Reducing the thickness of the insulation layer of building walls based on the study of their temperature and moisture regime

Abstract. The thickness of the building insulation is calculated with the table values of the moisture of building materials, which is included in the regulatory documents with a margin. The study proposes the method for determining the thickness of the insulation layer using the values of operational moisture. To determine the operational moisture of materials, the equation based on the moisture potential is applied, the solution of which is sought using a discrete-continuum approach. The proposed equation makes it possible to determine the distribution of the moisture potential function in the wall enclosing structure. Thereafter, using the moisture potential scale, the distribution of mass moisture over the thickness of the enclosure for the period of maximum moisture accumulation is determined. The equation for determining the thickness of the insulation layer, in which the calculated value of the operational moisture is explicitly substituted, is derived. An algorithm for calculating the thickness of the insulation layer, taking into account the unsteady-state heat and moisture regime of the building envelope, is given. The application of the proposed method on a wall enclosing structure with insulation from expanded polystyrene boards is illustrated. It was found that the thickness of the insulation can be reduced by 26 mm from 120 mm to 94 mm while maintaining the reduced heat transfer resistance of the building wall. It is noted that the proposed method can be applied in the design of new wall enclosing structures.

1 Introduction

During the operation of the building, building materials accumulate moisture in their thickness. Water in the material conducts heat well, therefore, it negatively affects the thermal protection of buildings [1]. Every building material has a dry thermal conductivity. This thermal conductivity is achieved by drying the building material in a special oven. According to modern regulations in the construction industry, building materials can be calculated under operating conditions A or B. The operating conditions of the building material depend on the construction area, humidity zone and the moisture regime of building premises. Humidity

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2 The Problem

3 Materials and Methods

3.1 Application of the discrete-continuous approach to calculate the operational mass moisture and operational thermal conductivity of the building materials of the building wall

\[
\frac{\partial \mu}{\partial \tau} = \kappa \cdot \frac{\partial^2 \mu}{\partial x^2}
\]
Fig. 1. Application of the discrete-continuum approach for discretization of the space-time domain of the building wall in a one-dimensional formulation.

The discrete-continuum approach makes it possible to derive the equation for the dependence of the moisture potential on time:

\[ f(t, t, t) - \tau \cdot \left( \left( \begin{array}{c} t_1 \\ t_2 \end{array} \right)^{-1} - \left( \begin{array}{c} t_1 \\ t_2 \end{array} \right)^{-1} \right) \cdot \left( \begin{array}{c} t_1 \\ t_2 \end{array} \right)^{-1} \cdot \left( \begin{array}{c} t_1 \\ t_2 \end{array} \right)^{-1} - \left( \begin{array}{c} t_1 \\ t_2 \end{array} \right)^{-1} \right) \cdot \left( \begin{array}{c} t_1 \\ t_2 \end{array} \right)^{-1} \cdot \begin{array}{c} \mathbf{B} + \mathbf{e} \end{array} \cdot \begin{array}{c} \mathbf{F} \end{array} \right)

3.2 Derivation of the equation for calculating the thickness of the insulation layer taking into account the unsteady-state moisture regime

The dependence of the thermal conductivity coefficient on mass moisture is determined by the formula:

\[ \lambda = \lambda_0 + \mathbf{\omega} \cdot \Delta \mathbf{L} \]

where \( \lambda \) and \( \lambda_0 \) – thermal conductivity of a building material in wet and dry conditions, \( \omega \) – mass moisture content of the building material, \( \mathbf{\omega} \); \( \Delta \lambda \) – increase in thermal conductivity with a change in the moisture of the building material, \( W/(m \cdot \mathbf{\degree C}) \).

The heat transfer resistance of the building wall is the sum of the thermal conductivity resistance of the material layers and the heat transfer resistance of air and material surfaces:

\[ R_\text{cond} = \sum R_i \]

where \( R_\text{cond} \) – conditional resistance to heat transfer, \( (m^2 \cdot \mathbf{\degree C})/W \).

Equation (4) is a conditional resistance to heat transfer which does not take into consideration the point (dowels, insulation fasteners) and linear (window and door slopes, etc.).
thermal inhomogeneities which is located in the wall structure. In order to take them into account, it is necessary to calculate the value of the reduced resistance to heat transfer: 

\[ R_{red} = \sum_{j} \sum_{k} R_{cond} \ln \frac{\varphi_{\alpha}}{\lambda_{\alpha}} \]

where \( R_{red} \) – reduced resistance to heat transfer, (m²·K)/W; \( \varphi_{\alpha} \) \( \lambda_{\alpha} \) – an expression describing the decrease in resistance to heat transfer due to the influence of point and linear thermal inhomogeneities, W/(m²·K).

According to regulatory requirements, the value of reduced heat transfer resistance must be more or equal than the value of required resistance to heat transfer:

\[ R_{red} \geq R_{req} \]

where \( R_{req} \) – required resistance to heat transfer, (m²·K)/W.

