

Bedload transport analysis using various methods

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Abstract. Mount Merapi, located between the provinces of Central Java and Yogyakarta, is one of the most active volcanoes, regularly spewing lahar material and supplying sediment to the surrounding rivers. These rivers are classified as volcanic rivers and some are utilized as irrigation water sources, flowing through densely populated areas that require the construction of river structures such as retaining walls, bridge pillars, weirs, and groundfills. Bedload transport calculation is an essential part of planning such as river constructions. As bedload transport calculations are continually evolving, scientists have developed several formulaic approaches. To determine the appropriate formula, evaluating existing formulae is necessary. In this study, we compared bedload transport calculations to laboratory measurements using sediment materials sourced from rivers originating from Mount Merapi. The formula used in the study was based on the sediment material's d₅₀ diameter parameter. Our analysis indicates that the equation proposed by Lajeunesse, Malverti, and Charru (2010) yields the closest results to those obtained in the laboratory.

1 Introduction

Mount Merapi is located in between the Central Java and Yogyakarta, Indonesia. As the recorded data since mid-1500s, Mount Merapi recognized as one of the most dangerous volcanoes and the most active in the world [1]. Mount Merapi has erupted 41 times in the last 200

years including 15 major eruptions [2] as shown in Fig. 1. The last In 2010, a huge eruption occurred reached VEI-4 and was known as the largest eruption in the 20th century. The eruption resulted in approximately 386 reported death, and more than 2,200 families lost their houses [3]. In addition, the river located across the Mount Merapi was covered by pyroclastic material.

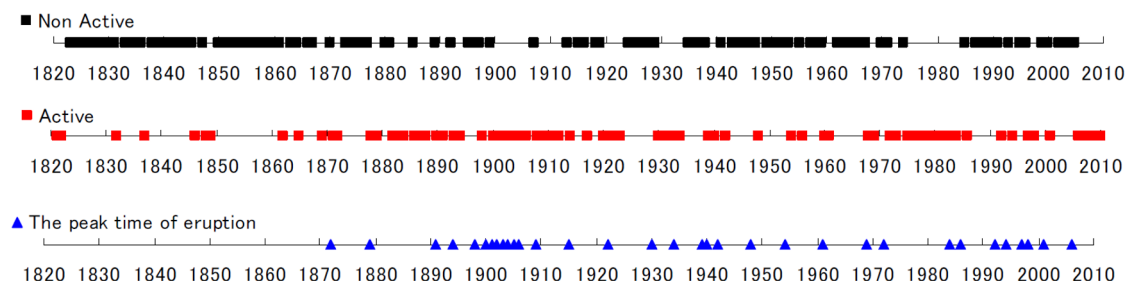


Fig. 1. History of the activity of Mount Merapi [2].

The pyroclastic material is become an issue when settled around the river slope, especially during the rainfall season [4]. The distribution of pyroclastic material deposits of Merapi shown in Fig. 2. The river-slope failure triggered by rainfall is a dangerous thing after the eruption. The pyroclastic material will be carried away by the flow and enter the river into bedload sediment in the river that originates around Merapi.

The Opak River, located in the provinces of Yogyakarta and Central Java, is significantly impacted by the sediment resulting from the eruption of Mount Merapi. As a result, the river experiences the process of bedload sediment transport, where sediment particles are transported by the water flow through rolling, sliding, or jumping [5]. The Opak River serves as a vital

irrigation source for agricultural lands and gardens, spanning across four districts in Yogyakarta Province and two districts in Central Java Province [6].

Sediments are composed of fragments of rock material, exhibiting a range of sizes and shapes, from larger boulders to extremely fine colloids. These sediment particles can take on rounded, oval, or square forms [7]. The transportation of bedload sediment often gives rise to issues, particularly for water structures situated in rivers, when there is an imbalance in sediment load. This happens when there is insufficient sediment supply upstream to match the sediment transport capacity. Consequently, the sediment carried by the flow tends to consist of finer particles rather than coarse ones, owing to selective transport [8]. This will

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result in erosion and subsidence of the riverbed and riverbanks. Therefore, it is necessary to do a research to determine the amount of bedload transport in the river. Analysis of bedload transport can now be done with various formulas. However, the current formula has not been able to produce a good value and can only be applied under certain conditions.

