Heat Transfer Analysis using Finite Element Method under Convective Boundary Condition

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Abstract. The model in this study scrutinizes the effect of convective boundary conditions on the flow of a nanofluid across permeable flat plate. The fundamental equations get altered into a nonlinear form through choosing appropriate similarity transformations. In the process, they are solved mathematically by substantiated FEM code through use of variational finite element method. The outcomes clearly show the characteristics of relevant parameters such as temperature and velocity profiles. When the numerical analysis is evaluated in context of formerly published information, the reliability of the numerical code is conformed. Its found that there is a surge in thermal conductivity when proportion of nanoparticles rises in the fluid. Permeability of plate has a significant influence on the heat transfer and skin friction. The investigation supports the possibility of extending the work to flows of non-Newtonian fluid, three dimensional and for consideration of pressure gradients on arbitrary surfaces. The results practically aid the design of heat transfer systems for futuristic technology involving heat enhancement.

1. Introduction

In the past decade, nanotechnology has gained significant attention in many engineering areas. Effective techniques for fluid cooling and heating are needed in numerous areas like power, transportation, manufacturing, and electronic devices. Choi [1] proposed the term nanofluid for the fluid containing nanometre sized (1-100 nm) particles known as nanoparticles. Masuda et al. [2] first studied the thermal conductivity enhancement using nanofluid. Nanoparticle size influence the transport capabilities of the fluid. The thermal characteristics of nanofluid were enhanced by maintaining minimum nanoparticle 5% volume fraction [3, 4].

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Nanofluids have potential applications in space technology [5], biomedical [6,7], material processing [8], solar devices [9] and combustion processes [10,11]. Recently Assael MJ et al. [12] presented several potential applications of nanofluids for heat transfer. In industrial applications such as chemical coating of vertical flat plates, hot rolling, wire drawing, etc., boundary layer flows past flat bodies are regularly encountered [13]. Stokes [14, 15] first studied the viscous incompressible flow over horizontal flat plate. Soundalgekar et al. [16] examined corresponding problem for vertical plate. Siegal [17] conducted the initial investigations over vertical plate. These results correspond with the experiments of Goldstein and Eckert [18]. A momentous volume of research on flow over flat plate has been testified [19-27]. Recently, Zainal et al. [28] investigated stagnation point, mixed convection flow past flat plate, Klazly and Bognar [29] examined the nanofluid flow using CFD simulations, Singh et al. [30] used technique of quasi-linearization to study viscous dissipation over plate, K. Mair et al. [31] has presented study based on entropy generation for Blasius flow.

In various engineering problems, the process of suction/injection is important in context of surface heat transfer characteristics applicable in chemical and mechanical design [32-35]. heat exchange is dominant in many critical industrial processes. Swapna et al. [36] presented their work for the impact of convective boundary conditions. Many studies considering convective heat transfer for different geometries are reported [37-42]. To investigate the behavior of nanofluids considering the solid volume fraction, the model developed by Tiwari and Das [43] has been widely used in literature [44-46]. The present work thus examines the heat transfer flow taking into account convective boundary surface condition and solid volume fraction over permeable flat plate employing finite element method. Highly efficient, robust, extensively validated, novel FEM code is applied here. This study adds to engineering science literature of computational multiphysical nanofluid dynamics simulation.

2. Model Formulation

Consider 2D, steady, laminar, incompressible nanofluid flow over permeable semi-infinite flat plate. We get coefficient of heat transfer $h_f$ by heating left side of the plate through convection from a hot fluid at temperature $T_f$, $T_w$ is the temperature of nanofluid. The velocity of the far flow from the plate is $U_f$, and mass flux velocity $S_w$ is presumed to be constant. The flow geometry is shown in Fig. 1.
Further, assuming valid Boussinesq and boundary layer approximations. Since flow is over a flat surface and hence the pressure gradient is zero, the basic steady equations following the work [47] are:

\[ \frac{\partial u_i}{\partial x_i} + \frac{\partial v_i}{\partial y_i} = 0 \]  

