Application of the method of mathematical statistics to study the wear of parts of modern machines

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Abstract. This article describes the method of mathematical statistics for studying the wear of parts of modern machines. It is examined how the service life of the parts is established. It was also assumed that the statistical data on the size of worn parts obey the law of normal distribution. The figure presented in the paper illustrates the scheme for determining the inter-repair service life of the part for this case according to statistical data. An example is considered that allows to establish the norm of the maintenance period of the support roller of a modern tractor VT-150. This article is recommended for agricultural specialists, researchers, teachers, postgraduates, undergraduates and students of agricultural universities in the field of training "Agroengineering".

1 Introduction

In recent years, methods of probability theory and mathematical statistics have been increasingly used in various fields of mechanical engineering and agriculture to study the wear of parts and interfaces of modern machines and mechanisms.

At the same time, it should be noted that probability theory is based on a number of basic concepts, with the help of which a logical definition of subsequent more complex ones is given.

The main concepts are an event and its quantitative characteristic – the probability of an event. An event is understood as any fact that may or may not occur as a result of an experiment, experiment or observation under a certain set of conditions.

For example, such random events as [1, 2, 3]:

– the measured value of the wear of the part is the result (outcome); the measurement operation performed is an event (test);

– overtaking a car on a selected road section (a set of conditions – the observed road section, the event is overtaking);

– failure–free operation of the car when running a certain mileage (condition – monitoring the operation of the car, event – failure–free operation);

– meeting an impassable area for the car (condition - studying the movement of the car, event – impassability of the car).

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Each of the above events has some kind of possibility, i.e. they can be quantified. So, if we take the operating time of the car longer, then the reliability of its operation will have less opportunity [4-6].

2 Materials and methods

In this article, methods of mathematical statistics are used to study the wear of parts of modern machines.

Mathematical statistics is understood as "a section of mathematics devoted to mathematical methods of collecting, systematizing, processing and interpreting statistical data, as well as using them for scientific or practical conclusions.

The rules and procedures of mathematical statistics are based on probability theory, which makes it possible to assess the accuracy and reliability of the conclusions obtained in each problem based on the available statistical material [7, 8, 9].

3 Discussion

In this article, the curves of the dependence of wear and the wear rate of parts on the wear rate are considered.

Since the quantitative characteristics of the studied object or phenomenon in the form of a number and a measure are of interest in probability theory and mathematical statistics, the concepts of experiment, test, observation, experiment, sample can be conditionally combined.

As a result of certain experiments or observations, a large amount of experimental data or statistical material is obtained.

For use in mathematical statistics, these concepts can also be conditionally combined and spoken of as statistical data.

There is a certain practical meaning associated with the concept of the probability of an event. Usually, based on experience or observations, more likely events are established, i.e. those that are observed more often, and less likely ones that occur less often.

Figure 1 shows the curves of the dependence of wear h and the wear rate of parts on the wear rate, where the abscissa axis shows the time of operation of the coupling in hours, and the ordinate axis shows the amount of wear in mm.

The presented curves are valid for most possible patterns of wear of parts and interfaces.

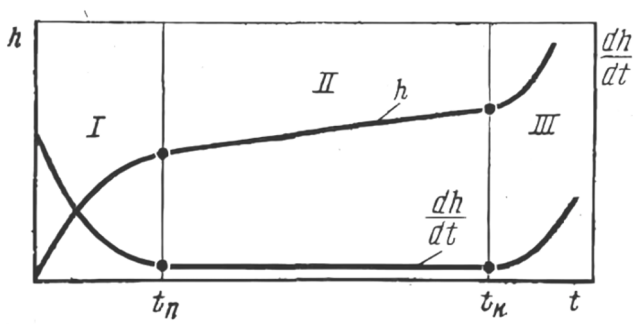

Fig. 1. The dependence of wear h and the wear rate of parts on the wear rate $\frac{dh}{dt}$

Table 1 shows three main areas of increasing wear of parts and interfaces: the initial one is a curved section *I*, the intermediate rectilinear section *II*, and the final one is also a curved section *III*.