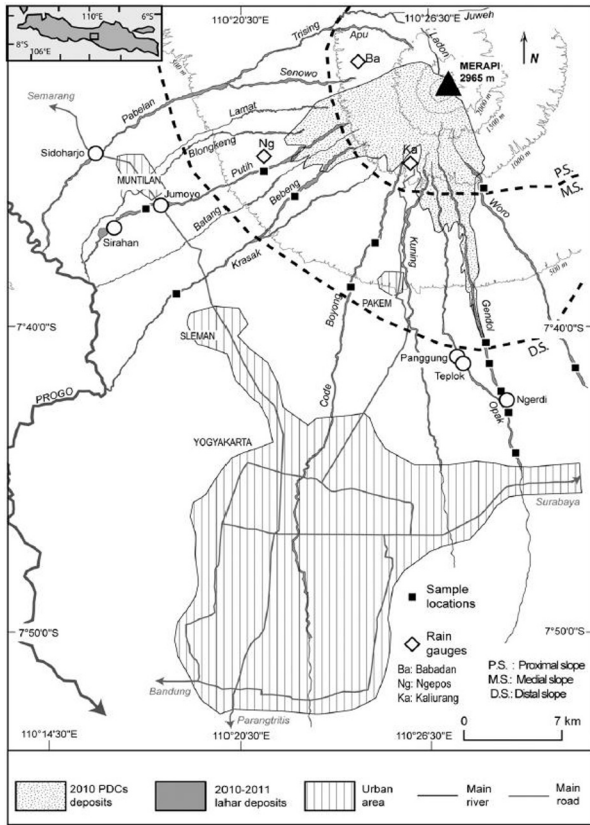


Fig. 2. Location of pyroclastic deposits of Mount Merapi [4].

Conducting research on bedload sediment transport in natural rivers presents significant challenges due to issues related to measurement accuracy, non-uniform sediment mixture composition of riverbed particles, and the fluctuating nature of river flow. To overcome these challenges, the measurement of hydraulic parameters and sediment parameters was carried out using the Multi-Purpose Teaching Flume in the Laboratory, providing a controlled and stable environment for accurate experimentation.

Numerous investigations have been conducted to quantify the rate of bedload sediment transport. Among these studies, Andrian and Pranoto [9] focused on analyzing the bedload sediment transport in the Cibeet River, located in West Java. Their research utilized a combination of laboratory testing and HEC-RAS modeling, employing various sediment transport formulas such as the Meyer-Peter and Müller, Engelund-Hansen, and Yang equations.

Moreover, various methods have been used by numerous researchers to calculate the bedload transport, such as Siregar et al. [10] used to the van Rijn, du Boys, and Rottner equations to calculate bedload transport in the Kuala Langsa Estuary by taking sediment samples for laboratory tests to be analyzed using, and Pabintan

et al. [11] that was using the Schoklitsch and Haywood equations to study about bedload transport in the lower reaches of the Kambu river, Kendari City.

The concept of bedload transport analysis was investigated by Du Boys in 1879 by assuming the sediment layers overlap. These layers experience movement driven by the traction force induced by uniform flow. The velocity of this layer movement exhibits a progressive increase from the base layer to the surface, as depicted in Fig. 3 [12].

