Mathematical modeling of the processes of combined heat and moisture transfer during storage and drying of raw cotton

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Abstract

The article developed a multidimensional mathematical model of joint heat and moisture transfer processes in inhomogeneous porous bodies, considering internal heat and moisture release, heat, and moisture exchange with the environment. Based on the usage of an implicit finite difference scheme with the second order of precision in time and space variables, an effective numerical solution for resolving issues with heat and moisture transmission has been created. Software was created for studying the processes of heat and moisture transfer during the storage and drying of raw cotton based on the developed numerical algorithm, making it possible to identify and forecast changes in temperature and humidity at arbitrary points of raw cotton with various sizes.

1 Introduction

The problems of developing the theoretical foundations and methodology for modelling complex processes of heat and mass transfer are devoted to the work of a number of prominent scientists, such as A.V. Lykov, Yu.A. Mikhailov[1], LI.YI and Zhu.Qingyong [2], D.A.Nield and A.Bejan [3] and others. The problems of mathematical modeling of the processes of drying and storage of various materials (including for raw cotton) are considered in the works of N. Ravshanov[4], A.Mamatov, A.Parpiev and A.Kayumov [5], T.J.Afolabi and S.E.Agarry [6], N.Wang and J.G.Brennan [7] et al. The Philippe and De Vries model [8], A.V.Lykov [9], and Whitaker [10] are now the most commonly studied models used to describe the processes of heat and moisture transfer in capillary-porous media. These models were created using the concepts of mass and energy conservation, Fourier's law of heat conduction, Fick's law of gas diffusion, and Darcy's law of liquid diffusion. The option of
control potentials, such as partial pressure, relative humidity, and water content in porous bodies, is a characteristic of these models.

A set of partial differential equations presented by A.V. Lykov describes the processes of linked heat and moisture transfer inside a wet porous body during drying and has the following form [9]:

\[
\frac{\partial u}{\partial \tau} = a_{m} \nabla u + a_{r} \nabla T
\]

\[
\frac{\partial T}{\partial \tau} = a_{m} \nabla T + a_{r} \nabla u
\]

where coefficients \(a_{m}, a_{r}, a_{m}, a_{r}\) are determined by the relations:

\[a_{m} = a_{m} = \frac{\lambda}{\rho c_{v}}\]

- diffusion coefficient of wet bodies (diffusion coefficient of moisture \(a_{m}\) can be called the coefficient of potential conductivity of moisture transfer);

\[a_{r} = a_{r} = a_{m} \delta\]

- coefficient of thermal diffusion of wet bodies;

\[a_{m} = a + a_{m} \frac{r_{m}}{c} = \frac{\lambda}{\rho c_{v}} + a_{m} \frac{r_{m}}{c}\]

- coefficient of diffusion of heat (in the physical sense, the coefficient of thermal diffusivity is the coefficient of diffusion of heat);

\[a_{r} = a_{m} \frac{r_{m}}{c}\]

- coefficient of thermal diffusion of wet bodies; \(r_{m}\) - capillary radius; \(\delta\) - relative moisture thermal diffusion coefficient, usually experimentally determined by the formula \(\delta = \frac{a_{r}}{a_{m}}\).

An analytical method for resolving the A.V. Lykov heat and mass transfer equations with time-dependent boundary conditions is presented in the works of Jen Y. Liu [11]. The solution consists of the sum of solutions of inhomogeneous equations. When the boundary conditions change, the homogeneous solutions remain the same, and only particular solutions change. Numerical results on the example of drying a porous material show that with a simultaneous increase in temperature and a decrease in the equilibrium mass transfer potential over time, the drying time and heat absorption can be reduced until the desired moisture content is reached.

A two-dimensional model for the investigation of heat and moisture transmission via porous wood construction materials is presented in the paper [12]. Researchers provide a non-stationary coupled model for heat transmission and moisture exchange in low-temperature wood materials. The two non-linear partial differential equations that come from the coupled model are then numerically solved using an implicit iterative method. Comparisons are made between experimental readings reported in the literature and the numerical results of the potential change in temperature and humidity.