(1)

\[ u_i \frac{\partial u_i}{\partial x_i} + v_i \frac{\partial u_i}{\partial y_i} = \nu_x \frac{\partial^2 u_i}{\partial y_i^2} \]  

(1)

\[ u_i \frac{\partial T_i}{\partial x_i} + v_i \frac{\partial T_i}{\partial y_i} = \frac{k_{nf}}{\rho C_p} \left( \frac{\partial^2 T_i}{\partial y_i^2} \right) \]  

(2)

subject to velocity field boundary conditions:

\[ u_i = 0, v_i = S_w(x_i) \quad \text{when} \quad y_i = 0 \]

\[ u_i \rightarrow U_\infty \quad \text{as} \quad y_i \rightarrow \infty \]  

(3)

where \( S_w(x_i) \) is the injection, suction velocity when \( S_w(x_i) > 0 \), \( S_w(x_i) < 0 \) and \( S_w(x_i) = 0 \) for an impermeable plate.

Thermal field boundary conditions are:

\[ y_i = 0: -k_{nf} \frac{\partial T_i}{\partial y_i} = h_f \left[ T_f - T_i \right] \]  

(4)

\[ y_i \rightarrow \infty: T_i \rightarrow T_\infty \]

Generally, \( \Psi_1(x_i,y_i) \) (stream function) is defined as \( u_i = \frac{\partial \Psi_1}{\partial y_i}, v_i = - \frac{\partial \Psi_1}{\partial x_i} \) so that it satisfies equation Error! Reference source not found..

Further, introducing nondimensional similarity variables:

\[ f_1(\eta_i) = \frac{\Psi_1(x_i,y_i)}{\sqrt{U_\infty} \nu_f x_i}, \quad \eta_i = y_i \sqrt{\frac{U_\infty}{\nu_f x_i}}, \quad \theta_i(\eta_i) = \frac{T_i - T_\infty}{T_f - T_\infty} \]  

(5)

From equation (5) we get:

\[ \Psi_1(x_i,y_i) = \sqrt{U_\infty} \nu_f x_i \cdot f_1(\eta) \]  

(6)

Therefore

\[ u_i = U_\infty f_1'(\eta_i) \]

\[ v_i = \frac{1}{2} \left( \frac{U_\infty \nu_f}{x_i} \right)^{1/2} \left( \eta_i f_1'(\eta_i) - f_1(\eta_i) \right) \]  

(7)
In view of equation (5), equations (1) and (2) get transformed to non-linear ODE

\[ 2f_i'' + \phi f_i f_i' = 0 \]  
\[ \theta_i'' + \frac{1}{2} \Pr \left( \frac{k_f}{k_{nf}} \right) \phi_2 f_i \theta_i' = 0 \]

Here, \( \phi_1 = (1 - \phi)^{2.5} \left[ 1 - \phi + \phi \left( \frac{\rho_s}{\rho_f} \right) \right] \), \( \phi_2 = \left[ 1 - \phi + \phi \left( \frac{\rho C_p}{\rho C_p} \right)_f \right] \) (11)

With boundary conditions:

\[ f_i(0) = s_w, \quad \theta_i'(0) = -a \left[ 1 - \theta_i(0) \right], \quad f_i'(0) = 0, \]
\[ f_i'(\eta_i) \to 1, \quad \theta_i(\eta_i) \to 0 \quad \text{as} \quad \eta_i \to \infty \]

where, \( a = \frac{c}{k_{nf} U_{\infty}} \left( \frac{1}{V_f} \right)^{1/3} \) is parameter of convective heat transfer, \( \phi \) is solid (nanoparticle) volume fraction, \( \Pr = \frac{\mu_f (C_p)_f}{k_f} \) is Prandtl number and \( s_w \) is rate of transpiration (8) at surface.

We must note, similarity solution is possible, if quantity ‘\( a \)’ is a constant and not a function of \( x_i \). Thus, we get, \( h_f = c x_i^{-1/2} \) where \( c \) is constant [39].