Table 1. Three main areas of increasing wear of parts and interfaces

Plot N°	Name of the area of increasing wear	Brief description of the areas of increasing wear of parts and interfaces
I	Curved section of the increase in wear of parts	The first, the initial wear area, where there is a slight increase in wear. At this stage, the process of developing a new interface is underway.
II	A rectilinear section of a slight increase in the wear of parts and interfaces	The second, intermediate section of the increase in wear of the interface, the largest in length, corresponds to the period of normal operation of the interface. On this rectilinear section of wear, there is a natural wear of parts and interfaces.
III	The final (curved) section of the catastrophic increase in wear of parts and interfaces	The third, final phase of the increase in wear of the interface, small in length, corresponding to the period of destruction of parts due to exceeding the permissible limit (emergency wear).

The wear rate is of great interest for engineering practice.

The wear rate is the ratio of the wear value of a part or mate to the time interval during which it occurred.

The wear rate of parts and interfaces is greatly influenced by the specific pressure P and the speed of relative sliding.

This dependence can be expressed by the following formula [10]:

$$I = kp^m v^n, \tag{2}$$

where *I*– is the wear rate of parts and interfaces;
k – is a coefficient that characterizes the influence of the material of the part and the quality of its surface;
m,n – are constants that characterize the type of lubricant, the quality of the lubricating layers.

Speaking about the application of the method of mathematical statistics to study the wear of modern machines, it should be noted that modern tractors, cars and agricultural machines are products of mass production, which is characterized by the processing of parts with automatic dimensioning, i.e., conditions are characteristic in which the worker does not affect the operation of the equipment during the manufacture of parts.

Numerous studies have shown that the size distribution curves of such parts obey the law of normal distribution (Gauss' law).

After the assembly of the machine, the parts are subjected to wear during their use. If we abstract from the actual working conditions of the parts, accept that a sufficiently large batch of machines of the same brand and the same technical condition are used under almost identical conditions and when used under steady natural wear, then we can assume that the parts we are interested in are subjected to further processing to the maximum permissible condition.

If further this batch of machines is turned off from work when the considered parts according to the accepted service life should reach the maximum permissible wear, and measure these parts, then, obviously, the distribution curve of the obtained sizes of them should also obey the law of normal distribution.

The actual working conditions of the parts and their wear differ in a great variety, which, of course, will lead to this or that distortion of the normal distribution curve.

Thus, the comparison of the obtained real curves of the size distribution of worn parts with the corresponding curves of the normal distribution allows us to judge the nature of deviations in the working conditions and wear of these parts from those adopted above.

Let's consider the possibilities of using the statistical method for such purposes as:

a) establishing the inter-repair service life of parts;
b) evaluation of the quality of the use of parts;
c) wear characteristics of parts.

Let's consider first of all how the service life of parts is established. Here it is necessary to state, first of all, the general position that statistical data reflect the current state of the issue under study, and using them to establish all kinds of standards requires mandatory consideration of the fact that the projected standards should be medium-progressive.

At first glance, it may seem that statistical methods are unsuitable for designing such norms. In reality, this is not the case. If a sufficiently extensive sample of the studied objects is made so that the objects included in the sample correctly reflect the corresponding properties of the entire population, then the required design of norms can be made based on the following considerations.

Let’s say for our case we have the following data:

τ_n – the projected inter-repair service life of this part;

$\tau_{\min}$, τ_c, $\tau_{\max}$ – the smallest, average and longest inter-repair service life of the part obtained from statistical data.

If we accept τ_n - τ_c, the norm will reflect the indicators already achieved by practice and will not be average progressive.

In order for the norm to be average progressive, the following inequality must be observed:

$$\tau_c \le \tau_n \le \tau_{\max}. \tag{3}$$

Let's assume that the statistical data on the size of worn parts obey the law of normal distribution. Fig. 2 illustrates the scheme for determining the inter-repair service life of the part for this case according to statistical data.

Based on the known provisions of probability theory for the accepted conditions, we have an analytical expression of Gauss' law:

$$y = \varphi(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-a)^2}{2\sigma^2}}, \tag{4}$$

where σ – the average square deviation of the service life;

a –the abscissa of the vertex of the Gauss curve, in our case τ_c.

