Heat exchange in fuel rods at different cross sections

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Abstract. The work is devoted to the process of heat exchange in fuel rods of different cross-sections. It considers a stationary process without internal heat sources under boundary conditions of the first kind. The nature of the change in the temperature field during the transition from a cylindrical to an elliptical section is shown. A formula was also obtained for finding the temperature in a fuel element formed by elliptical surfaces of co-focal ellipses.

1 Introduction

In the theory of heat transfer, one of the main tasks is to determine the temperature field, thereby determining the spatio-temporal distribution of temperature in the area under study. The distribution is determined by the differential equation of thermal conductivity, which follows from the law of conservation and transformation of energy.

As you know, the fuel cell is the main element of the core of any nuclear reactor. However, the presence of nuclear fuel requires special attention and caution when operating it. Therefore, one of the key points of the NPP safety analysis is the investigation of the occurrence of emergency processes and confirmation that during the accident the main parameters do not exceed the permissible limits [1-7].

A unified approach to the thermal calculation of various nuclear reactors is explained by the presence of internal heat generation in them due to a nuclear reaction [8-10]. The problem of the thermal conductivity of a cylindrical wall is of great practical interest. The solution of such a problem makes it possible to calculate heat transfer in various types of heat exchangers [11-16].

It is not surprising, therefore, that the theory of heat transfer has developed intensively, especially in recent decades. This is due to the needs of thermal power engineering, nuclear energy, and cosmonautics. The intensification of various technological processes, as well as the creation of optimal installations from the point of view of energy consumption, is unthinkable without a deep study of the thermophysical processes that take place in these installations. In connection with the improvement of the thermal equipment of energy-consuming and producing devices, a more accurate calculation of heat transfer processes in thermal networks is required. Therefore, it seems advisable to improve the methods of calculating heat transfer in such systems. It is known that for better cooling of fuel elements (electrical conductors, rods of nuclear reactors, etc.), it is necessary to have a large heat

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transfer surface. An increase in the surface can be achieved either by finning, or by replacing rods of circular cross-section, which have a minimum surface of the heat sink, with rods of another cross-section, for example, oval or elliptical. Bodies with an elliptical cross-section occupy a special place. Their peculiarity lies in the fact that by manipulating the change in the length of the semi-axes of the ellipse, it is possible to obtain accurate analytical solutions to stationary thermal conductivity problems for a very wide range of shape changes: from a cylinder to a thin plate.

To perform such a calculation, data on thermal resistances on the outer surface of the foam are required. At the same time, cooling must be effective so that the temperatures of the materials of the structural elements are acceptable. There are many computer programs for modelling processes in the core of a nuclear reactor.

2 Main part

The main fuel elements traditionally used in nuclear power plants can be classified into three types, namely a plate-type element (all shapes), a cylindrical element elongated in the direction of the axis (usually having a round or annular section), which forms a rod element, and a spherical element, usually in the form of a small particle diameter.

Cylindrical fuel elements are, for example, cylindrical containers with nuclear fuel used in graphite-gas reactors, rods used in water-water power reactors, or rod-type fuel elements in fast neutron nuclear reactors.

In order to find the distribution of the temperature field, it is necessary to solve the equation of thermal conductivity. In the case of a cylindrical wall, the equation has the form:

\[ \frac{1}{r} \frac{d}{dr} \left( r \frac{dT}{dr} \right) = 0 \]  \hspace{1cm} (1)

where from:

\[ \frac{d}{dr} \left( r \frac{dT}{dr} \right) = 0 \]  \hspace{1cm} (2)

We will look for a solution at the given values of the temperatures of the inner and outer surfaces. Let's integrate both parts of the equation:

\[ r \frac{dT}{dr} = C \]  \hspace{1cm} (3)

or:

\[ \frac{dT}{dr} = \frac{C}{r} \]  \hspace{1cm} (4)

Reintegrate:

\[ \int_{t_i}^{t} dT = \int_{r_i}^{r} \frac{C dr}{r} \]  \hspace{1cm} (5)

where from:
We find the integration constant $C$ from the condition that for $r = r_2, t = t_2$:

$$t_2 - t_1 = C \ln \left( \frac{r_2}{r_1} \right)$$

or:

$$C = \frac{t_2 - t_1}{\ln \left( \frac{r_2}{r_1} \right)}$$

Finally we get:

$$t = t_1 + \frac{t_2 - t_1}{\ln \left( \frac{r_2}{r_1} \right)} \ln \left( \frac{r}{r_1} \right)$$

Thus, the temperature dependence on the radius in the cylindrical wall is represented as a logarithmic curve (fig. 1).

**Fig. 1.** Temperature dependence on the radius in the cylindrical wall.

To determine the temperature field in an elliptical section fuel element, we will consider sophocal ellipses for the sake of calculations. At the same time, the problem is solved by the fact that there are no internal heat sources. The heat-conduction equation in the Cartesian coordinate system has the form:
\[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \]  
\[(10)\]

Then we will look for the solution itself in the form:
\[ t = C_1 \left( \frac{x^2}{a^2} + \frac{y^2}{b^2} \right) + C_2 \]
\[(11)\]

We will find constants from the following boundary conditions of the condition. The temperature \( t_1 \) is set on the inner surface \( (\alpha_1) \):
\[ \alpha_1 : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \]
\[(12)\]

substituting (12) into (11), we get:
\[ t_1 = C_1 + C_2 \]
\[(13)\]

where from:
\[ C_2 = t_1 - C_1 \]
\[(14)\]

then the law of temperature distribution will take the form:
\[ t = t_1 + C_1 \left( \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 \right) \]
\[(15)\]

The temperature \( t_2 \) is set on the outer surface \( (\alpha_2) \):
\[ \alpha_2 : \frac{x^2}{a_1^2} + \frac{y^2}{b_1^2} = 1 \]
\[(16)\]

Since the ellipses are so perceptual, when moving from the inner to the outer surface:
\[ a_1 = ka \quad b_1 = kb \]
\[(17)\]

When substituting expression (17) into (16), we get:
\[ \alpha_2 : \frac{x^2}{a^2 k^2} + \frac{y^2}{b^2 k^2} = 1 \]
\[(18)\]

then:
\[ t_2 = t_1 + C_1 \left( k^2 - 1 \right) \]
\[(19)\]

From where the second constant is equal to
\[ C_1 = \frac{t_2 - t_1}{k^2 - 1} \]
\[(20)\]

Finally we get:
Thus, a formula was obtained for finding the temperature in the fuel element formed by elliptical surfaces of sophocal ellipses. In order to determine the nature of the behaviour of the temperature field, we need to consider points belonging to elliptical contours inside the fuel element itself:

\[
a_2 = ra \quad b_2 = rb
\]

where \(1 < r < b\). The expression in this case (21) will take the form

\[
t = t_1 + \frac{t_2 - t_1}{k^2 - 1} \left( \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 \right)
\]

It follows from expression (23) that the dependence of the temperature field is characterized as a parabolic curve (fig. 2).

**Fig. 2.** Elliptical section fuel element temperature distribution.

### 3 Conclusion

Thus, the work is devoted to the issues of heat transfer in fuel rods under boundary conditions of the first kind. Two sections were considered, as well as the nature of the behaviour of the temperature field during the transition from one section to another. A formula was also obtained for finding the temperature in the fuel element formed by elliptical surfaces of sophocal ellipses. The result obtained can be useful for further theoretical research in this field.

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