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Heat exchange between the fuel element of a nuclear reactor made of uranium oxide and its shell under boundary conditions of the fourth kind

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Abstract. The work is devoted to the heat exchange of a nuclear reactor. Attention was paid to the mathematical modeling of the temperature field of the fuel element and the thin gas layer. It is assumed that the thermal contact between the solid–interlayer system is ideal. Methods of differentiation, integration and approximate methods were used for numerical modeling. The dependence of the thermal conductivity of uranium oxide on temperature was also taken into account. Based on the obtained expression, the linear heat transfer coefficient and linear thermal resistance were also found.

1 Introduction

As you know, the fuel cell is the main element of the core of any nuclear reactor. However, the presence of nuclear fuel requires special attention and caution when operating it. Therefore, one of the key points of the NPP safety analysis is the investigation of the occurrence of emergency processes and confirmation that during the accident the main parameters do not exceed the permissible limits.

A unified approach to the thermal calculation of various nuclear reactors is explained by the presence of internal heat generation in them due to a nuclear reaction [1-7]. To date, a lot of work has been devoted to the calculation of fuel elements made of uranium oxide [8-14], but little attention has been paid to the heat exchange of these elements with their surrounding shell.

The paper considers boundary conditions of the fourth kind. Boundary conditions of the fourth kind define the conditions of heat exchange at the boundary of the ideal contact of two bodies consisting of different substances with different physical properties. Therefore, they simulate the so-called ideal thermal contact between tightly touching bodies, and have a simple physical meaning: what amount of heat is supplied from the depth of the first body to its boundary, the same amount of heat is diverted into the depth of the second body. In this work, using the methods of differentiation and integration, the problem of heat

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exchange between a cylindrical heat-generating element and its shell under boundary conditions of the first and fourth kind was solved. As a result, the laws of change of temperature fields in both bodies are obtained. At the same time, the dependence of the thermal conductivity of uranium oxide on temperature was taken into account. Also, based on the obtained mathematical modeling, the linear coefficient of heat transfer through the shell and the linear thermal resistance for this case were determined.

2 Main part

There is a small gap in the design of the nuclear element itself. It is located between the fuel rod and the shell. The shell itself is a thin gas layer of helium, which is chemically neutral and highly conductive (figure 1).

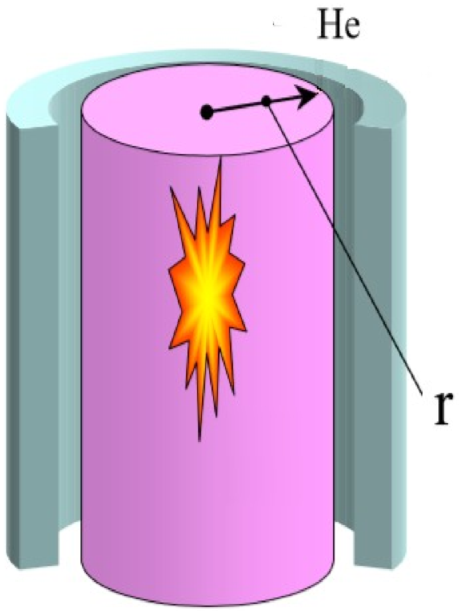

Fig. 1. The design of the fuel element of a nuclear reactor.

The temperature field of a cylindrical fuel cell with a specific power q_v is described by the Poisson equation. In this case, the equation of thermal conductivity will take the form:

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)=-\frac{q_v}{\lambda_1} \tag{1}$$

Where: λ_1 is the thermal conductivity coefficient of the fuel rod. At the beginning, we will separate the variables:

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right)=-\frac{q_v}{\lambda_1}r \tag{2}$$

Integrating both parts of the equation, we get:

$$r\frac{dT}{dr} = -\frac{q_v r^2}{2\lambda_1} + C_1 \tag{3}$$

Let's separate the variables again:

$$\frac{dT}{dr} = -\frac{q_v r}{2\lambda} + \frac{C_1}{r} \tag{4}$$

Let's apply the integration method again:

$$T_1 = -\frac{q_v r^2}{4\lambda_1} - C_1 \ln r + C_2 \tag{5}$$

Constant C1 = 0 due to the final value of the temperature in the center. Therefore:

$$T_1 = -\frac{q_v r^2}{4\lambda_1} + C_2 \tag{6}$$

The temperature field of the shell is described by the following equation:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\lambda_2 r \frac{\partial T}{\partial r}\right) = 0 \tag{7}$$

Where: λ2 is the coefficient of thermal conductivity of the material from which the shell is made. Integrating the expression (7):

$$\frac{dT}{dr} = \frac{C_3}{\lambda_2 r} \tag{8}$$

Re-integrating (8), we get:

$$T_2 = \frac{C_3}{\lambda} \ln r + C_4 \tag{9}$$

We find the constant C3 from the following boundary condition. In this heat exchange, the boundary conditions are boundary conditions of the fourth kind. In this case, the temperatures of the contacting surfaces are the same:

$$T_1\big|_{r=R_1} = T_2\big|_{r=R_1} \tag{10}$$

Where: T1 and T2 are the temperatures of the fuel element and the interlayer. And the heat flows at the boundary of their separation should be the same:

$$-\lambda_1 \frac{dT_1}{dr}\bigg|_{r=R_1} = -\lambda_2 \frac{dT_2}{dr}\bigg|_{r=R_1} \tag{11}$$