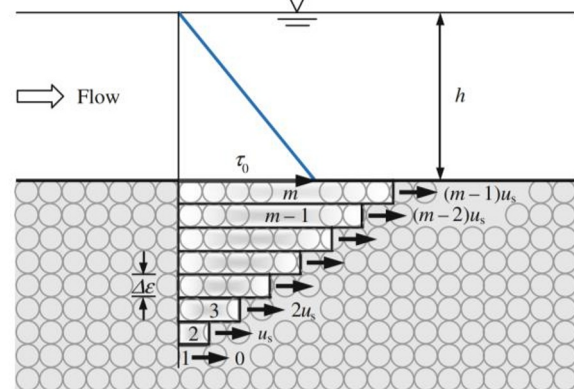


Fig. 3. Du Boys' bedload model sketch definition [12].

Where m is the number of layers, then the layer that moves the fastest occurs in the top layer with a speed of $(m-1)u_s$, where u_s is the speed of the layer below it. Then the shear stress that occurs in the surface layer is defined as in Equation 1.

$$\tau = \rho_w g D S \quad (1)$$

Shields [13] conducted a bedload transport experiment using uniform grains at uniform flow. According to [13], in uniform flow there will only be frictional and compressive forces due to the weight of the water. Then the shear stress that occurs was defined as in Equation 2.

$$\tau = \rho_w g R_h S \quad (2)$$

The basic shear stress equation was then derived into a dimensionless equation known as the Shields parameter, which is stated in Equation 3 [13]. In his experiments, Shields [13] gave a threshold for relatively uniform grain motion as a function of the dimensionless critical shear stress or the critical Shields parameter. Where the critical Shield parameter can be analysed using Equation 4 because it gives better results [14].

$$\theta = \frac{\tau}{(\rho_s - \rho_w) g d} \quad (3)$$

$$\theta_c = 0.27 S^{0.46} \quad (4)$$

2 Methods

2.1 Preparation

The material used was sand that was available in the laboratory. The prepared sand was then subjected to a gradation test with an ASTM Standard sieve using a sieve shaker. The sand used is sand retained by sieve no. 8 (2.26 mm), 12 (1.70 mm), 16 (1.18 mm), 20 (0.85 mm), 30 (0.59 mm) 40 (0.425 mm), 50 (0.30 mm), and 60 (0.25 mm). The setup tool used is a Multi-Purpose Teaching Flume with a channel width of 10 cm and a channel length of 5 m and a sediment catcher box was provided at the outlet channel.

2.2 Flow calibration on multi-purpose teaching flume

The channel in Flume is an open channel type. Flow conditions in open channels tend to change according to time and space, so that changes in flow parameters will affect flow velocity, where the flow rate will determine the state and nature of the flow [15]. Calibration was done by measuring the flow velocity and flow depth to calculate the Froude number. If the value of the Froude number < 1 , then the flow is of subcritical type. If the value of the Froude number $= 1$, then the flow is of critical type. If the value of the Froude number > 1 , then the flow is of supercritical type [15]. In this study, the Froude number determined is $Fr > 1$ or supercritical flow. This calibration was carried out to ensure that the flow through the flume channel is uniform with the experiments carried out the previous day.

2.3 Bedload sediment transport experiment

Experiments in this study were carried out by measuring hydraulic parameters, namely flow velocity, flow width, and flow depth. Then a simulation of bedload transport was carried out by flowing uniform granular sand that was carried out for each grain size 5 times in an experiment with the sediment feeding method, which is dropping sand material (bedload) into the flow as debris.

Sediment moving in the flow is then caught using a sediment catcher box that has been prepared in the outlet channel of the Multi Purpose Teaching Flume by recording the time it takes to fill the sediment catcher box with the debris flow until it is full. Then the caught sediment was then dried in an oven and weighed.

2.4 Analysis of bedload sediment transport rate

Analysis of bedload transport rates in this study used equation from Meyer-Peter and Müller [17], Fernandez-Luque and Van Beek [18], Smart [19], Low [20], Wong and Parker [21], Lajeunesse, Malverti, and Charru [22]. Equation can be seen through Equations 5-11.

