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Natural waves in a spatial viscoelastic cylinder with a radial crack

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Abstract. The investigation of the dispersion of waves in elastic bodies with different cutouts is of great interest in various scientific and technological fields. The objective of this research is to examine the propagation of free damped waves in elastic dissipative bodies with longitudinal notches. The dynamic behavior of cylindrical bodies with different cutouts is described by the equations of viscoelasticity. The solution of a system of differential equations is expressed by cylindrical Bessel and Hankel functions. The frequency equation is solved using the Muller, Gauss, and orthogonal run methods. It is found that with a decrease in the oscillation frequency in a cylinder of a given radius, the phase and group velocities of the longitudinal wave tend to the common limit - the phase velocity of the rod waves. It is found that as the oscillation frequency reduces in a cylinder of a given radius, the phase and group velocities of the longitudinal wave tend to converge towards the common limit. Additionally, it was found that for a specific value of the Poisson's ratio (0.2833), the frequencies of the shear and radial longitudinal resonances coincide in the cylinder.

1 Introduction

The challenges of wave propagation in continuous multilayer systems attract the attention of numerous researchers in our country and abroad. This is because in many fields of science and technology, there is an increasing need to calculate stress and strain fields that occur in layered bodies with different rheological properties when exposed to various kinds of dynamic loads. Dynamic problems of dissipative (viscoelastic) dynamic systems are solved by methods of mathematical physics. The complexity of solving these problems is associated with various reasons, for example, different rheological properties of media (anisotropy, viscosity, creep, plasticity, heterogeneity, etc.), which requires the use of a variety of models [1,2]. Despite the large number of mathematical models of a mechanical system,

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mathematical methods for solving problems have been developed mainly for such systems as acoustic, elastic movements [3-5].

The main difficulty here is to establish dependencies between stresses and deformations of a viscoelastic body. There are several approaches to establish these dependencies [5]. One of them is based on simplified mechanical models (Feucht and Maxwell models), in which elastic properties are described based on Hooke's law, and viscous properties are described based on Newton's rheological law on the flow of a viscous fluid. Meanwhile, it is known that these models do not reflect the real behavior of a loaded body made of viscoelastic material and do not give a satisfactory agreement with the experiment.

This work is devoted to research on wave processes in extended plates and cylindrical bodies, examples of which in practice can be road surfaces, well casing, or any sufficiently long straight structure (plate or pipe) of constant or variable cross-section. The issues of wave propagation in rectangular plates and rods were studied in the works [6-8].

In some works, a rod wave is used to control extended objects in the torsional wave mode, in which there is no wave dispersion [9, 10]. Typically, the reflection coefficient is used to test waveguides of extended objects. This parameter contributes to the detection of defects. Consideration of the damping ability of the waveguide material plays a special role [11,12]. This noticeably reduces its own oscillations, significantly reduces amplitudes and voltages [13,14]. It is usually difficult to solve these problems, which makes it difficult to create a mathematical model of an object [15-18].

2 Methods

To validate the reliability of the developed algorithm, we solve a test problem. The issue of propagation of natural waves in a viscoelastic continuous cylinder is investigated. The harmonic waves of the cylinder are described using the viscoelasticity equations in a cylindrical coordinate system.

The outer surface of the cylinder is free from forces (or stresses). To describe the rheological (or viscoelastic) properties of the material, two types of cores are used: the first is the Rzhanitsyn – Koltunov core, and the second is the fractional-exponential Rabonov core.

The Lame differential equation, for the case under consideration, takes the following form [12].

$$(k_F^2 - 1) graddiv\vec{u} + \nabla^2\vec{u} - \Omega^2\vec{u} = 0, \tag{1}$$

where

$$k_F^{-2} = \frac{2(1-\nu^F)}{1-2\nu^F}; \Omega^2 = \omega^2 \frac{1+\nu^F)}{(1+\nu^F)E^F}.$$