Here, the dynamic viscosity \( \mu_{nf} \), heat capacity \( \left( \rho C_p \right)_{nf} \), thermal conductivity \( k_{nf} \) and the density \( \rho_{nf} \) of nanofluid are as follows [44, 47]:

\[ \mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}} \]
\[ \left( \rho C_p \right)_{nf} = (1 - \phi) \left( \rho C_p \right)_f + \phi \left( \rho C_p \right)_s \]
\[ k_{nf} = k_s + 2k_f - 2\phi \left( k_f - k_s \right) \]
\[ k_f = k_s + 2k_f + \phi \left( k_f - k_s \right) \]
\[ \rho_{nf} = (1 - \phi) \rho_f + \phi \rho_s \]

The values of rate of heat transfer \( -\theta_i'(0) \) and the rate of shear stress \( f_i''(0) \) at the surface (the local Nusselt number, the skin friction coefficient) are of practical interest.
3. Mathematical Analysis

3.1 Finite element modelling

Details of method used here are available in the work by Reddy [50].

The finite-element modelling stages are given by,

3.1.1. Discretization of Finite-element

The entire area is alienated into a fixed number of subdomains known as an element. Finite-element mesh is obtained by assembling elements.

3.1.2. Element equations generation

i. For given problem, variational formulation is constructed by isolating a typical element from the mesh.

ii. An estimated answer of the variational problem is presumed. Using this solution in the system obtained element equations are written.

iii. The element interpolation functions are used to construct element matrix.

3.1.3. Element equations assembling

Applying continuity conditions for interelement, numerical equations are assembled. The entire domain is governed by global finite-element model constructed from these numerical equations.

3.1.4. Applying boundary conditions

The constructed equations are imposed by natural and essential boundary conditions.

3.1.5. Solving assembled equations

The numerical techniques, such as LU decomposition, Gauss elimination, etc. are used to solve assembled equations. Here, actual functions are approximated by shape functions.

To solve the non-linear ODE (9) and (10) subject to boundary conditions (12), let first assume:

\[ f_i' = h_i \]  \hspace{1cm} (14)

The system gets reduce to:

\[ 2h_i'' + \phi_i f_i' h_i' = 0 \]  \hspace{1cm} (15)

\[ \theta_i'' + \frac{1}{2} \Pr \left( \frac{k_f}{k_{sf}} \right) \phi_i f_i' \theta_i' = 0 \]  \hspace{1cm} (16)

and boundary conditions become:
3.2 Variational form

The variational form linked with equations (14)-(16) over element \((\eta_e, \eta_{e+1})\) is:

\[
\int_{\eta_e}^{\eta_{e+1}} w_1 \left( f_i' - h_i \right) d\eta_i = 0
\]  \hspace{1cm} (18)

\[
\int_{\eta_e}^{\eta_{e+1}} w_2 \left( 2h_i'' + \phi_x f_i h_i' \right) d\eta_i = 0
\]  \hspace{1cm} (19)

\[
\int_{\eta_e}^{\eta_{e+1}} w_3 \left( \phi_x + \frac{1}{2} \Pr \left( \frac{k_f}{k_{nf}} \right) \phi_x \phi_i \phi_i' \right) d\eta_i = 0
\]  \hspace{1cm} (20)

Here, \(w_1, w_2, w_3\) are linearly independent functions known as weight functions corresponding to the variation in the functions \(f_i, h_i, \phi_i\) respectively.

3.3 Finite element form

To obtain the finite element model, we substitute following approximations into equations (18)-(20).

\[
f_i = \sum_{j=1}^{2} (f_i) \psi_j, \quad h_i = \sum_{j=1}^{2} (h_i) \psi_j, \quad \phi_i = \sum_{j=1}^{2} (\phi_i) \psi_j
\]  \hspace{1cm} (21)

Here \(\psi_i = w_1 = w_2 = w_3\) \((i = 1, 2)\),

Linear interpolation functions:

\[
\psi_1 = \left( \eta_{e+1} - \eta_i \right) \left( \eta_{e+1} - \eta_e \right)^{-1}, \quad \psi_2 = \left( \eta_i - \eta_e \right) \left( \eta_{e+1} - \eta_e \right)^{-1}, \quad \eta_e \leq \eta_i \leq \eta_{e+1}
\]  \hspace{1cm} (22)