It has been made possible to thoroughly study the processes of heat and moisture exchange that take place during the processing and storage of agricultural products as a result of the work of the aforementioned scientists as well as that of numerous other researchers. It has not yet been thoroughly studied how to develop and enhance mathematical models that account for such internal and external phenomena as self-heating, ambient temperature, and solar radiation, which have a significant impact on the processes of heat and moisture exchange in agricultural products.
2 Problem statement

The most general set of equations are (1) and (2), which hold true for all kinds of heat and moisture transmission as well as drying wet materials. The following system of differential equations is proposed as a mathematical model of heat and moisture transfer, which takes into account moisture and heat exchange with the environment, sources of heat and moisture release inside a heterogeneous porous medium, and flow insolation solar radiation. It takes into account the variability of the main thermophysical indicators of the process of drying and storing of heterogeneous porous bodies:

$$\frac{\partial T}{\partial \tau} = div(a \nabla T) + div(\delta \nabla u) + f, \quad (3)$$

$$\frac{\partial u}{\partial \tau} = div(\delta \nabla u) + div(a \nabla T) + q, \quad (4)$$

with initial

$$T(x,y,z) = T_0(x,y,z) \quad u(x,y,z) = u_0(x,y,z) \quad (5)$$

and boundary conditions

$$\lambda \frac{\partial T}{\partial x} \bigg|_{x=x_{oc}} = -\beta \left(T_{oc} - T(x,y,z)\right) - \eta \rho \gamma R(\tau), \quad (6)$$

$$\lambda \left. \frac{\partial T}{\partial x} \right|_{x=L_x} = -\beta \left(T_{oc} - T(L_x,y,z)\right) - \eta \rho \gamma R(\tau), \quad (7)$$

$$\lambda \left. \frac{\partial T}{\partial y} \right|_{y=y_{oc}} = -\beta \left(T_{oc} - T(x,y_{oc},z)\right) - \eta \rho \gamma R(\tau), \quad (8)$$

$$\lambda \left. \frac{\partial T}{\partial y} \right|_{y=L_y} = -\beta \left(T_{oc} - T(x,L_y,z)\right) - \eta \rho \gamma R(\tau), \quad (9)$$

$$\left. \frac{\partial T}{\partial z} \right|_{z=0} = 0, \quad (10)$$

$$\lambda \left. \frac{\partial T}{\partial z} \right|_{z=L_z} = -\beta \left(T_{oc} - T(x,y,L_z)\right) - \eta \rho \gamma R(\tau), \quad (11)$$

$$\lambda \left. \frac{\partial u}{\partial x} \right|_{x=x_{oc}} = -\beta \left(u_{oc} - u(x,y,z)\right), \quad (12)$$

$$\lambda \left. \frac{\partial u}{\partial x} \right|_{x=L_x} = -\beta \left(u_{oc} - u(L_x,y,z)\right), \quad (13)$$

$$\lambda \left. \frac{\partial u}{\partial y} \right|_{y=y_{oc}} = -\beta \left(u_{oc} - u(x,y_{oc},z)\right), \quad (14)$$

$$\lambda \left. \frac{\partial u}{\partial y} \right|_{y=L_y} = -\beta \left(u_{oc} - u(x,L_y,z)\right), \quad (15)$$
Here \(T(x,y,z,t)\) - is the temperature at the point \(x,y,z\in \Omega\) at time \(t \geq 0\); \(u(x,y,z)\) - change of moisture over time; \(a(x,y,z)\) - coefficient of thermal diffusivity; \(\delta(x,y,z)\) - coefficient of moisture conductivity; \(\nabla = \left\{ \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\}\) - nabla operator; \(f(x,y,z)\) - is the intensity of the internal heat release of the mass; \(b = \frac{u}{c}\) - heat release coefficient; \(c\) - specific heat capacity; \(\alpha\) - empirical parameter; \(q(x,y,z)\) - the intensity of internal sources of moisture, at constant values of the density of the material - \(\rho\); \(\xi\) - drying coefficient; \(m\) - maximum evaporation rate; \(\beta\) - heat transfer coefficient; \(\theta_c\) - ambient temperature; \(\eta\) - coefficients for carrying out the boundary condition to the dimensional form; \(\gamma\) - absorption coefficient; \(R(\tau)\) - flux of solar radiation; \(\beta\) - coefficient of moisture return; \(u_c\) - ambient humidity.

