hinge. The section is represented as consisting of $n$ number of ring-shaped sections (layers), which are heated to different temperatures. Within each section the temperature, strength and strain characteristics are constant.

The deformation diagrams of concrete sections heated to different temperatures (Fig. 2) are obtained by introducing the coefficients $\gamma_{bt(s)}$ and $\beta_{bt(s)}$ to the standard bilinear and three-linear diagrams, by which the design resistance and elastic moduli of concrete and reinforcement are multiplied respectively (SP 468.1325800.2019).

The nonlinear response of the column will be described by means of a bilinear deformation diagram (Fig. 1). For this purpose, it is necessary and sufficient to set three parameters: ultimate moment $M_u$, effective stiffness $EI_{eff}$ and ultimate curvature $\rho_u$.

The carrying capacity of a reinforced concrete column (ultimate moment $M_u$) will be determined by considering the stage of failure of an eccentrically compressed element. In this case we consider that there is a case of large eccentricities $\xi \leq \xi_R$, i.e. the destruction begins after reaching the ultimate stresses $R_b$ in the stretched reinforcement - the plastic mechanism.

Then, as the bending moment increases, the ultimate stresses in the compressed concrete $R_b$ and the reinforcement $R_{sc}$ are reached. At this stage, the concrete cross-section can be divided into two main zones: a protective layer and a confined concrete core. The protective layer will deform as unconfined concrete - without stress growth. The core acts as concrete
confined by transverse hoops, due to which there is an increase in stresses and as a consequence an increase in the load-bearing capacity of the column.

![Fig. 2. Deformation diagrams of (a) reinforcement; (b) unconfined concrete; (c) confined concrete.](image)

Thus, the failure stage of a reinforced concrete column is associated with the achievement of stresses equal to the confined concrete ultimate stresses $R_{b, tr}$ at the appropriate temperature in all layers of the core. The ultimate stresses for the confined concrete are determined by the moment of yielding in the transverse hoops, i.e. reaching the ultimate stresses $R_{s, tr}$ in them.

In the confined concrete diagram, these stresses will correspond to the relative deformation $\varepsilon_{b, tr}$. Since the values of deformations $\varepsilon_{b, tr}$ will increase with increasing temperature, the stage of element failure will be associated with reaching the specified deformations in the least heated concrete layer of the compressed zone.

As studies [16] show, by the time the ultimate bearing capacity is reached, the concrete of the protective layer completely collapses, which is associated with the achievement of ultimate deformations in it $\varepsilon_{b, 2}$. Only an insignificant section of the protective layer near the neutral axis will be involved, the contribution of which is insignificant. Therefore, the protective layer will be neglected in determining the bearing capacity of the section.

In accordance with the above described, the diagram of stress and strain distribution in the column cross-section for the fracture stage is shown in Fig. 3.

The ultimate bending moment $M_u$ is determined by composing and solving equations of equilibrium of internal forces in the section.

The equation of the sum of force projections on the longitudinal axis of the element is written in the form of

$$N + R_y A_y - R_{ax} A_x \eta_a - N_b = 0,$$

where $N_b$ is the total axial force taken by the compressed zone of concrete;

$\eta_a$ is the coefficient that takes into account the loss of stability of the compressed bars. The coefficient $\eta_a$ takes values from 0 to 1.

The force $N_b$ is found by summing up the stresses in $n$ sections of the compressed zone, taking into account their heating temperature.

$$N_b = R_{b, tr(i)} x(b - 2a_{tr}) + \sum_{i=1}^{n} \left[ R_{b, tr(i+1)} - R_{b, tr(i)} \right] (b - 2a_{tr} - 2ia_{sl})(x - ia_{sl})$$,

where $R_{b, tr(i)}$ is the strength of the confined concrete in the $i$-th layer;

$a_{tr}$ is distance from the edge of the element to the transverse hoops;

$\eta_{sl}$ is thickness of the $i$-th layer of concrete.