Quadratic interpolation functions:

\[
\psi_1 = \left( \eta_{e+1} + \eta_e - 2\eta_i \right) \left( \eta_{e+1} - \eta_i \right) \left( \eta_{e+1} - \eta_e \right)^{-2}, \quad \psi_2 = 4 \left( \eta_i - \eta_e \right) \left( \eta_{e+1} - \eta_i \right) \left( \eta_{e+1} - \eta_e \right)^{-2}, \quad \psi_3 = \left( \eta_{e+1} + \eta_e - 2\eta_i \right) \left( \eta_i - \eta_e \right) \left( \eta_{e+1} - \eta_e \right)^{-2}, \quad \eta_e \leq \eta_i \leq \eta_{e+1}
\]  \hspace{1cm} (23)

The developed finite element model is stated as:
Here \( [K^{mn}] \) are \( 2 \times 2 \) and \( \{r^m\} \) are \( 2 \times 1 \) order matrices \((m,n=1,2,3)\).

Matrices are given by:

\[
\begin{align*}
K_{ij}^{11} &= \int_{\eta_i}^{\eta_j} \psi_j \frac{d\psi_i}{d\eta_i} d\eta_i, \quad K_{ij}^{12} = -\int_{\eta_i}^{\eta_j} \psi_j \frac{d\psi_j}{d\eta_i} d\eta_i, \quad K_{ij}^{13} = 0 \\
K_{ij}^{21} &= 0, \quad K_{ij}^{22} = -2 \int_{\eta_i}^{\eta_j} \psi_j \frac{d^2\psi_j}{d\eta_i^2} d\eta_i + \int_{\eta_i}^{\eta_j} \phi_i \frac{d\psi_j}{d\eta_i} d\eta_i, \quad K_{ij}^{23} = 0 \\
K_{ij}^{31} &= 0, \quad K_{ij}^{32} = 0, \quad K_{ij}^{33} = -\int_{\eta_i}^{\eta_j} \psi_j \frac{d^2\psi_j}{d\eta_i^2} d\eta_i + \frac{1}{2} \Pr \left( \frac{k_f}{k_r} \right) \phi_i \int_{\eta_i}^{\eta_j} \frac{d\psi_j}{d\eta_i} d\eta_i
\end{align*}
\] (24)

\[
\begin{align*}
r_i^1 &= 0, \quad r_i^2 = -\left[ \psi_j \frac{d\psi_j}{d\eta_i} \right]_{\eta_{i-1}}^{\eta_i}, \quad r_i^3 = -\left[ \psi_j \frac{d\psi_j}{d\eta_i} \right]_{\eta_{i-1}}^{\eta_i}
\end{align*}
\] (25)

with \( \overline{f_i} = \sum_{i=1}^{n} (f_i) \psi_i \) .

No significant change in the results has been observed for large values of \( \eta_i > 10 \). Thus, without loss of generality for the computational purposes, \( \infty \) can be fixed at 10. The total domain is split into \( n \)-linear elements having width \( h_i = \frac{10}{n} \). The calculations are carried out for 200, 250, 300, 350, 400, 500 and 1000 elements and \( n = 400 \) is fixed.

The order of the element matrix (23) is \( 6 \times 6 \). If we split domain into 400 equal linear elements, after assembling all element equations we get 1203\( \times \)1203 order matrix. The system obtained is highly nonlinear and linearized by incorporating the function \( \overline{f_i} \). By sustaining an accuracy of 0.0001, an iterative scheme for \( f_i, h_i \) and \( \theta_i \) is used to solve the remaining system of non-linear equations.