The value *a* included in expression (4) has the following value:

$$a = \tau_c = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{+\infty} x e^{-\frac{(x-a)^2}{2\sigma^2}} dx. \tag{5}$$

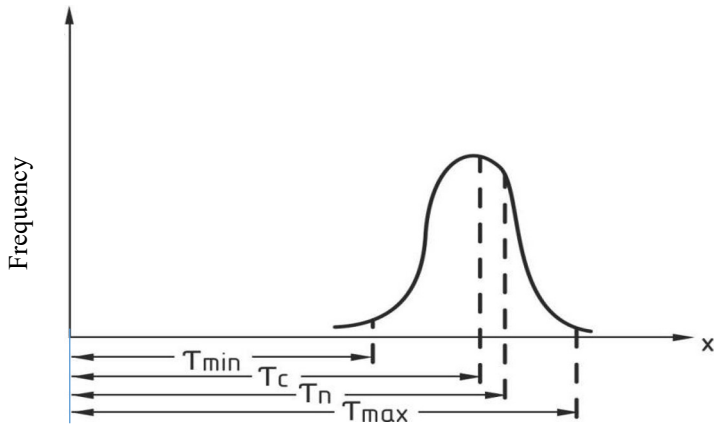

Fig. 2. The scheme of determining the repair service life by the Gauss curve

The resulting value is an analytical expression of the arithmetic mean in the law of normal distribution, i.e.

$$a = \tau = \bar{x}, \tag{6}$$

where*x*– the commonly accepted notation of the arithmetic mean.

So how next $\tau_{\max} - \tau_{\min}$ there is a scattering field and it is equal to the following values:

$$\tau_{\max} - \tau_{\min} = 6\sigma, \tag{7}$$

then from the expression (7) we get:

$$\tau_{\max} = \tau_0 - 3\sigma = \bar{x} - 3\sigma, \tag{8}$$

while:

$$\tau_{min} = \tau_0 + 3\sigma = \bar{x} + 3\sigma. \tag{9}$$

To determine τ_n it is necessary to meet the condition (3). We will accept the projected inter-repair service life τ_n equal to the abscissa corresponding to the half of the area enclosed between the right symmetrical half of the curve and the axis of the abscissa.

The area enclosed between the entire curve and the abscissa axis (if the grouping center coincides with the origin) is equal to:

$$\Phi(x) = \int_{-\infty}^{+\infty} y dx = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-\frac{(x-a)^2}{2\sigma^2}} dx = 1. \tag{10}$$

Denoting $\frac{x}{\sigma} = z$, let 's imagine the area value in the following form:

$$\Phi(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-\frac{x^2}{2\sigma^2}} dx = \Phi(z) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-\frac{x^2}{2}} dz. \quad (11)$$

According to the corresponding table of the textbook [10], we find that a quarter of the area [0,25Φ(z)]corresponds to the abscissa$z = \frac{2}{3}$ = 0,67, that is, when $z = \frac{2}{3}\sigma$.

Thus, the norm of the inter-repair service life of the part that interests us will be equal to:

$$\tau_0 = \tau_c + \frac{2}{3}\sigma = \bar{x} + \frac{2}{3}\sigma. = 6\sigma, \quad (12)$$

The correctness of such a decision is justified by the following.

Half of the positive area cut off by the abscissa is equal to $\frac{2}{3}\sigma$, corresponds to half of the cases of over-fulfillment of the average period of the statistical sample made and therefore is the most acceptable for the design of the average progressive norm.

If we take the abscissa $\leq \frac{2}{3}\sigma$, the norm will be underestimated, since in practice half of all cases of over-fulfillment of the average term will correspond to an abscissa greater than the accepted one.

If, on the contrary, the abscissa is $\geq \frac{2}{3}\sigma$, the norm as a whole will turn out to be impossible, since in practice half of all cases of over-fulfillment of the average term will correspond to an abscissa smaller than the accepted one.

Consider the following example. It is necessary to establish the norm of the maintenance period of the support roller (node 100.31.132A) of the tractor VT-150 according to the first two graphs Table 2 [11].