The height of the compressed zone $x$ is determined from formula (1). Thus, as mentioned above, the condition of equality of the greatest deformations in the least heated $n$-th layer of the compressed zone ultimate deformations of confined concrete at a given temperature should be satisfied

$$\varepsilon_{b, tr(n)} = \varepsilon_{b, 3, tr(n)}$$.
The number of considered layers $n$ is determined iteratively until the condition (1) converges. At first, it is assumed that the compressed zone includes all layers of the section above the symmetry axis, if condition (1) in this case is not definable, then it is necessary to exclude the bottom layer and repeat the procedure.

The ultimate moment $M_u$ is found from the condition of equality to zero of the sum of moments relative to the center of gravity of the stretched reinforcement in the form

$$Ne = R_{sc} A_s' \eta_b (h_o - a') + M_b,$$

(4)

where $M_b$ is the resultant bending moment taken by the compressed zone of concrete.

The moment $M_b$ is determined by the formula

$$M_b = M_{b1} + M_{b2},$$

(5)

where

$$M_{b1} = R_{b,tr(1)} x (b - 2a_{tr})(h_o - a_{tr} - 0.5x),$$

$$M_{b2} = \sum_{i=1}^{n} \left( R_{b,tr(i+1)} - R_{b,tr(i)} \right) (b - 2a_{tr} - 2ia_{sd}) (x - ia_{sd})(h_o - a_{tr} - \frac{x - ia_{sd}}{2}) .$$

(6) (7)

Determine the limiting bending moment $M_u$ relative to the center of gravity, taking into account the effects of second order effects

$$M_u = \frac{Ne - N h_o - a'}{2 \eta},$$

(8)

where $\eta$ is the coefficient of longitudinal bending;

$$\eta = \frac{I - N}{I - N_{cr}},$$

(9)

where $N_{cr}$ is the Euler critical force.

The ultimate curvature of a reinforced concrete column $\rho_u$ is determined in accordance with the hypothesis of flat sections from the consideration of the distribution of relative deformations in the fracture stage. Due to the smallness of the section rotation angle, the curvature $\rho_u$ is found by the formula

$$\rho_u = \frac{\varepsilon_{b,tr(n)}}{x - (n - 1)a_{sd}},$$

(10)

where $\varepsilon_{b,tr(n)}$ are the ultimate relative strains for the least heated n-th layer of the compressed zone;

To determine the slope of the first section of the bilinear diagram of deformation of a reinforced concrete column, we will use the approach used in [17], which showed a fairly good correlation with the results of numerical simulation. This approach is based on the bilinear model of Elwood and Eberhard (2009) [18] and is generalized to the case of eccentrically compressed elements after a fire.

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**Fig. 3.** Distribution of strains and stresses in the column cross section for the fracture stage
The slope of the elastic part of the diagram is characterized by the value of the effective stiffness $EI_{eff}$. It is assumed that the experimental curve and the first section of the diagram have an intersection point which corresponds to the bending moment at the first yield point $M_{1,y}$ - reaching the yield stresses in the reinforcement or reaching the relative deformations in the concrete of 0.002.

The effective stiffness of the column $EI_{eff}$ is determined by the formula

$$0.2 \leq \frac{EI_{eff}}{EI_g} = \frac{0.45 + \frac{2.5N}{l} \sum A_{g,i} R_{b,i}(l)}{1 + \frac{110(d/h)(h/a)}} \leq 1,$$

where $A_{g,i}$, $R_{b,i}(l)$ is the area and ultimate strength of the $i$-th section of the concrete gross without taking into account the formation of cracks in the tensile zone; $d$ is the nominal diameter of the reinforcement; $h$, $a=l/2$ is size of the cross-section along the plane of bending moment and the distance from the column embedment to the point of inflection of its longitudinal axis.

The stiffness of the concrete section of the column after fire $EI_g$ is defined as

$$EI_g = \sum_i E_{b,i} I_{g,i},$$

where $E_{b,i}$, $I_{g,i}$ is the elastic module and moment of inertia of the gross $i$-th section of concrete.