The processes of heat and moisture transfer in porous media during the storage and drying of heterogeneous bodies can be studied, observed, and predicted using a mathematical model of this kind. This model accounts for the heterogeneity of the medium, heat and moisture exchange with the environment, daily variation in solar radiation, and internal heat and moisture release of the material.

### 3 Solution method

It is difficult to arrive at an analytical solution because, as can be observed from the statement of the problem, the object of research is defined by a system of partial differential equations with a source of heat and moisture release. Given the foregoing, we utilize the finite difference method to solve problems (3) through (17), substituting a grid for the region of the continuous solution.

Let's introduce the space-time grid:

\[
\Omega_{xt} = \left\{ (x_i = i\Delta x, y_j = j\Delta y, z_k = k\Delta z, \tau_n = n \Delta \tau) \right\}
\]

\(i = \overline{1,N_i}, \overline{1,M_j}, \overline{1,L_k}, \overline{1,N_t}, \overline{1,N\Delta \tau} \}

and replace the differential operators of equation (3) with difference operators in \(Ox\):

\[
\frac{T_{i,j,k}^{n+1} - T_{i,j,k}^n}{\Delta \tau} + \frac{T_{i+1,j,k}^n - T_{i,j,k}^n}{\Delta \tau} = a_{i,j,k} \left( T_{i+1,j,k}^n - a_{i-1,j,k} T_{i-1,j,k}^n \right) + \frac{\Delta x}{\Delta y} T_{i,j+1,k}^n - \left( a_{i,j+1,k} + a_{i,j-1,k} \right) T_{i,j,k}^n + \frac{\Delta x}{\Delta y} T_{i+1,j,k}^n + a_{i,j,k} T_{i-1,j,k}^n
\]

\[
\frac{T_{i,j+1,k}^n - T_{i,j,k}^n}{\Delta \tau} + \frac{T_{i,j+1,k}^n - T_{i,j,k}^n}{\Delta \tau} = a_{i,j,k} \left( T_{i,j+1,k}^n - a_{i,j-1,k} T_{i,j-1,k}^n \right) + \frac{\Delta x}{\Delta y} T_{i,j+1,k}^n - \left( a_{i,j+1,k} + a_{i,j-1,k} \right) T_{i,j,k}^n + \frac{\Delta x}{\Delta y} T_{i,j+1,k}^n + a_{i,j,k} T_{i,j-1,k}^n
\]
Applying the sweep method for the sequence at $N$, $N-1$ and $N-2$, we find $T^{n+1}_{N-ijk}$ and $T^{n+1}_{N-ijk}$:

$$T^{n+1}_{N-ijk} = \alpha_{T:N-ijk} T^{n+1}_{N-ijk} + \beta_{T:N-ijk}$$

where

$$\varphi = \eta \rho \gamma R(\tau)$$

Further, we approximate the boundary condition (6) with respect to $Ox$ and obtain:

$$\frac{\Delta}{\Delta x} \left( T^{n+1}_{ij} - T^{n+1}_{ij} - T^{n+1}_{ij} \right) = -\beta T_{oc} + \beta T^{n+1}_{ij} - \varphi^{n+1}, \quad (20)$$

where $\varphi = \eta \rho \gamma R(\tau)$.
Putting $T_{N-1,j,k}^{n+1}$ from (24) and $T_{N-1,j,k}^{n+1}$ from (25) to (23), we find $T_{N,j,k}^{n+1}$:

$$T_{N,j,k}^{n+1} = -\lambda \alpha_{T,N-1,j,k} \beta_{T,N-1,j,k} - \lambda \beta_{T,N-1,j,k} + \lambda \alpha_{T,N-1,j,k} \beta_{T,N-1,j,k} - \Delta x \beta T_0 - \Delta x \varphi$$