Then the system of Navier equations has the from:

$$\begin{cases}
w'\Gamma_{\lambda 1} = \dfrac{\sigma}{k} - \dfrac{\lambda_{01}}{k}\left(i\gamma_z u + \dfrac{1}{r}\left(w + \dfrac{\partial v}{\partial \varphi}\right)\right) \\
v'\Gamma_{\mu 1} = \dfrac{\tau_\varphi}{\mu_{01}} + \dfrac{\Gamma_{\mu 1}}{r}\left(v - \dfrac{\partial w}{\partial \varphi}\right) \\
u'\Gamma_{\mu 1} = \dfrac{\tau_z}{\mu_{01}} + \Gamma_{\mu 1} i\gamma_z w \\
\sigma' = -\omega^2\rho w + \dfrac{1}{r}\left(A - \dfrac{\partial \tau_\varphi}{\partial \varphi}\right) - i\gamma_z\tau_z \\
\tau'_\varphi = -\omega^2\rho v - \dfrac{1}{r}\left(\dfrac{\partial (A+\sigma)}{\partial \varphi} + 2\tau_\varphi\right) - i\gamma_z B \\
\tau'_z = -\omega^2\rho u - \dfrac{1}{r}\left(\dfrac{\partial B}{\partial \varphi} + \tau_z\right) + i\gamma_z\left(\sigma + 2\mu_{01}\Gamma_{\mu 1}(ku - w')\right)
\end{cases}$$

where

$$A = 2\mu_{01}\Gamma_{\mu 1}\left(\frac{1}{2}\left(\frac{\partial v}{\partial \phi} + w\right) - w'\right), B = \mu_{01}\Gamma_{\mu 1}\left(\frac{1}{r}\frac{\partial u}{\partial \phi} - i\gamma_{z1}v\right).$$

Thus, after transformation (1), we obtain similarly transformed boundary conditions on the axis of the cylindrical body and the free surface in the form

$$r = 0, R: \quad \sigma = \tau_\phi = \tau_z = 0 \ . \tag{2}$$

Conditions are placed on the future surface of the cylinder:

$$r = a, b: \sigma_{rr} = \sigma_{r\varphi} = \sigma_{rz} = 0. \tag{3}$$

Then the dispersion equation for an extended cylindrical body has the following form

$$\left(\Omega^2 - 2\tilde{\chi}^2\right)^2 J_0(a)J_1(b) + 4\tilde{\chi}^2 ab J_0(b)J_1(a) - 2\Omega^2 a J_1(a)J_1(b) = 0. \tag{4}$$

Here a and b are the radii of the shell, $\delta > 0$ - attenuation coefficient, $\tilde{\chi} = \chi - i\delta, \chi$ - wave number,

$$\Omega^2 = \omega^2 \frac{1 + \nu^F}{(1 + \nu)E^F}.$$

$$E^F = 1 - \frac{k}{\beta + (i\omega)^{1+\alpha}}, \nu^F = \nu + \frac{1 - 2\nu}{2}\frac{k}{\beta + (i\omega)^{1+\alpha}},$$

$-\pi < arg(i\omega) < \pi.\ k, \beta$ - material parameters.

The dispersion equation (transcendental equation) (4) is solved numerically by Muller methods. During the calculation, several modes (complex roots) are determined. The results are illustrated in Figure 1 and Figure 2 when $k = 0.45, \beta = 1.0, \nu = 0.25$.

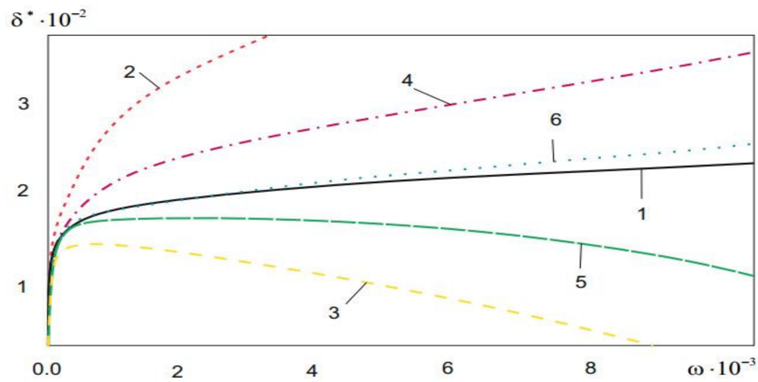

Fig. 1. The change in the damping coefficient corresponding to the first mode from the frequency, when α=-2/9.