Using the obtained values of $M_u$, $\rho_u$ and $EI_{eff}$, a bilinear diagram of the deformation of a reinforced concrete column after a fire is constructed according to Fig. 1.

### 3 Results

On the basis of the obtained bilinear diagram of deformation of a reinforced concrete column, taking into account the damage resulting from the fire action, we conduct a comparative analysis of the performance of three columns. The design of all columns is identical, except for the duration of the standard fire to which they are preliminarily exposed.

The cross-section of the columns is square $300x300$ mm, the geometric dimensions are given in Fig. 4a. The longitudinal reinforcement of 4 bars $\varnothing 25$ reinforcement class A500, $A_s=A_s'=982 \text{ mm}^2$. The design length of the column is assumed to be $l_o=3 \text{ m}$. All reinforcement is considered to be effectively braced against loss of stability $\eta_s=1$. Concrete of the columns of class B20. Transverse reinforcement of closed hoops $\varnothing 8$ class A500, in the most loaded zone of the column (in the zone of plastic hinge) the hoops are installed with a spacing $s_w=100 \text{ mm}$. The transverse reinforcement coefficient by volume will be $\rho_s=0.005$. 

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**Fig. 4.** Calculation results: (a) design cross-section of the column; (b) bilinear "moment-curvature" diagrams for columns at different duration of a standard fire.
Table 1. Calculation results of the bilinear model \((N=300 \text{ kN}; k=0.3)\).

<table>
<thead>
<tr>
<th>№</th>
<th>Standard fire time (ISO-834) (t)</th>
<th>Ultimate moment, (M_u \text{ kNm})</th>
<th>Inflection curvature, (\rho_y \text{ 1/m})</th>
<th>Ultimate curvature, (\rho_u \text{ 1/m})</th>
<th>Effective stiffness, (E_{\text{eff}} \text{ MPa})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0 min</td>
<td>132.33</td>
<td>0.0041</td>
<td>0.0072</td>
<td>33.15</td>
</tr>
<tr>
<td>2</td>
<td>30 min</td>
<td>101.12</td>
<td>0.0044</td>
<td>0.0099</td>
<td>23.11</td>
</tr>
<tr>
<td>3</td>
<td>60 min</td>
<td>70.66</td>
<td>0.0047</td>
<td>0.0124</td>
<td>15.46</td>
</tr>
</tbody>
</table>

The strength of confined concrete \(R_{b,\text{tr}}\) depends on the strength of unconfined concrete \(R_b\) and the effective lateral pressure \(R_e\), which results from the resistance of the collars to the transverse deformations of concrete. According to [14], the strength of confined concrete can be determined as

\[ R_{b,\text{tr}} = R_b + 4.1R_e. \] (13)

The effective lateral pressure \(R_e\) for a square section is found by the formula

\[ R_e = k_e\rho_yR_{s,\text{tr}}, \] (14)

where \(R_{s,\text{tr}}\) is the yield strength of the transverse reinforcement; \(k_e\) coefficient of retention efficiency, which takes into account the non-uniform compression of concrete in sections other than circular.

The value of the axial force is assumed to be \(N=300 \text{ kN}\). This corresponds to the coefficient of axial force for the column undamaged by fire \(k=0.3\), which takes place for the columns of the middle floors of the frame structure.

\[ k = \sum_i \frac{N_i}{A_{g,i}R_{b,\text{tr}(i)}}. \] (15)

The first column is taken as a control column, that is, it is not subjected to fire impact. As the fire exposure for the other two columns adopted standard fire according to ISO-834 duration, respectively, 30 and 60 minutes.

Thermal calculation is done on the basis of partial differential Fourier equations. The solution is obtained by means of the finite element method. The thickness of layers into which the section is divided is taken to be \(a_s=25 \text{ mm}\).

The mechanical characteristics of the reinforcement and concrete are corrected depending on the temperature of heating of a given layer by introducing the coefficients \(\gamma_{b(i)}\) and \(\beta_{b(i)}\) to the design resistance and the modulus of elasticity, respectively.