The values of the temperature sequence $T_{N+1,j,k}^{n+1}$, $T_{N-1,j,k}^{n+1}$, ..., $T_{j,k}^{n+1}$ are determined by the method of back-sweep by decreasing $i$:

$$T_{j,k}^{n+1} = \alpha_{T,j,k} T_{j+1,k}^{n+1} + \beta_{T,j,k}$$

Similarly, equation (4) is approximated by $O \times$ finite difference relations and grouping similar terms, we obtain a system of tridiagonal algebraic equations with respect to the required variables:

$$a_{u,j,k} u_{j-1,j,k}^{n+1} - b_{u,j,k} u_{j,j,k}^{n+1} + c_{u,j,k} u_{j+1,j,k}^{n+1} = -d_{u,j,k}$$

Further, we approximate the boundary condition (12) with the second order of accuracy in $O \times$ and obtain:

$$\lambda_{\Delta x} u_{j,j,k}^{n+1} + u_{j,j,k}^{n+1} - u_{j,j,k}^{n+1} = -\beta \left( u_{oc} + \beta u_{j,j,k}^{n+1} \right)$$

From the system of equations (28), for $i=1$, we get:

$$a_{u,j,k} u_{j,j,k}^{n+1} - b_{u,j,k} u_{j,j,k}^{n+1} + c_{u,j,k} u_{j,j,k}^{n+1} = -d_{u,j,k}$$

Putting $u_{j,j,k}^{n+1}$ from (30) into (29), we find the value of $u_{j,j,k}^{n+1}$:

$$u_{j,j,k}^{n+1} = \alpha_{u,j,k} u_{j,j,k}^{n+1} + \beta_{u,j,k}$$

From relation (31), the sweep coefficients are determined as:

$$\alpha_{u,j,k} = \frac{\lambda_{\Delta x} \lambda c_{u,j,k} - \lambda \lambda c_{u,j,k}}{a_{u,j,k} \lambda c_{u,j,k} - \lambda c_{u,j,k} \lambda c_{u,j,k} \beta_{u,j,k}}$$

$$\beta_{u,j,k} = \frac{-d_{u,j,k} \lambda - \Delta x c_{u,j,k}}{a_{u,j,k} \lambda - \lambda c_{u,j,k} \lambda c_{u,j,k} \beta_{u,j,k}}$$

Similarly, approximating the boundary condition (13) with respect to $O \times$, we obtain:

$$\lambda_{\Delta x} u_{j,j,k}^{n+1} + u_{j,j,k}^{n+1} - u_{j,j,k}^{n+1} = -\beta \left( u_{oc} + \beta u_{j,j,k}^{n+1} \right)$$

Applying the sweep method for the sequence $N$, $N-1$ and $N-2$, we find $u_{N+1,j,k}^{n+1}$ and $u_{N-1,j,k}^{n+1}$.
\[ u_{i j k}^{n+1} = \alpha_{u_{i j k}} u_{i j k}^n + \beta_{u_{i j k}} \]

Where \( \alpha_{u_{i j k}} \) and \( \beta_{u_{i j k}} \) are the coefficients for the variable \( u_{i j k} \). Putting \( u_{i j k}^{n+1} \) from (33) and \( u_{i j k}^n \) from (34) to (32), we find \( u_{i j k}^n \):

\[ u_{i j k}^n = \frac{-\lambda - \Delta x \beta + \lambda \alpha_{u_{i j k}} - \Delta z \beta}{\lambda - \Delta z \beta + \lambda \alpha_{u_{i j k}}} \]

The values of the moisture sequence \( u_{i j k}^{n+1}, u_{i j k}^n, \ldots, u_{i j k} \) are determined by the back-sweep method to decrease \( i \):

\[ u_{i j k} = \alpha_{u_{i j k}} u_{i j k}^{n+1} + \beta_{u_{i j k}} \]

Further, a similar action is performed by \( O_y \) and then on \( O_z \), we find \( T_{i j k}^{n+1} \):

\[ T_{i j k}^{n+1} = \frac{-\lambda - \Delta z \beta + \lambda \alpha_{T_{i j k}} - \Delta z \omega_{u_{i j k}}}{\lambda - \Delta z \beta + \lambda \alpha_{T_{i j k}}} \]

Accordingly, we get \( u_{i j k}^{n+1} \):

\[ u_{i j k}^{n+1} = \frac{-\lambda - \Delta z \beta + \lambda \alpha_{u_{i j k}} - \Delta z \beta}{\lambda - \Delta z \beta + \lambda \alpha_{u_{i j k}}} \]

4 Computing experiments and analysis of results

Based on the proposed mathematical models and numerical algorithms, the object-oriented software program "HMT-Calc" was built in C# to monitor and predict the processes of heat and moisture transmission in porous media.