2.4.1 Meyer-Peter dan Müller [17]

$$q_b = 8(\theta - \theta_c)^{1.5} \sqrt{\left(\frac{\rho_s - \rho_w}{\rho_w}\right) g d^3} \quad (5)$$

2.4.2 Fernandez-Luque dan Van Beek [18]

$$q_b = 5.7(\theta - \theta_c)^{1.5} \sqrt{\left(\frac{\rho_s - \rho_w}{\rho_w}\right) g d^3} \quad (6)$$

2.4.3 Smart [19]

$$q_b = 4.2 S^{0.6} C \theta^{0.5} (\theta - \theta_c) \sqrt{\left(\frac{\rho_s - \rho_w}{\rho_w}\right) g d^3} \quad (7)$$

$$C = \frac{u}{\sqrt{g D S}} \quad (8)$$

2.4.4 Low [20]

$$q_b = \frac{6.42}{\left(\frac{\rho_s - \rho_w}{\rho_w}\right)^{0.5}} (\theta - \theta_c) d u S^{0.6} \quad (9)$$

2.4.5 Wong dan Parker [21]

$$q_b = 3.97(\theta - \theta_c)^{1.5} \sqrt{\left(\frac{\rho_s - \rho_w}{\rho_w}\right) g d^3} \quad (10)$$

2.4.6 Lajeunesse, Malverti, and Charru [22]

$$q_b = 10.6(\theta - \theta_c) (\sqrt{\theta} - \sqrt{\theta_c} + 0.025) \sqrt{\left(\frac{\rho_s - \rho_w}{\rho_w}\right) g d^3} \quad (11)$$

The above equation assumes that the density of water is $\rho_w = 1000 \text{ kg/m}^3$, the density of sediment is $\rho_s = 2650 \text{ kg/m}^3$, and the acceleration due to gravity is $g = 9.81 \text{ m/s}^2$. The equation with the concept of shear stress is a classic equation that is commonly used. Therefore, many researchers have developed the equation of bedload transport rate with the concept of shear stress.

3 Results

Based on the analysis of bedload transport rate with the 6 equations above, a recapitulation of the results of bedload transport rate for each experiment is obtained, as shown in Table 1.

Table 1. Recapitulation of bedload transport rates.

