Simulation of forced vibration of a wedge pair under the impact of variable force

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Abstract. The forced oscillations of a wedge pair under the effect of a variable force are investigated in the article. The general solution to the differential equation of forced oscillations of a wedge pair is chosen in the form depending on some unknown function. An equation for this function is obtained for the cosine form of the virtual displacement of the wedge under longitudinal displacement. This equation is obtained on the basis of the principle of virtual work of inertial forces, elastic forces, and external dynamic forces with virtual displacement. As a result, the solution to the differential equation of forced vibrations of a wedge pair under a variable longitudinal external force was obtained, and calculations were performed on the basis of the obtained solution. Research methods are based on the principle of virtual displacement, the method of mathematical modeling, and the analytical method of their solutions.

1 Introduction

There are various technologies for tillage and several options of the working bodies for its realization [1]. The main working element of all working bodies for soil cultivation has a wedge-shaped structure. The static and dynamic states of wedge-shaped structures can be studied on the basis of the theory of elasticity. The study of the process of contact interactions of mechanism elements with a wedge-shaped structure under active pressure was also conducted within the framework of problems of statics based on the theory of elasticity [1]. Using the developed calculation model, numerical solutions were obtained for various values of geometric parameters and elasticity parameters. Important integral dependences were obtained that relate the stress state function to the operating characteristics of the drive mechanism. The problem of the flat wedge penetration into a plastic medium obeying the double-shearing model was solved in [2]. It was shown that in the case of the penetration of a wedge, there is a significant qualitative difference between the solutions obtained by different models. In particular, the velocity field is continuous at the rigid plastic boundary.

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2 Method

Research methods are based on the principle of virtual displacements of theoretical mechanics; the method of mathematical modeling and the analytical method of their solutions.

3 Material

Let us consider a homogeneous, elastic wedge pair from an isotropic medium, which has the form of an elongated prismatic rod (Fig. 1). Studies of free longitudinal oscillations of such a wedge pair under the influence of a constant force were performed in [6]. Here we consider the forced oscillations of this wedge pair under the action of a variable force that changes its value over time as a time-dependent function.

Fig. 1. Edge pair shape.

Let a constant dynamic force \( Q(t) \) depending on time be subjected to an axisymmetric wedge. The differential equation for forced oscillations of such a wedge has the following form:
The general solution of differential equation (1) of forced oscillations of the wedge is sought in the following form:

$$u(t) = \sum_{k=1,3,5,...}^{\infty} \phi_k(t) \frac{\pi k}{l} z$$

where

$$\phi_k(t) = \frac{\pi k}{l}$$

is the time function depending on the initial conditions and external impact. To determine this function, we use the principle of virtual displacement. Let us assume that the virtual displacement of the wedge under longitudinal displacement has the following form:

$$u = \sum_{k=1,3,5,...}^{\infty} \phi_k(t) \frac{\pi k}{l} z$$

Then the expressions for the virtual work of the forces of inertia, the forces of elasticity, and the external dynamic force under virtual displacement are, respectively:

$$\delta W_u = \int \rho F dz \delta u = -\rho F \int u \phi_k(t) \frac{\pi k}{l} z dz = -\rho F \int C_k \phi_k(t)$$

$$\delta W_z = \int (EFu \frac{d}{dz}) \delta u = -EF \phi_k(t) \left( \frac{\pi k}{l} \right) \int C_k \phi_k(t) \frac{\pi k}{l} z dz = -\frac{k}{l} \pi EF \int C_k \phi_k(t)$$

$$\delta W_Q = QC_k \frac{\pi k}{l} z \bigg|_{z=0} = QC_k$$

$$\delta W_u + \delta W_z + \delta W_Q = 0$$

$$\phi_k(t) = \frac{\pi k}{l}$$

$$\frac{\pi k}{l} = \frac{Q(t-t)}{\rho Fl} \left[ \int Q(t-t) \left( t - \tau \right) d\tau \right]$$

$$\phi_k(t) = \frac{\pi ak}{\rho Fl} \int Q(t-t) \left[ k \pi a \frac{t}{l} \right] d\tau$$
Let us consider the case when the external dynamic longitudinal force \( Q(t) \) is harmonically variable. In this case, from expression (8) we obtain:

\[
\varphi_i = \frac{Q_0}{\pi ak \rho F} \int_0^l Q_o \omega t \cdot \left[ k \pi a \left( t - \tau \right) \right] d\tau = \frac{Q_0}{\pi ak \rho F \left( \omega - p_k \right)} \int_0^l \left[ \omega p_k t - p_k \omega t \right] \]

and the law of forced longitudinal oscillations of an axisymmetric wedge based on (2) takes the following form:

\[
u = \frac{Q_0}{\pi a \rho F} \sum_{k=1,3,5,...} \frac{(\pi k) z}{l} \left( \omega p_k t - p_k \omega t \right)
\]

Thus, the general solution to the differential equation of forced oscillations of an axisymmetric wedge with a harmonically variable external longitudinal force \( Q(t) \) is obtained.

### 4 Results

Calculations were conducted by solving the differential equation of forced oscillations of an axisymmetric wedge under a variable longitudinal force. The calculation results are shown in Figs. 1 and 2.

As seen from these graphs, when the external force changes according to the harmonic law, the forced oscillations of the pole differ somewhat from the case when this force has a constant value. That is, in this case, due to the frequency of the external force, the amplitude of the forced oscillatory motion changes significantly. From solution (10), it can be seen that the resonance phenomenon occurs when the circular frequency \( \omega \) of the external force approaches the frequency \( p_k \) of free oscillations. To eliminate this situation, it is appropriate to choose a low frequency of the external network. In most cases, since the speed of sound propagation along the length of the wedge is quite high, the probability of resonance is very low.

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**Fig. 1.** The law of forced oscillations of an axisymmetric wedge under external harmonic force

\[ a = \sqrt{\frac{E}{\rho}} \]
It should be noted that from the solutions of two specific cases (8) and (10), i.e. from the law of forced longitudinal oscillations of the wedge, it is possible to find all dynamic quantities, in particular elastic strains, forces, and stresses, with high accuracy for any point of time and arbitrary section of the wedge. Besides, even if the patterns and distribution of external force $Q(t)$ acting on the wedge are different, using the Duhamel integral formula it is possible to find the dynamic displacements of the wedge particles, that is, the laws of forced oscillations can be determined with high accuracy.

5 Conclusion

To avoid the phenomenon of resonance in the process of crushing the soil structure by the impacts of the wedge-shaped device used in tillage, it is appropriate to choose the external force of a low frequency.

References