The interpolation functions for 1D and 2D problems, can be linear, quadratic and higher order. Though, the fitness of the shape functions differs with the problem. Linear and quadratic shape functions are used owing to the simple and effective usage in computations. Table 1 gives evaluation for both types of interpolation function, and observed that the outcomes do not differ much, indicating that both elements provide almost the same precision.
Table 1: Results of $f_i$, $\partial f_i / \partial \eta_i$ and $\theta_i$ for linear as well as quadratic elements when step size = 0.1, Pr = 6.2, $\phi = 0.01$, $\alpha = 0.5$, $s_w = 1$.

<table>
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<tr>
<th>$\eta$</th>
<th>Quadratic</th>
<th>Linear</th>
<th>Quadratic</th>
<th>Linear</th>
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<td>0.2641</td>
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<td>0.0355</td>
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<td>0.5671</td>
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</tr>
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4. Code Validation

To evaluate the accuracy and the efficiency of the FEM code, the numerical results of $-\frac{\partial \theta_i}{\partial \phi}(0)$ for values of $a$ and numerical values of $f_i$, $(f_i)'$ and $(f_i)''$ with variation in $\eta_i$ are depicted in Table 2 and 3 respectively. The results are in good correlation with previous studies, testifying to the validity of the code. In order to further validate the trends of temperature and velocity profiles, for various values of $\phi$ the developed temperature profiles have been graphically shown in Fig. 2 and they are depicting the same trend as those analysed previously by Anjali Devi and Julie Andrews [47]. Further, profiles of the temperature and velocity for $s_w$ are graphically depicted in Fig. 3 and Fig. 4 and the trends are qualitatively similar with those of Ishak [38]. Thus, it confirms the authenticity of our numerical code.

Table 2: Results for $-\frac{\partial \theta_i}{\partial \phi}(0)$ by varying $a$ when $\phi = 0$, $s_w = 0$, Pr = 0.1.

<table>
<thead>
<tr>
<th>$a$</th>
<th>Aziz [37]</th>
<th>Ishak [38]</th>
<th>Present result</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.0594</td>
<td>0.058338</td>
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</tr>
<tr>
<td>0.2</td>
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Table 3: Results for $f_i$, $(f_i)'$, $(f_i)''$ by varying $\eta$ when $\phi = 0$, $s_w = 0$.

<table>
<thead>
<tr>
<th>$\eta_i$</th>
<th>Howarth [49]</th>
<th>Cortell [48]</th>
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<td>9</td>
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</table>
Fig. 2. Variation in temperature profiles $\theta_1(\eta_1)$ with $\phi, f_c = 0, \text{Pr} = 6.2, n=1000$.

Fig. 3. Variation in Velocity profiles $f'_1(\eta_1)$ with $s_v, \phi = 0, \alpha = 1, \text{Pr} = 1$.

Fig. 4. Variation in Temperature profiles $\theta'_1(\eta_1)$ with $s_v, \text{Pr} = 1, \alpha = 1, \phi = 0$. 

5. Results and Discussion

Impact on the temperature and velocity distributions with variation in thermophysical parameters i.e. $\phi$, $s_w$, and $a$ are shown in Fig. 5-8. The numerical results for the skin friction coefficient $f_i''(0)$ and rate of heat transfer $-\theta_i'(0)$ are reported in Table 4 and Table 5 for selected parameters.

In Fig. 5, it is observed that, the temperature increases if volume fraction is increased. Owing to the high thermal conductivity of the $CuH_2O$ nanofluid and further with increase in volume fraction, the resistance between the nanofluid and the wall becomes high which tends to rise thickness of thermal boundary layer.

Fig. 6, 7 exhibit the evolution of velocity and temperature for the effect of injection/suction, respectively. It is observed from Fig. 6, as suction ($s_w > 0$) increases, the velocity also increases. It is also noted that the flow is strongly decelerated with suction $s_w = 2$, while it is accelerated with increasing injection $s_w = -0.5$. This is because suction causes the boundary layer to cling more firmly to the plate, thereby decreasing the momentum boundary layer thickness. Conversely, injection assists in momentum development as more nanofluid particles are added through the plate thereby enhancing the velocity and increasing the thickness of momentum boundary layer. It is distinctly observed from Fig. 7 that the temperature decreases with increasing suction. On the other hand, injection or blowing is found to enhance the temperature.
In Fig. 8, relation between \( a \) and \( \theta_l(0) \) is depicted. It is observed that as convective heat transfer parameter increases, surface temperature \( \theta_l(0) \) increases. This is due to the hot fluid side thermal resistance and \( h_f \) are inversely proportional to each other. Further, it is seen from the figure that \( \theta_l(0) = 1 \) as \( a \to \infty \). It can be observed from equation (9) that \( a \) has no effect on the flow field.