Let us substitute the conditional resistance to heat transfer from equation (4) into equation (5):

\[ R_{red} = \sum_{j} \sum_{k} \left( \delta_{\alpha} + \sum_{\alpha} \psi_{\alpha} \right) \sum_{\alpha} \chi_{\alpha} \]

We substitute the reduced resistance to heat transfer from equation (7) into inequality (6).

\[ \sum_{j} \sum_{k} \left( \delta_{\alpha} + \sum_{\alpha} \psi_{\alpha} \right) \sum_{\alpha} \chi_{\alpha} \geq \sum_{j} \sum_{k} \left( \delta_{\alpha} + \sum_{\alpha} \psi_{\alpha} \right) \sum_{\alpha} \chi_{\alpha} \]

Let us express the thickness of the insulation from inequality (8):

\[ \delta_{\alpha} \geq \sum_{\alpha} \psi_{\alpha} + \sum_{\alpha} \chi_{\alpha} \]

In inequality 9, the value \( \delta_{\alpha} \) \( \psi_{\alpha} \) \( \chi_{\alpha} \) does not include the parameters of the insulation layer, since the layer thickness is moved to the left side of the equation.
To consider the unsteady-state moisture regime, it is necessary to enter the operational mass moisture into equation (9). To do this, let us substitute the operational thermal conductivity (3) into inequality (9):

\[
\delta_{\text{in}} \geq \frac{\lambda_{\text{req}} + \Delta\lambda_{\text{req}}}{\alpha + \sum \psi + \sum \chi} - \left( \sum \psi_0 + \sum \chi_0 \right)
\]

3.3 Algorithm for calculating the thickness of the insulation layer using the unsteady-state heat and moisture regime of the building envelope

4 Results and Discussion

4.1 Determining the maximum moisture content time moment in the building wall and the value of moisture content of building materials

Table 1.

<table>
<thead>
<tr>
<th>A layer number</th>
<th>A layer material</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Layer of internal plaster</td>
</tr>
<tr>
<td>2</td>
<td>Aerated concrete base layer</td>
</tr>
<tr>
<td>3</td>
<td>Polystyrene insulation layer</td>
</tr>
<tr>
<td>4</td>
<td>External plaster layer</td>
</tr>
</tbody>
</table>

The calculation of this wall according to the regulatory documents showed that with the thickness of the base layer of 0.3 m, the thickness of the insulation layer should be 0.12 m.
Mathematical modeling of the unsteady-state moisture regime was performed in order to determine the maximum moisture period.

Change in the insulation operational moisture of the building wall during the year is presented (Figure 3).

The moisture distribution in the investigated wall of the building at the moment of maximum moisture accumulation is presented. (Figure 4).

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**Fig. 2.** Enclosing structure of a residential building (1 – external plaster; 2 – expanded polystyrene 94 mm; 3 – aerated concrete 300 mm; 4 – internal plaster).

**Fig. 3.** Change in insulation operational moisture of the building wall during the year.

**Fig. 4.** The moisture distribution of the investigated wall of the building at the moment of maximum moisture accumulation.
The achieved values of operational humidity and standard values of moisture content of building wall materials are presented (Table 2).

Table 2. Achieved values of operational moisture and standard values of moisture content of building wall materials

<table>
<thead>
<tr>
<th>Wall material</th>
<th>Calculated operational moisture</th>
<th>Moisture according to the regulatory documents</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base</td>
<td>4.3 % by mass</td>
<td>12.0 % by mass</td>
</tr>
<tr>
<td>Insulation</td>
<td>4.3 % by mass</td>
<td>10.0 % by mass</td>
</tr>
</tbody>
</table>

Determining the thickness of the insulation layer taking into account the unsteady-state moisture regime of the building envelope

The thickness of the insulation layer was determined by the formula (10) using the values of the calculated moisture in accordance with the table 2. As a result, the thickness of the building wall insulation was obtained as 94 mm, which is 26 mm less than the value of the insulation thickness according to the normative moisture of building materials.

The enclosing structure of a residential building after clarifying its heat and moisture behavior is presented (Figure 5).

Fig. 5. The enclosing structure of a residential building after clarifying its heat and moisture state (1 – external plaster; 2 – insulation saving in 26 mm; 3 – expanded polystyrene 94 mm; 4 – aerated concrete 300 mm; 5 – internal plaster).

Insulation savings are obtained due to the modeling of the unsteady-state heat and moisture regime of the building wall as well as consideration of variable climatic influences on the fence, the physical properties of building materials and the inertia of the moistening process are achieved.

5 Conclusion

The new effective method taking into account point and linear thermal inhomogeneities and the unsteady-state moisture regime of building materials caused by climatic influences and the inertia of the moistening process for determining the thickness of the insulation layer for the building wall was developed. The moisture content of a building material is determined using the discrete-continuum approach. Determination of the value of...
The operational thermal conductivity is calculated according to the well-known formula for the dependence of thermal conductivity on mass moisture. The results of the calculations showed the possibility of achieving the same values of the reduced resistance to heat transfer with the savings of 26 mm of the thickness of the insulation layer due to unsteady-state heat and moisture regime of the fence. This method can be applied in practical engineering work for the design of new enclosing structures.

Acknowledgments

This publication has been supported by the RUDN University Scientific Projects Grant System, project № 202248-2-000 "Study of the temperature and moisture regime of building walls in order to reduce the thickness of the insulation".

References


3. I. Vorobyeva, The prognosis of the diabetic retinopathy using computer science and biotechnology, E3S Web of Conferences, 203, 01028 (2020) https://doi.org/10.1051/e3sconf/202020301028

4. I. V. Vorobyeva, Mathematical modeling in diabetic retinopathy, E3S Web of Conferences, 224, 03020 (2020) https://doi.org/10.1051/e3sconf/202022403020

5. I. V. Vorobyeva, Prediction of the course of primary open-angle glaucoma in combination with diabetic retinopathy using a mathematical model, E3S Web of Conferences, 224, 03021 (2020) https://doi.org/10.1051/e3sconf/202022403021

6. I. V. Vorobyeva, Assessment of the development of primary open-angle glaucoma and diabetic retinopathy using digital medicine, E3S Web of Conferences, 224, 03022 (2020) https://doi.org/10.1051/e3sconf/202022403022


