Features of the interaction of stationary oscillating charges

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Abstract. Systems of stationary oscillators determine the structure of a large number of natural materials by the model of many physical processes. In particular, they are the main physical model of the energy transfer mechanism in communication lines and power transmission systems. The interaction of oscillators usually leads to a change in their mode of oscillation, in particular, to synchronization of oscillations. In the present work, the possibilities of another effect of the interaction of oscillators are considered, which, at a stable frequency of oscillations of each of the system of coherent oscillators, leads to their mutual displacement to the points of stable equilibrium. It has been established that the violation of the coherence of the radiation of oscillators leads to the motion of both individual oscillators and a system of oscillators. Conditions for the equilibrium state of a system of oscillators are obtained.

1 Introduction

The physical properties of individual oscillators and systems of oscillators of various nature are described in the literature [1–3] and relatively well studied in classical and quantum [4,5] systems. They are of great practical interest in terms of many properties: physical, chemical [6, 7], photochemical [8], with local or distributed communication [9], biological [10–12], medical [12] parameters, etc. Attention is paid to the observed effect of synchronization of harmonic [13–15] and anharmonic [16] oscillators. Features of synchronization of an ensemble of phase oscillators under the action of general noise and global coupling are considered in [17–20]. For non-identical oscillators, a state of quasi-synchronization occurs. The effect of incoherence is considered in [20–22]. The physical mechanism of this effect is still unclear [22]. The interaction of a system of oscillators essentially depends on the physical properties of these oscillators:

1) in the first case, the interaction of oscillators causes a change in the parameters of each,
2) in the second case, the intrinsic parameters of the oscillators do not depend on external conditions and are determined only by internal properties. In this case, the interaction of oscillators causes their mutual movement in the region where their interaction is minimal (in this case, the potential energy of the system is minimal). In this paper, we consider for the first time the properties of the second configuration, the interaction of coherent and quasi-coherent oscillators with a stable oscillation frequency.

2 Materials and methods

The paper investigates the interaction of stationary charge oscillators in the form of charges with Coulomb coupling. Consider the interaction of two oscillators located at a distance \( r \) from each other (Fig. 1). The interaction of charges is described by the Coulomb law:

\[
F = k \frac{q_1 q_2}{r^2}
\]

At the same time, like charges stationary in time repel each other, and unlike charges attract.

Fig. 1. Interaction of two charges

Consider now non-stationary charges. Let first the value of the first charge change according to the harmonic law, while the second charge has a constant value:

\[
q_1 = q_1 \cos(\omega t + \phi_1) \quad q_2 = \text{const}
\]

In this case, the force acting on the second charge from the first charge changes according to the harmonic law:

\[
F = k \frac{q_1 \cos(\omega t) - kr + \phi_1}{r^2}
\]

Where

\[
k = \frac{\omega}{v} = \frac{\pi}{\lambda}
\]

Now let's assume that both charges change according to the harmonic law with the same frequency (coherent oscillators) and are described by the functions:

\[
q_1 = q_1 \cos(\omega t + \phi_1) \quad q_2 = q_2 \cos(\omega t + \phi_2)
\]

In this case, the force acting on the second charge from the first charge at the values of the initial phases \( \phi_1, \phi_2 \) is equal to:

\[
F = k \frac{q_1 \cos(\omega t) - kr + \phi_1}{r^2}
\]
This relation takes into account the delay of the field created by the first charge in the region where the second charge is located (the passage of the electric field of interaction of distance \( r \)). The transformation gives the force impact ratio:

\[
F = \frac{k q}{r^2} \left\{ \sin(kr + \phi_r - \phi_r) + \sin(\omega t - kr + \phi_r + \phi_r) \right\}
\]

Thus, the force acting on the second charge from the first charge includes a constant component and a wave action with a frequency \( \omega \), and the interaction of oscillators depends significantly on the ratio of the phases of oscillations of independent oscillators.

### 3 Main results and discussion

**Condition for the stability of a system of stationary oscillators**

The first component of the impact force on the second oscillator is equal to zero when the condition is met:

\[
kr + \phi_r - \phi_r = (n + \frac{1}{2})\pi
\]

where \( n = 0, 1, 2, \ldots \)

\[
r_n = (n + \frac{1}{2})\lambda + \frac{\phi_r - \phi_r}{\pi}\lambda
\]

Thus, when the distances between the charges are equal to the values \( r_n \), a stable equilibrium of the system of two coherent oscillating charges is observed. In this case, the state with the smallest value \( r_n \) is the most stable. Points of stable position are shifted relative to each other at a distance \( \sqrt{2}\lambda \).

When the distance between the charges is smaller \( r < r_n \), the time constant component of the force has a positive value. In this case, the interaction of like charges is observed. With a distance between the oscillators \( r \geq r_n \), this component of the force changes direction, becomes a negative value (under these conditions, the interaction of charges of the opposite sign is observed). This force acts on the charge up to the establishment of a stable equilibrium distance between the charges \( r = r_n \). The first oscillator is subjected to the same influence from the side of the second oscillator. Thus, there is a countable number of distances of stable states of a system of two coherent oscillators. When the frequency of charge oscillations changes, the distance of stable equilibrium also changes (in the visible range in free space in the range from 0.2 µm to 0.1 µm). The coordinates of the stable equilibrium depend on the initial phases of the oscillators' oscillations and can be regulated within wide limits.
Unlike constant charges of the same name, where the force of interaction of charges is always directed in one direction at any distance, in a system of synchronously oscillating charges at distances between them in the intervals of values $r_5, r_4, r_3, r_2, r_1, r_0$, repulsion of oscillators is observed. At distances between charges in the ranges of values $r_5, r_4, r_3, r_2, r_1, r_0$, the repulsion of oscillators changes into their mutual attraction. The process of establishing a stable equilibrium is oscillatory, described by the equation:

$$ma + \frac{k\cdot q}{r}\{\cos(kr)\} = \cdots$$

It follows that the oscillation process is not harmonic.