The dependence of the change in the imaginary part of the phase velocity on the frequency is obtained for two nuclei (Rzhanitsyn – Koltunov and Robotnov) the period is almost the same (Figure 1 – 1 and 6; 4 and 2; 3 and 5 - correspond to aperiodic fluctuations), results without taking into account viscosity (Figure 1). On Figure 2 illustrates the change in the imaginary part of the phase velocity for various values (1-5, 2-7, 3-9, 4-11) of the expansion terms of the special Bessel and Neumann functions. They differ from each other up to 6% at high frequencies. The solid lines show when$\beta = 1.0$, and the dotted lines-when $\beta = 0.80$. Numerical analysis has established that the behavior of the imaginary part of the phase velocity strongly depends on the singularity of the parameter α.

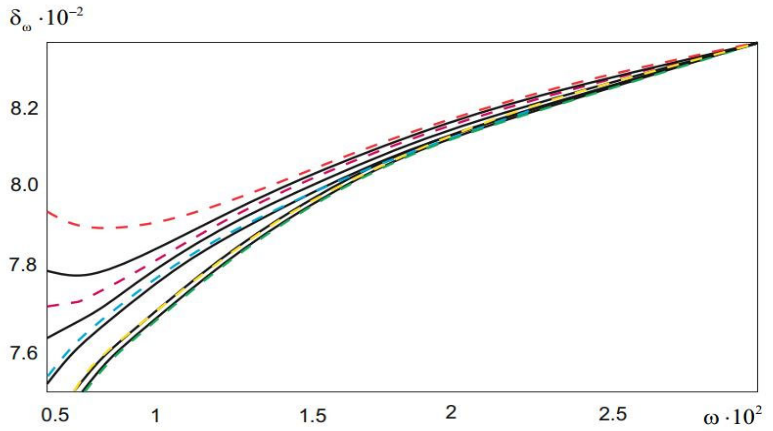

Fig. 2. The change in the damping coefficient corresponding to the first mode of the frequency, when $\alpha = -1/6$ $(n = 2,3,..)$.

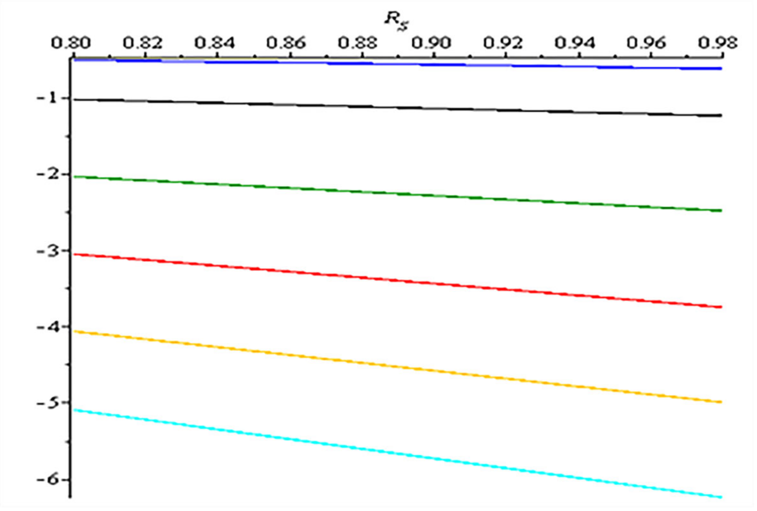

Fig. 3. The change in the attenuation rate as a function of the viscosity amplitude for the following values of the transverse wave velocity:1. $C_S := 50$ 2. $C_S := 100$ 3. $C_S := 200$ 4. $C_S := 300$ 5. $C_S := 400$ 6. $C_S := 500$.