The first column shows the observed service life of xi (the middle of the interval of the service life), in the second – the corresponding frequencies n_i .

At the same time, the average value can be determined by the following expression:

$$\bar{x} = \tau_c = \frac{1}{n}\sum_{i=1}^{i=k} n_i x_i, \quad (13)$$

where $\bar{x}$ – the average value of the inter-repair service life of the support roller;

n – the number of support rollers taken under observation.

n_i– frequency.

Based on the columns of the third and second table 2 , we have the following value of the average value of the maintenance period of the support roller:

$$\bar{x} = \tau_c = \frac{253450}{74} \approx 3420 \text{ moto-ch.}$$

Table 2. Calculation of the norm of the maintenance period of the support roller

x_i	n_i	$x_i n_i$	$x_i - \overline{x}$	$(x_i - \overline{x})^2 10^6$	$n_i(x_i - \overline{x})^2 10^6$
1	2	3	4	5	6
1750	1	1750	1670	2.79	2.79
2250	4	9000	1170	1.37	5.48
2750	8	22000	670	0.45	3.60
3250	29	94300	170	0.03	0.87
3750	21	78700	330	0.11	2.31
4250	9	38200	830	0.69	6.21
4750	2	9500	1330	1.77	3.54
	74	253450		7.21	24.80

The value of the standard deviation (at n > 10) can be calculated using the following formula:

$$\sigma = \sqrt{\frac{\sum_{i-1}^{i=k} n_i (x_i - \overline{x})^2}{n}}, \tag{14}$$

where σ – standard deviation, moto-h;
$\overline{x}$ – the average value of the inter-repair service life of the tractor support roller VT-150, moto-h;
n – the number of support rollers taken under observation, pcs.;
n_i– frequency.
Using the last three columns of Table 2, we obtain the following value of the standard deviation:

$$\sigma = \sqrt{\frac{24,80 \cdot 10^6}{74}} = 580 \text{ moto-h.}$$

As a result, the required average progressive rate of the inter-repair service life of the VT-150 tractor support roller can be determined by the following expression:

$$\tau_n = \tau_c + \frac{2}{3}\sigma = \overline{x} + \frac{2}{3}\sigma, \tag{15}$$

where τ_n – the average progressive rate of the inter-repair service life of the tractor support roller VT-150, moto-h;
$\overline{x}$ – the average value of the inter-repair service life of the tractor support roller VT-150, moto-h;
σ – standard deviation, moto-h.

Hence, the required average progressive rate of the inter-repair service life of the support roller of a modern tractor VT-150 is the following value:

$$\tau_n = = 3420 + \frac{2}{3} 580 \approx 3800 \text{ moto-h.}$$

To control to what extent the data in Table 2 satisfy the normal distribution, we use the commonly practiced criterion of A. N. Kolmogorov's agreement, the so-called criterion λ

(lambda), which gives fairly accurate results with a sufficiently large number of observations (at least several dozen).

For this purpose, we will use the auxiliary table 3, which allows us to make the check we are interested in.

In the first two graphs of this table, the data of the first two graphs of Table 2 are rewrittenn_xи n_x^1 correspond to the observed and theoretical frequencies and valuesN_xи N_x^1 – observed and theoretical (according to Gauss' law) accumulated frequencies.

Determination of the value n_x^1 produced according to the following mathematical dependence:

$$n_x^1 = \frac{n\Delta x}{\sigma}\Phi^1(z), \tag{16}$$

where Δx – the value of the same groups or classes into which the observed values are divided$x$.