Grain Diameter mm	Experiment No.	Meyer-Peter dan Müller (1948) m ³ /s/m	Fernandez-Luque dan van Beek (1976) m ³ /s/m	Smart (1984) m ³ /s/m	Low (1989) m ³ /s/m	Wong dan Parker (2006) m ³ /s/m	Lajeunesse, Malverti, dan Charu (2010) m ³ /s/m
2.36	1	6.78919E-07	4.8373E-07	6.7563E-07	1.0074E-06	3.36914E-07	7.89237E-07
	2	6.78919E-07	4.8373E-07	6.7563E-07	1.0074E-06	3.36914E-07	7.89237E-07
	3	6.78919E-07	4.8373E-07	6.7563E-07	1.0074E-06	3.36914E-07	7.89237E-07
	4	6.78919E-07	4.8373E-07	6.7563E-07	1.0074E-06	3.36914E-07	7.89237E-07
	5	6.78919E-07	4.8373E-07	6.7563E-07	1.0074E-06	3.36914E-07	7.89237E-07
1.70	1	4.05681E-06	2.89048E-06	2.14749E-06	3.23248E-06	2.01319E-06	2.39222E-06
	2	4.05681E-06	2.89048E-06	2.14749E-06	3.23248E-06	2.01319E-06	2.39222E-06
	3	4.05681E-06	2.89048E-06	2.14749E-06	3.23248E-06	2.01319E-06	2.39222E-06
	4	4.05681E-06	2.89048E-06	2.14749E-06	3.23248E-06	2.01319E-06	2.39222E-06
	5	4.05681E-06	2.89048E-06	2.14749E-06	3.23248E-06	2.01319E-06	2.39222E-06
1.18	1	9.64053E-06	6.86888E-06	4.29819E-06	6.46979E-06	4.78411E-06	4.14404E-06
	2	9.64053E-06	6.86888E-06	4.29819E-06	6.46979E-06	4.78411E-06	4.14404E-06
	3	9.64053E-06	6.86888E-06	4.29819E-06	6.46979E-06	4.78411E-06	4.14404E-06
	4	9.64053E-06	6.86888E-06	4.29819E-06	6.46979E-06	4.78411E-06	4.14404E-06
	5	9.64053E-06	6.86888E-06	4.29819E-06	6.46979E-06	4.78411E-06	4.14404E-06
0.85	1	1.26689E-05	9.02663E-06	5.15679E-06	7.76217E-06	6.28697E-06	4.69112E-06
	2	1.26689E-05	9.02663E-06	5.15679E-06	7.76217E-06	6.28697E-06	4.69112E-06
	3	1.26689E-05	9.02663E-06	5.15679E-06	7.76217E-06	6.28697E-06	4.69112E-06
	4	1.03954E-05	7.40669E-06	4.02131E-06	6.05302E-06	5.15869E-06	3.90781E-06
	5	1.03954E-05	7.40669E-06	4.02131E-06	6.05302E-06	5.15869E-06	3.90781E-06
0.59	1	1.26953E-05	9.04539E-06	4.59448E-06	6.91577E-06	6.30003E-06	4.20163E-06
	2	1.28525E-05	9.15744E-06	4.79906E-06	7.15566E-06	6.37808E-06	4.2812E-06
	3	1.28525E-05	9.15744E-06	4.79906E-06	7.15566E-06	6.37808E-06	4.2812E-06
	4	1.28525E-05	9.15744E-06	4.79906E-06	7.15566E-06	6.37808E-06	4.2812E-06
	5	1.28525E-05	9.15744E-06	4.79906E-06	7.15566E-06	6.37808E-06	4.2812E-06
0.425	1	1.70313E-05	1.21348E-05	6.35437E-06	9.53487E-06	8.45176E-06	5.17118E-06
	2	1.70313E-05	1.21348E-05	6.35437E-06	9.53487E-06	8.45176E-06	5.17118E-06
	3	1.70313E-05	1.21348E-05	6.35437E-06	9.53487E-06	8.45176E-06	5.17118E-06
	4	1.70313E-05	1.21348E-05	6.35437E-06	9.53487E-06	8.45176E-06	5.17118E-06
	5	1.70313E-05	1.21348E-05	6.35437E-06	9.53487E-06	8.45176E-06	5.17118E-06
0.30	1	1.83798E-05	1.30956E-05	6.68551E-06	1.00318E-05	9.12096E-06	5.21254E-06
	2	1.83798E-05	1.30956E-05	6.68551E-06	1.00318E-05	9.12096E-06	5.21254E-06
	3	1.83798E-05	1.30956E-05	6.68551E-06	1.00318E-05	9.12096E-06	5.21254E-06
	4	1.83798E-05	1.30956E-05	6.68551E-06	1.00318E-05	9.12096E-06	5.21254E-06
	5	1.83798E-05	1.30956E-05	6.68551E-06	1.00318E-05	9.12096E-06	5.21254E-06
0.25	1	1.89287E-05	1.34867E-05	6.81797E-06	1.02305E-05	9.39336E-06	5.20927E-06
	2	1.89287E-05	1.34867E-05	6.81797E-06	1.02305E-05	9.39336E-06	5.20927E-06
	3	1.89287E-05	1.34867E-05	6.81797E-06	1.02305E-05	9.39336E-06	5.20927E-06
	4	1.89287E-05	1.34867E-05	6.81797E-06	1.02305E-05	9.39336E-06	5.20927E-06
	5	1.89287E-05	1.34867E-05	6.81797E-06	1.02305E-05	9.39336E-06	5.20927E-06

Through the analysis of bedload transport rates using the six equations mentioned earlier, the study provides a comprehensive summary of the results obtained from each experiment, as presented in Table 1. Based on it, the volume of bedload transport per second per meter (m³/s/m) was determined. These results were averaged and compared to the measured laboratory experimental data, as depicted in Table 2. To visually validate the findings and identify the closest resemblance to the laboratory measurements, the data from Table 2 were plotted in a graphical form, illustrating the relationship between grain diameter and bedload transport rate, as shown in Fig. 4.