Substitute expression (9) in (11):

$$-2q_v R_1 = \frac{C_3}{R_1} \tag{12}$$

From where the constant C3 is equal to:

$$C_3 = -2q_v R_1^{\,2} \tag{13}$$

Then the temperature T2 is equal to:

$$T_2 = \frac{-2q_v R_1^{\,2}}{\lambda_2}\ln r + C_4 \tag{14}$$

Next, we will use the boundary condition (10):

$$-\frac{q_v R_1^{\,2}}{4\lambda_1} + C_2 = \frac{-2q_v R_1^{\,2}}{\lambda_2}\ln R_1 + C_4 \tag{15}$$

Where from:

$$C_2 = q_v R_1^{\,2}\left(\frac{1}{4\lambda_1} - \frac{2\ln R_1}{\lambda_2}\right) + C_4 \tag{16}$$

We find the constant C4 from the boundary condition of the third kind: convective heat exchange with the external environment occurs on the surface of the shell:

$$-\lambda_2 \frac{dT_2}{dr}\bigg|_{r=R_2} = \alpha(T_2 - T_0) \tag{17}$$

Where T0 is the ambient temperature. Substituting expression (14) into (17) we get:

$$4q_v R_2 = \alpha\left(C_4 - T_0 - \frac{2q_v R_1^{\,2}}{\lambda_2}\ln R_2\right) \tag{18}$$

From where the constant C4 is equal to:

$$C_4 = T_0 + \frac{2q_v R_1^{\,2}}{\lambda_2}\ln R_2 + \frac{4q_v R_2}{\alpha} \tag{19}$$

When substituting this expression into equation (16), we find the constant C2:

$$C_2 = T_0 + q_v R_1^{\,2}\left[\frac{1}{4\lambda_1} + \frac{2\ln\left(R_2 / R_1\right)}{\lambda_2}\right] + \frac{4q_v R_2}{\alpha} \tag{20}$$

Then the expression for determining the temperature field of the fuel cell will take the form:

$$T_1 = T_0 + \frac{q_v (R_1^{\,2} - r^2)}{4\lambda_1} + \frac{2q_v R_1^{\,2}}{\lambda_2}\ln\left(R_2 / R_1\right) + \frac{4q_v R_2}{\alpha} \tag{21}$$

And for the shell:

$$T_2 = T_0 + \frac{2q_v R_1^{\,2}}{\lambda_2}\ln\left(R_2 / r\right) + \frac{4q_v R_2}{\alpha} \tag{22}$$

The resulting field of the system under study is shown in figure 2.

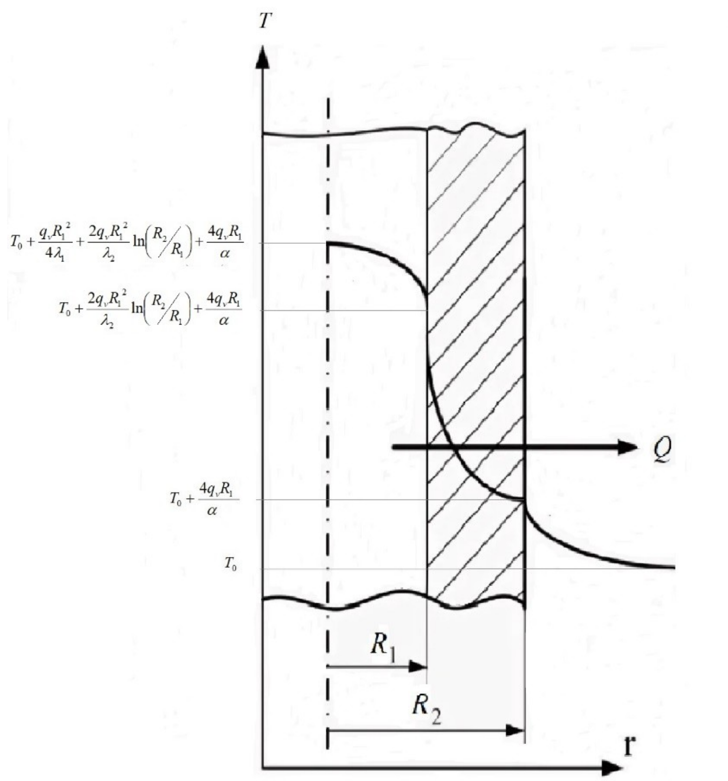

Fig. 2. Graph of the behavior of the temperature field of the element and shell.

As follows from expression (21), the temperature of the fuel cell varies according to the parabolic law in the direction of decrease from the center. From the second expression obtained (22) it follows that the temperature field of the shell decreases according to the logarithmic law. The obtained dependences are due to the given boundary conditions.

3 Conclusion

Thus, using the methods of differentiation and integration, the stationary problem of finding the temperature field of both the fuel element of a nuclear reactor and the helium layer was solved, taking into account the temperature dependence of the thermal conductivity of uranium oxide. Numerical simulation of the heat release process allows you to choose the power of internal sources, which ensures the optimal operation of a nuclear reactor.

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