The computer modeling and analysis of complex dynamic structures of robotics from composite materials

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Abstract. The methods of designing and modeling dynamic stands from a composite material are considered. One of the complex elements of the stands is the elements that ensure the dynamic behavior of the channels of the stand: bearing supports, gear rims, gearboxes, motors, servo drives, etc. Approximation of these elements in numerical methods for calculating and studying dynamic structures (finite element method) is a difficult task. Direct approximation of these elements does not satisfy the adequacy. Various approaches are used to adequately approximate such elements. In this paper, a methodology is proposed that consists in determining the rigidity parameters of bearing supports, gear rims, gearboxes and motors. In the future, these elements are replaced by rod systems corresponding to the replaced elements in terms of stiffness parameters. The effectiveness of the proposed method is confirmed by a comparison with available experimental methods. The shell structures and stands were modeled in ANSYS modeling module. The considered methods for designing stands can be applied to robotic systems, weaving machines and dynamic systems for various purposes, since they contain all the elements inherent in the considered stand. The finite element approximation of the stand was carried out by elements of various types.

1 Introduction

The purpose of this study is a method for calculating robotic systems from composite materials using the example of calculating a semi-natural simulation bench. The significance of the work is determined by the fact that, until now, stands have not been made from composite materials with higher specific strength characteristics compared to magnesium alloys traditionally used in the manufacture of stands.

For a numerical study of the mechanical behavior and stress-strain state of the stand, it is necessary to develop a methodology for designing a model in one of the automated simulation systems. In the present study, the simulation of the stand was carried out in ANSYS computer-aided design system [1], designed for modeling and analysis. As an object of study, we consider the bench shown in the Figure 1 [2] containing surfaces of double curvature, plates and shells, elements consisting of hollow structures designed to protect electrical equipment and motor units from high-speed air flow from the compressor.
2 Materials and methods

2.1 Equation

\[
\frac{\partial}{\partial t} \frac{\partial T}{\partial q_k} + \frac{\partial U}{\partial q_k} = Q
\]

\( T \) is the kinetic energy of deformation, \( U \) is the potential energy of deformation, \( q \) is the vector of generalized displacements, the index \( k \) is the number of degrees of freedom, \( t \) is time, the dot above the letter means differentiation with respect to time.

2.2 Relationship between stresses-strains

In the present study, the stand material is a composite material [8, 9]. There are two approaches to identify the relationship between stresses and strains for a multilayer composite material, when the base of the layers is located at different angles: the method of reduced stiffnesses and taking into account the dependence for each layer separately. The reduced stiffness method used in this study calculates the generalized characteristics of a multilayer composite material characteristic of a homogeneous material. In the method under consideration, the relationship between stresses and strains for each layer of a multilayer composite material, the number of resolving equations depends on the number of layers of the multilayer composite, which leads to an increase in the resolving equations as for the method of reduced characteristics multiplied by the number of layers of the multilayer composite. This approach leads to an unreasonable large number of resolving equations, an increase in computational errors, and is practically not used when considering large structures, which also includes a multi-degree bench for semi-natural simulation.
2.3 Method for determining the structure of a composite material

The multilayer composite material changes its characteristics depending on the location of the warp of the composite yarns. To obtain the maximum rigidity of a multilayer composite material, it is necessary to position the base of the composite material in the direction of the maximum stresses acting in the structure under study under the action of operational loads. In this case, part of the layers in the multilayer composite material should be made at an angle to the maximum stresses to absorb shear stresses. The ratio of the arrangement of layers in a multilayer composite material is a complex task that requires a large number of theoretical and experimental studies. And it depends on the structure under study and the operating loads.

This study proposes an iterative method for arranging layers of a multilayer composite material along lines of maximum stresses. At the first stage, a model of a structure made of a homogeneous material under the action of operational loads is investigated. As a result of the study, we obtain the trajectories of maximum stresses. At the second stage, the structure model is made of a composite material with the arrangement of layers of a multilayer composite material along the lines of maximum stresses. The trajectories of maximum stresses obtained as a result of the studies make it possible to correct the location of the composite base and carry out the calculation of the structure. In this way, the location of the base of the composite material along the lines of maximum stresses is achieved and, thereby, a model of the structure of maximum strength and rigidity is obtained. Layers with an angle to the trajectories of maximum stresses for the perception of shear stresses are determined from the results of the analysis of the stress-strain state of the structure.