The data in Table 2 was then also processed to find the deviation of the bedload transport rate in each

equation with the measurement results, so that the difference between the analysis results and the measurement results can be known, as can be seen in Table 3.

4 Discussions

Differences from the analysis results and measurement results must be viewed from the investigation method used by the researchers in defining the bedload transport equation, which is described as follows:

- Fernandez-Luque and Van Beek [18] investigated a closed rectangular channel with a length of 8 m and a width of 10 cm. The investigation was carried out

with variations of slope 12° , 18° , and 22° with 5 materials used as sediment samples, namely 2 types of sand 0.90 mm and 1.8 mm ($\rho_s = 2640 \text{ kg/m}^3$),

gravel 3.3 mm ($\rho_s = 2640 \text{ kg/m}^3$), magnetite 1.8 mm ($\rho_s = 4580 \text{ kg/m}^3$), and a walnut kernel 1.5 mm ($\rho_s = 1340 \text{ kg/m}^3$).

Table 2. Average bedload transport results from analysis data and measurement data.

Grain Diameter mm	Laboratory Experiment m ³ /s/m	Meyer-Peter dan Müller (1948) m ³ /s/m	Fernandez-Luque dan van Beek (1976) m ³ /s/m	Smart (1984) m ³ /s/m	Low (1989) m ³ /s/m	Wong dan Parker (2006) m ³ /s/m	Lajeunesse, Malverti, dan Charru (2010) m ³ /s/m
2.36	4.78E-06	6.79E-07	4.84E-07	6.76E-07	1.01E-06	3.37E-07	7.89E-07
1.70	3.70E-06	4.06E-06	2.89E-06	2.15E-06	3.23E-06	2.01E-06	2.39E-06
1.18	3.00E-06	9.64E-06	6.87E-06	4.30E-06	6.47E-06	4.78E-06	4.14E-06
0.85	3.66E-06	1.18E-05	8.38E-06	4.70E-06	7.08E-06	5.84E-06	4.38E-06
0.59	4.37E-06	1.28E-05	9.14E-06	4.76E-06	7.11E-06	6.36E-06	4.27E-06
0.425	4.60E-06	1.70E-05	1.21E-05	6.35E-06	9.53E-06	8.45E-06	5.17E-06
0.30	4.62E-06	1.84E-05	1.31E-05	6.69E-06	1.00E-05	9.12E-06	5.21E-06
0.25	4.19E-06	1.89E-05	1.35E-05	6.82E-06	1.02E-05	9.39E-06	5.21E-06