The impact of suction/injection and convective heat transfer parameter has been calculated on the \( f_i'(0), -\theta_l'(0) \) and depicted in Table 4 and Table 5, respectively. It is noted from Table 4 that \( f_i'(0) \) increases with increasing \( s_w \). The surface shear stress is more for suction (\( s_w > 0 \)) in contrast to injection (\( s_w < 0 \)). It is also detected the heat transfer rate at the surface \( -\theta_l'(0) \) is higher for the suction. This phenomenon is due to introduction of suction causing rise in surface shear stress along with surge in local Nusselt number. It is noted from Table 5 that heat transfer rate is boosted as \( a \) increases. Further, the plate high inner thermal resistance is due to higher value of the convective parameter.
Table 4: Influence of $s_w$ on $f_1'' (0)$, $-\theta'_1 (0)$ when $\phi = 0.01$, $a = 1$.

<table>
<thead>
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<th>$-\theta'_1 (0)$</th>
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<tr>
<td>2</td>
<td>1.1693</td>
<td>0.8556</td>
</tr>
<tr>
<td>3</td>
<td>1.6317</td>
<td>0.8920</td>
</tr>
</tbody>
</table>

Table 5: Influence of $a$ on $-\theta'_1 (0)$ when $\phi = 0.01$, $s_w = 1$.

<table>
<thead>
<tr>
<th>$a$</th>
<th>$-\theta'_1 (0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.05</td>
<td>0.0492</td>
</tr>
<tr>
<td>0.1</td>
<td>0.0968</td>
</tr>
<tr>
<td>0.5</td>
<td>0.4316</td>
</tr>
<tr>
<td>1</td>
<td>0.7600</td>
</tr>
<tr>
<td>3</td>
<td>1.5428</td>
</tr>
<tr>
<td>10</td>
<td>2.4124</td>
</tr>
<tr>
<td>20</td>
<td>2.7439</td>
</tr>
</tbody>
</table>

6. Conclusions

A mathematical study for heat transfer of a nanofluid flow over flat plate in the presence of suction/injection with a convective surface boundary condition has been conducted. The altered system is solved by using a valid variational FEM demonstrating agreement with previous studies. The present solutions reveal that:

- Surge in the nanoparticles enhances CuH₂O nanofluid thermal conductivity.
- Increasing suction retards the nanofluid flow and decreases the temperature, whereas injection parameter does contrary.
- Heat transfer and skin friction are sturdily influenced by the suction/injection.
- The temperature, heat transfer are boosted with an increase in the convective boundary parameter.
- Vigorous cooling of vertical plate may be effectively attained by judiciously implementing the parameters studied in this paper.
- The current study can be extended to flows considering pressure gradients on arbitrary surfaces and also to cases of three-dimensional flows, non-Newtonian flows.
- These findings are of practical importance in designing heat transfer systems that employ nano-fluids as their principal heat exchange medium.
Nomenclature

\( x_1, y_1 \): Cartesian coordinates along the plate and normal to it

\( T_1 \): temperature of fluid in boundary layer

\( \nu_\text{sf} \): kinematic viscosity

\( f_1 \): dimensionless velocity

\( \eta_1 \): dimensionless coordinate

\( \theta_1 \): dimensionless temperature

\( \mu_f \): viscosity of a base fluid

\( k_s, k_f \): thermal conductivities of the solid fractions and of the fluid

\( \rho_s, \rho_f \): densities of the solid fractions and of the fluid

Superscript

\( ^\prime \): differentiation with respect to \( \eta_1 \)

References