The calculation results were obtained using the developed algorithm (Figure 1 and Figure 3). The results of the phase velocity change are compared with the results of [7]. The calculation results differ by up to 10%. The following relations are proposed for a viscoelastic cylinder:

$$c_{phR} = 1{,}3552 R_s c_s, c_{phI} = -1{,}273 \cdot 10^{-2} R_s c_s, \tag{5}$$

where $R_s \in (1 - k\Gamma^s, 1 - \Gamma^s), 0 < k < 1$. Phase velocities, depending on the viscosity coefficient, decrease to 6-9 %. The calculation results are illustrated in Figure 3. Figure 3 shows that with increasing phase velocities, they increase linearly. An example of a stationary state of harmonic smoking vibrations of a cylindrical rod is given in [8]. The torque M is determined by the following ratio$M(z,t) = (\pi/2)(b^4 - a^4)(\rho\omega^2 h/\Omega)[A\,cos(\Omega z/h) - B\,sin(\Omega z/h)]e^{i\omega t}$, where $a, b -$ is the inner and outer radii, respectively, h is the length of the cylinder, A and B are arbitrary constants.

During the solution process, complex natural frequencies are determined for a finite cylinder. Thus, the developed methodology and algorithm for solving the tasks set enable the identification of dynamic characteristics (natural frequencies, damping coefficients, group velocities, and waveforms).

3 Results and discussion

The Rzhanitsyn-Koltunov core $R_k(t) = A_k e^{-\beta_k t}/t^{1-\alpha_k}$ is chosen for the viscoelastic material, the values of the Poisson's ratio are: $\nu_1 - \nu_2 = 0{,}25$, and with parameters: $A_k = 0{,}048$; $\alpha_k = 0{,}1$; $\beta_k = 0{,}05$, (k=1,2). To obtain numerical results for a two-layer viscoelastic pipe, the following parameter values are assumed: D_1=50mm, D_2=5mm, the outer pipe is made of steel X12: $\mu_T = 80 \cdot 10^9 Pa, \rho_T = 7800\ kg/m^3$, the inner pipe is made of steel 30 l: $\mu_I = 70 \cdot 10^9 Pa, \rho_I = 7500\ kg/m^3$.

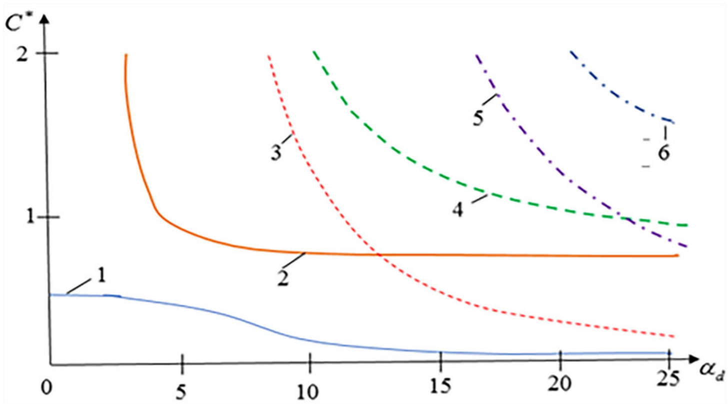

Fig. 4. Change of the phase velocity from the wave number.

In Figure 4 shows the change in phase velocity: 1 and 3 - for a homogeneous pipe (excluding dispersion), 2,4,5 – different modes for two-layer homogeneous pipes.

$$R_1 = r_1/r_3 = 0.2{,}0.4{,}0.6{,}0.8; R_2 = r_2/r_3 = 0.6{,}0.6{,}0.8; 0.9; R_3 = 1.0{,}1.0{,}1.0{,}1.0.$$

From here it can be seen that with an increase in the wave number, the real parts of the phase velocity are decrease and approach the asymptotic. The patterns of change of the imaginary parts of the phase velocities of all modes, for viscoelastic cylindrical bodies, are almost the same.

4 Conclusions

It is found that as the oscillation frequency reduces in a cylinder of a given radius, the phase and group velocities of the longitudinal wave tend to converge towards the common limit, which is the phase velocity of the rod waves. For a specific value of the Poisson's ratio (0.2833), the frequencies of the shear and radial longitudinal resonances coincide in the cylinder. It is established that as the frequency changes, the phase velocity of the normal wave also changes and approaches the phase velocity of the Rayleigh wave as the frequency increases.

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