In our example , we get the following value n_x^1 :

$$n_x^1 = \frac{74 \cdot 500}{580}\Phi^1(z) = 63,8\Phi^1(z).$$

Table 3. Auxiliary table for calculating the criterion of consent of A. N. Kolmogorov

x	n_x	N_x	$z = \frac{x-\bar{x}}{\sigma}$	$\Phi(z) = \frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}}$	n_x^1	N_x^1	$N_x - N_x^1$
1750	1	1	2.88	0.0063	0.4	0.4	0.6
2250	4	5	2.02	0.0519	3.3	3.7	1.3
2750	8	13	1.18	0.2036	13.0	16.7	3.7
3250	29	42	0.29	0.3825	24.6	41.3	0.7
3750	21	63	0.57	0.3391	21.6	62.9	0.1
4250	9	72	1.43	0.1435	9.2	72.1	0.1
4750	2	74	2.29	0.0290	1.8	73.9	0.1
	74				73.9		

The value of the function below:

$$\Phi^1(z) = \frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}}$$

it is taken from a special table, for example, from the source [12] for $z = \frac{x-\bar{x}}{\sigma}$.

For the first value$x$ = 1750 the absolute value is obtained as follows:

$$z = \frac{1750-3420}{580} = 2,88.$$

For the value$z$ = 2,88according to the table of the literary source [13] we find$\Phi^l(z)$ = 0,0063 and we get the following value n_x^1 :

$$n_x^1 = 63{,}8\Phi^1(z) = 63{,}8 \cdot 0{,}0063 = 0{,}4, \text{ etc}$$

For the accumulated frequencies we have:N_x: 5 = 1 + 4; 13 = 1 + 4 + 8 etc.

Respectively for frequencies N_x^1 we will also have: 3,7 = 0,4 + 3,3; 16,7 = 0,4 + 3,3 + 13,0 etc.

A. N. Kolmogorov's criterion of consent λ calculated according to the following dependency:

$$\lambda = D_{\max}\sqrt{n}, \tag{17}$$

where λ – A. N. Kolmogorov's criterion of consent;

D_{max} – based on the data of the last column, it is calculated by the expression (18).

$$D_{\max} = \frac{(N_x - N_x^1)_{\max}}{n}. \tag{18}$$

In our case, we will have the valueD_{max}:

$$D_{\max} = \frac{3{,}7}{74}.$$

In turn, the value of the criterion:

$$\lambda = \frac{3{,}7}{74}\sqrt{74} = 0{,}43.$$

According to the table, for example, a literary source [14], the values of probabilities are found$p(\lambda)$, corresponding to different values λ, and on this basis, an estimate is made of the discrepancy between the assumed theoretical distribution (in this case, normal) and the observed one.

For our example, the table of the literary source [15] shows that the value of λ = 0,43corresponds to the probability$p(\lambda)$ = 0,991, that is, the probability is very close to one.

Sinceunlikelyeventsareconsideredtobepracticallyimpossible, theresultingproximity$p(\lambda)$to unity indicates the randomness of discrepancies between the empirical distribution and the theoretical one and, consequently, a fairly good approximation of the observed distribution to the assumed theoretical, in our case normal.

Otherwise, with proximity$p(\lambda)$to zero (practically no more than 0.05), it would be necessary to recognize the specified discrepancy as not accidental, but significant and, consequently, the observed distribution does not correspond to the assumed theoretical one.

4 Conclusions

Thus, it can be concluded that in probability theory and mathematical statistics, quantitative characteristics of the studied object or phenomenon in the form of a number and a measure are of interest, then the concepts of experiment, test, observation, experiment, sample can be conditionally combined.

As a result of certain experiments or observations, a large amount of experimental data or statistical material is obtained.

In this paper, it was examined how the service life of parts is established. It was generally stated here that statistical data reflect the current state of the issue under study,

and using them to establish all kinds of standards requires mandatory consideration of the fact that the projected standards should be average progressive.

The assumption was made that statistical data on the size of worn parts obey the law of normal distribution. The presented figure illustrates the scheme of determining the inter-repair service life of the part for this case according to statistical data.

An example is considered that allows to establish the norm of the maintenance period of the support roller of a modern tractor VT-150.

In this example, the value of the criterion of consent of A. N. Kolmogorov λ = 0,43corresponds to the probability$p(\lambda)$ = 0,991, i.e. the probability is very close to one.

Due to the fact that unlikely events are considered to be practically impossible, the resulting proximity $p(\lambda)$to unity indicates the randomness of discrepancies between the empirical distribution and the theoretical one and, consequently, a fairly good approximation of the observed distribution to the assumed theoretical, in our case normal.

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