The resulting bilinear diagrams are shown in Fig. 4b.

4 Discussion

It follows from the calculation results (table 1) that the load-carrying capacity of reinforced concrete columns sharply decreases with increasing fire duration. The value of ultimate moment \(M_u\) for an undamaged column was 1.3 and 1.9 times higher than after a standard fire duration of 30 and 60 minutes, respectively.

The decrease in carrying capacity in this case is associated not only with a decrease in the strength of concrete and longitudinal reinforcement, but also with a significant decrease in the strength of transverse hoops, which are located quite close to the heated faces of the section. This, in turn, leads to a reduction in the role of transvers reinforcement in retaining
the concrete core. With a fire duration of 60 minutes, the effect of retention in increasing the bearing capacity is almost completely lost.

In this connection, it is possible to recommend more rational schemes of setting transverse hoops and longitudinal reinforcement, for example, in addition to the installation of hoops along the outer edges, it is possible to install additional hoops, tying longitudinal rods, located not in the corners, but in the middle of the column edges. In addition, the use of welded meshes, when the indirect reinforcement is located more evenly across the cross-section of the element, seems promising in this case.

The effective stiffness of the columns $EI_{eff}$ also decreases after the fire, the changes in relation to the control column are even higher. The stiffness $EI_{eff}$ for the undamaged column was 1.43 and 2.15 times higher than after fire of 30 and 60 min, respectively.

In contrast, the plastic properties of fire-damaged columns increased, as shown by the limiting curvature $\rho_u$. The curvature for the column after the 30 min standard fire is 1.4 times higher than that of the control column, and after the 60 min fire is almost 1.7 times higher.

This is substantiated by an increase in the plastic properties of both concrete and reinforcement with an increase in the heating temperature. In addition, the effect of cross clamps on ductility, as opposed to load-bearing capacity, remains quite high.

The latter circumstance allows us to judge about certain reserves of carrying capacity of reinforced concrete columns, damaged by fire, when working as part of the frame structure.

As a result of the impact of the fire in one of the compartments of the building, there will be weakening of the structures that were in the hearth of the fire. The seismic response of the building will then change. Fire-damaged elements will enter the plastic stage and partially shut down. However, the undamaged elements will be overloaded due to the redistribution of forces from the weakened elements.

Thus, it is extremely important to further develop nonlinear methods of calculation of frame frames for seismic effects after a fire, which would take into account the mutual influence of damaged and undamaged elements on each other.

## 5 Conclusions

1. A bilinear model of operation of a reinforced concrete column subjected to high-intensity lateral load after a fire action is obtained. Only three parameters are used to set the model: ultimate moment, ultimate curvature and effective stiffness, which is relevant for practical seismic calculations by engineering methods, which have a requirement to reduce computational costs. The model takes into account different levels of axial loading, transverse reinforcement, second-order effects and uneven stress distribution in the compressed zone of concrete.

2. The results of calculations using the obtained model show a significant reduction in the load-carrying capacity and stiffness of the columns as a result of the impact of fire. However, plastic properties increase after fire action. Considering that a fire in the building occurs in its local area, in order to establish the true nature of the work of the load-bearing frame, it is necessary to apply numerical methods of calculation, at that, the mechanical characteristics of individual elements (columns and beams) in the section with a plastic hinge can be obtained analytically by the model suggested in the present paper.

3. The obtained model can be used as a basis for hysteresis diagram for low-cycle loading which is necessary for analysis of structural system in a time domain. When constructing the hysteresis diagram, the stiffness characteristics of the structure under unloading, pinching effect, cyclic degradation of strength and stiffness should be taken into account.
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References

2. Savin S. Yu., Kolchunov V. I., Fedorova N. V., Reinforced Concrete Structures 1(1) 46–54 (2023)
5. Alekseytsev A. V. Reinforced Concrete Structures 2(2) 3–12 (2023)
7. Shuna N., Birely A. C., Data in Brief 19 1650-1657 (2018)
17. Chernik V. I. Science Prospects. 5 82-86 (2022)