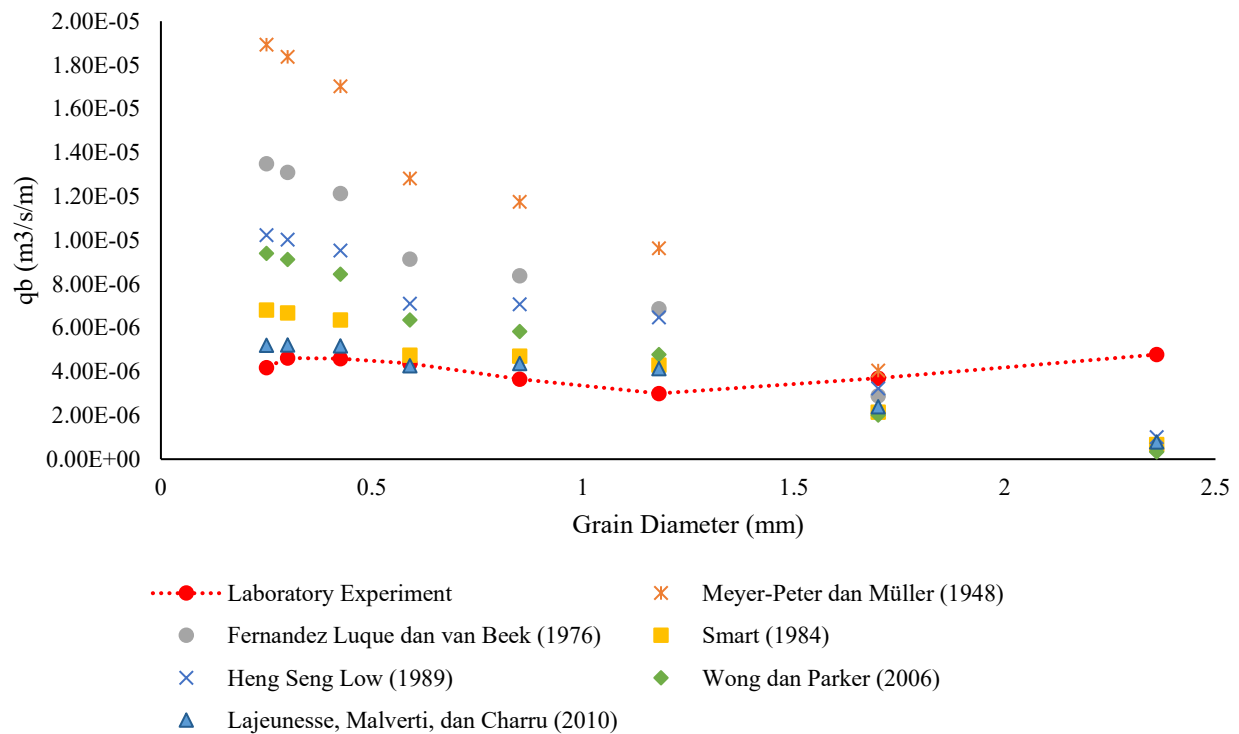


Fig. 4. Graph of bedload transport comparison based on analysis results and measurement results.

Table 3. Deviation of bedload transport results from analysis and measurement.

Grain Diameter mm	Meyer-Peter dan Müller (1948) m ³ /s/m	Fernandez-Luque dan van Beek (1976) m ³ /s/m	Smart (1984) m ³ /s/m	Low (1989) m ³ /s/m	Wong dan Parker (2006) m ³ /s/m	Lajeunesse, Malverti, dan Charru (2010) m ³ /s/m
2.36	4.10E-06	4.29E-06	4.10E-06	3.77E-06	4.44E-06	3.99E-06
1.70	3.57E-07	8.10E-07	1.55E-06	4.68E-07	1.69E-06	1.31E-06
1.18	6.64E-06	3.87E-06	1.30E-06	3.47E-06	1.78E-06	1.14E-06
0.85	8.10E-06	4.72E-06	1.04E-06	3.42E-06	2.17E-06	7.16E-07
0.59	8.46E-06	4.77E-06	3.93E-07	2.74E-06	2.00E-06	1.00E-07
0.425	1.24E-05	7.54E-06	1.76E-06	4.94E-06	3.85E-06	5.74E-07
0.30	1.38E-05	8.47E-06	2.06E-06	5.41E-06	4.50E-06	5.88E-07
0.25	1.47E-05	9.30E-06	2.63E-06	6.05E-06	5.21E-06	1.02E-06