Let us consider another particular case, when the difference between the initial phases of the oscillators is $\varphi_2 - \varphi_1 = \frac{\pi}{2}$. In this case we have:

$$F = \frac{k\cdot q}{r}\left\{\cos\left(kr + \frac{\pi}{2}\right) + \cos\left(\omega t - kr + \varphi_2 + \frac{\pi}{2}\right)\right\} =$$

$$= \frac{k\cdot q}{r}\{\sin(kr) + \sin(\omega t - kr + \varphi_2)\}$$

$$r_n = \frac{\lambda}{\pi}$$
The first term of the force makes it possible to determine the position of the stable equilibrium of the oscillators. At this force component is equal to zero at the coordinates moving in time at a speed in the direction of an oscillator with a lower frequency. If we consider the force acting on the first oscillator from the second, then we can show that a force also acts on it, tending to move it in the same direction in which the second oscillator moves. Thus, the frequency difference of the oscillators, leading to a linear change in time of the phase difference of the oscillators, leads to the movement of the entire system of oscillators in the direction of oscillators with a lower frequency (to the region of stable equilibrium). The second term characterizes the wave with the frequency created by the first oscillator in the region where the second oscillator is located. Thus, if at the system can be in a stable stationary state, then at the system of oscillators will move in the direction of the stable position of the oscillators, which itself is constantly shifting at a constant speed determined by the frequency difference of the oscillators.

\[
F = k \frac{q}{r} \left[ (\omega + \Delta \omega) t - (k + \Delta k) r + \phi \right] = k \frac{q}{r} \left\{ \Delta \omega t - (k + \Delta k) r + \phi - \phi' + \left[ (\omega + \Delta \omega) t - (k + \Delta k) r + \phi + \phi' \right] \right\}
\]

\[
\Delta \omega t - (k + \Delta k) r + \phi - \phi' = \left( n + 1 \right) \pi
\]

\[
r_n = \frac{\Delta \omega + \phi - \phi' - \left( n + 1 \right) \pi}{(k + \Delta k)}
\]

\[
\nu = \frac{\Delta \omega}{k + \Delta k}
\]
If the oscillators are located along the same line, the first-second and second-third can be in pairs with each other at stable equilibrium distances \( r = r_1 \) and do not interact. However, the distance between the first and third charges is equal to \( 2r_1 \), at which the charges, although with less force due to the increased distance, interact, displacing the first and third oscillators from their positions of stable equilibrium. In general, the system of oscillators becomes less stable.

If the oscillators are located at the vertices of an equilateral triangle (Figure 2b), then the distance between each pair of oscillators can correspond to the distance of stable equilibrium. The entire system of oscillators is in a state of stable equilibrium, forming a triangle configuration, the shape of which will depend on the initial phases of the oscillators. For example, with equal initial phases, an equilateral triangle is formed. If the initial phase of one of the oscillators changes, then the distance of stable equilibrium from this oscillator to neighboring oscillators will also change, the distance between which will remain the same. The triangle is deformed to maintain a stable state (Fig. 3).

In this case, the third (external) oscillator, due to an increase or decrease in the oscillation frequency, can cause the entire system to move towards this oscillator (at \( \omega_1 = \omega_2 = \omega \) or \( \omega_3 < \omega_2 = \omega_1 \)). The system of coherent oscillators located at the vertices of the octahedron also has a stable form, since pairwise interaction of oscillators is observed at equal stability distances. In a system of four oscillators located at the vertices of a tetrahedron (Fig. 3), by controlling the oscillation frequencies, it is possible to control the movement of the system in four directions. A change in the frequency of an oscillator can, as in mechanical oscillators, be associated with the magnitude of its mass, i.e. a more massive oscillator may have oscillation frequencies lower than those of other oscillators and thus be an "attraction" region.
The system of a large number of oscillators is shown in Fig. 5. An oscillator is highlighted in the center, from which 6 oscillators are located on the line of the first circle (1) at distances of pairwise stable equilibrium. On the second circle, the oscillators are located at distances that do not correspond to the distances of stable equilibrium and interact with the central oscillator, but these interactions are compensated by the symmetrical arrangement of the oscillators (2) relative to the central oscillator. A similar compensated interaction decreasing with distance from the central oscillator is observed with the oscillators of the third, fourth, etc. circles.

Thus, the stationary frequency of oscillations of individual oscillators forms a stationary distribution of oscillators in space. The motion of a stationary system of oscillators can be provided by controlling the oscillation frequency of elementary oscillators.

4 Conclusion
Oscillation of charges changes the nature of their Coulomb interaction: repulsion of oscillating charges is observed only at certain distances between them, depending on their phase difference, in other ranges of values, repulsion changes to attraction. There are distances of stable equilibrium when the oscillating charges do not interact on average over the period of oscillation. The mismatch of the oscillation frequencies leads to the spatial displacement of the system in the direction of oscillators with a lower oscillation frequency. Thus, the movement is a consequence of the phase shift of the interacting oscillators. When the frequencies of the oscillators are mismatched, the phase shift depends on time and a uniform movement is observed, the speed of which depends on the difference in the frequencies of the oscillators. Changing the frequency difference in time leads to accelerated motion. The results obtained are used to model the structure of substances and to create new ways to control the movement of material objects.

References