Since the harvesting of raw cotton is seasonal, this raw material is stored in open space for a certain period. Raw cotton is used to form riots, covered with a tarpaulin on top. To enhance ventilation, form a through-the-hole. With long-term storage, the commercial quality of raw cotton is lost, and there are cases of its spontaneous combustion. In this regard, temperature prediction and determination of the maximum allowable storage periods are essential to the storage technology of raw cotton.

Computational experiments were carried out for October 2022 in the Bukhara region, where the average ambient temperature was +25°C (+26°C during the day, +23°C at night), and the average ambient humidity was 36%.
Fig. 1. Change in temperature in the riot of raw cotton in three dimensions after 10 days of storage.

In figures 2–5, layers representing the outcomes of numerical computations done on a computer are displayed to provide a visual representation of changes in temperature and moisture in riots of raw cotton.

Fig. 2. Shows the humidity change in a riot of raw cotton after 10 days in a layer at z=5 m.

$$T_{oc} = C \cdot T(x, y, z) = C \cdot u_{oc} = u(x, y, z)$$
Fig. 3. Shows the temperature change in a riot of raw cotton after 10 days in a layer at z=5 m.

\[ T_{oc} = C_T(x, y, z) = C_u(x, y, z) = \]

Numerical experiments were carried out at various values of thermal diffusivity, moisture conductivity, various values of humidity and temperature of cotton riot including its properties. The size of the riot of raw cotton is taken as \( L_x = L_y = L_z = m \).

The cotton riot is erected so that the large sides of the rectangle are parallel to the north-south lines.

The results of the computational experiments on the change in temperature and moisture along the plane depending on x and y are shown in figures 2-3, where the temperature rises over time to 48 °C and the humidity inside the riot falls to 42%.

Fig. 4.\\n
\[ T_{oc} = C_T(x, y, z) = C_u(x, y, z) = \]

Figures 4-5 show the results of computational experiments on the change in temperature and moisture along the plane depending on x and y. Based on the results, it can be said that after 30 days of storage, the temperature inside the cotton riot reaches 72°C and the humidity inside the cotton riot will decrease to 52%.
According to computational experiments, the cotton riot peak temperature after 50 days rises by at least 15°C in comparison to the original value due to the intensification of heat transmission between the cotton riot and the surrounding atmospheric air. As a result, the maximum temperature at specific locations within the porous mass of the raw cotton riot can reach 90.7 °C, which is too high in terms of storage technology.

Fig. 5. Shows the temperature change in a riot of raw cotton after 30 days in a layer at z=6 m.

During long-term storage of raw cotton, the essential features of the fibre change, for example, density, moisture, contamination, oil content, germination, etc., resulting in a decrease in the quality of the fibre. The distribution of heat and moisture in raw cotton represents a great theoretical and practical need for solving the problems of storage and drying.

5 Conclusion

In order to study, predict, and make management decisions regarding the storage and processing of raw cotton, a multidimensional mathematical model and numerical algorithm for resolving the issue of joint heat and moisture transfer in heterogeneous porous bodies have been developed. These models and algorithms take into account internal heat and moisture release as well as heat and moisture exchange with the environment.

The findings demonstrated that one cannot ignore the significance of internal heat and moisture release when considering the initial moisture content of the raw cotton mass and the length of storage because these factors cause debate and self-ignition, which have a significant negative impact on the quality of the cotton fibre.

According to experimental investigations, when the temperature and relative humidity of raw cotton approach 80°C and 24%, respectively, the quality of the cotton fibre starts to deteriorate.

References