- b. Lajeunesse et al. [22] conducted an investigation on a rectangular channel with a length of 240 cm and a width of 9.6 cm with a variation of the slope value $0.0017 \leq S \leq 0.12$. The materials used as sedimentary materials are sand with diameters of 1.15 mm, 2.24 mm, and 5.50 mm with $\rho_s = 2650 \text{ kg/m}^3$.
- c. Low [20] conducted an investigation of bedload transport in a rectangular channel with a length of 6 m and a width of 15.5 cm with a variation of the slope value $0.0046 \leq S \leq 0.0149$. The investigation was carried out with long extruded plastic particles with a diameter of 3.5 mm with a specific weight of sediment $1 < S < 2.5$ with a flow velocity of 3.89-7.66 m/s.
- d. Smart [19] conducted an investigation of bedload transport in a rectangular channel with a length of 6 m and a width of 0.2 with a slope range of 3-25%, namely the value $0.03 \leq S \leq 0.020$. The investigation was carried out on grains of diameter $2 \leq d \leq 10.5 \text{ mm}$ with $2670 \leq \rho_s \leq 2680 \text{ kg/m}^3$.
- e. Meyer-Peter and Müller [17] conducted a series of bedload transport investigations in a rectangular channel with a channel width of 1 m. The probe was carried out with uniform sand with a diameter of $1.86 \leq d \leq 10 \text{ mm}$ with $\rho_s = 2680 \text{ kg/m}^3$.
- f. Wong and Parker [21] only re-analysed the Meyer-Peter and Müller equations based on existing references.

Based on the description above, it can be assumed that several factors influence the difference in the results of the analysis based on the equations of Meyer-Peter and Müller (1948), Fernandez-Luque and van Beek (1976), Smart (1984), Low (1989), Wong and Parker (2006), Lajeunesse, Malverti, and Charru (2010) [17-22] with the following measurement results:

- a. Differences in sediment particle material will affect the mass density of sediment particles, this will affect the weight value of sediment submerged in water even with the same sediment particle size. There is a significant difference in the density of different sediment values in the investigation conducted by Fernandez-Luque and van Beek [18].
- b. The slope value is very decisive in this study, where the slope value will affect the value of the critical Shield parameter as an indicator for the initiation of the initial movement of sediment particles. The difference in the slope is quite high in the investigation conducted by Low [20] and Smart [19], the value of the high slope will also affect the flow velocity. Therefore, the Low [20] and Smart [19] equations will be optimal if used in channels with steep slope conditions and heavy flow.
- c. Differences in the results of bedload transport rate are also influenced by the diameter size of the particles used. Where the size of the diameter of the sediment particles used by the researchers above in defining the bedload transport equation has a larger size ($1.15 \leq d \leq 10.5 \text{ mm}$) than that used in this

study ($0.25 \leq d \leq 2.36 \text{ mm}$), so the above equation is less sensitive to calculate the bedload transport rate.

From the comparison of research methods and comparison of bedload transport results based on the analysis and measurements above. The Lajeunesse, Malverti, and Charru [22] equation has the result of bedload transport that is closest to the measurement results. Because the bedload transport study conducted by Lajeunesse, Malverti, and Charru [22] has a flume setup and the sediment particle material used is almost the same as that used in this study.

In this study there were also errors that could result in inaccurate measurements of bedload transport. Therefore, a lower value was obtained when compared to the results of the analysis using the equation above, including the following:

- a. The inaccuracy of the analysis results with the measurement results can also be caused by the lack of information from the 6 methods above to take into account bedload transport for sediments in wet or dry conditions. Because the results of measurements of bedload transport carried out in the laboratory used sediment data in dry conditions.
- b. Sampling of sediment with a sediment catcher box was carried out at the beginning when new sediment comes out of the outlet channel. Therefore, it can be an indicator that causes the measurement of sediment transport rate to produce a smaller value than the results of analysis using the basic sediment transport equation.

5 Conclusions

Based on the results of bedload transport research that has been carried out at the Civil Engineering Laboratory of Muhammadiyah University of Yogyakarta, it can be concluded as follows:

- a. From the analysis of bedload transport, it was found that the Lajeunesse, Malverti, and Charru [22] equations were closest to the results of laboratory measurements.
- b. The important parameters used in the analysis of sediment transport are the diameter of the sediment particles, the density of the sediment particles, and the slope.

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