2.4 Modeling technique for gear rims, gearboxes and bearings

In this study, a method for approximating such elements of robotic systems as bearings, gear rims and bearing supports is proposed. The approximation technique is as follows. The rigidity of the specified elements is determined on the basis of the developed program using the formulas of the mechanics of machines and mechanisms. Next, a bar structure of the corresponding stiffness is modeled and the ring gear, bearing or ring gear is replaced in the model with a bar structure of the same stiffness. The performed calculations and studies have shown the validity of this technique. The results of comparison with the available experimental studies showed good agreement.

2.5 Modeling and approximation of the stand

The stand is a three-layer shell structure, consisting of a stand model made of lightweight composite material (Table 1) with outer load-bearing layers of a five-layer composite material with a layer orientation of 0/45/0/45/0 degrees. The stand was approximated by finite elements. Gear rims, bearing supports and gearboxes were approximated by bar structures of appropriate stiffness. The convergence of the results was determined by condensing the number of finite elements: the structure model was divided into \( n \) finite elements, the calculation was carried out, and the results obtained were compared with the results obtained by splitting into \( 2n \) finite elements. If the results differed by no more than 3%, splitting into \( n \) finite elements was considered sufficient to obtain acceptable results.

2.6 Algorithm for determining stiffness

We determine the rigidity of the gearbox using the methods of the theory of machines and mechanisms. One of the main parameters that reducers must satisfy is stiffness, gear ratio
The determination of the stiffness parameters of the gearboxes is carried out experimentally on the gearbox and theoretically. The experimentally determined gearbox parameters correspond most closely to the true stiffness of the gearbox. At the same time, this approach does not allow changing the gearbox parameters if the stiffness does not meet the operating requirements. Therefore, to determine the stiffness and dynamic characteristics of gearboxes, an algorithm was developed, and a program was compiled.

The stiffness parameters of the gearbox are determined using an application program written in Microsoft Excel. The program allows determining gear ratios, total gear ratio, moments of inertia reduced to the shaft, moments on the shaft, angular velocities and accelerations, moments of inertia, angular error, reduced stiffness, natural frequency, stiffness reduced to the input shaft of each shaft separately and gearbox as a whole. The developed program reduces the calculation time by more than 5...7 times in comparison with the analytical calculation. The program allows easily varying the components of the gearbox in order to obtain the optimal model. The reliability of the obtained results was compared with the experimental results. An error (~10%) is acceptable for this type of structure.

To use the program, it is enough to set the input parameters of the constituent parts of the gearbox: bearings, tolerances and fits, acceleration, number of gear teeth, etc.

In accordance with the ideology of the concept of the complex for the design of specific elements of multi-stage dynamic stands, such as gearboxes, the calculation of which cannot be carried out in ANSYS analytical block, a program for calculating gearboxes was developed. The program allows conducting a comprehensive analysis of gearboxes, taking into account moments of inertia, angular velocities, angular accelerations, gear ratios, bearing stiffness and other characteristics, and calculating the stiffness and natural frequency of the gearbox.

For a better understanding of the foregoing, we will describe what a reducer is. The gearbox in multistage dynamic stands is designed to change the angular velocity, accelerations or engine effort. And it is a structure consisting of shafts and gears of various diameters. In turn, each shaft is a multi-stage body of rotation containing gears of various characteristics (number of teeth, diameters, gear ratios, etc.).

When calculating the gearbox, the concept of equivalent diameter is used. A shaft of equivalent diameter is a shaft having a constant diameter along its length, the maximum deflection of which is equal to the deflection of a shaft comprising different diameters. To determine this parameter, a program was developed to determine the equivalent diameter of the gearbox shafts using Excel spreadsheets.

We describe the procedure for determining the equivalent shaft diameter. To determine the equivalent diameter of each shaft of the gearbox, firstly, using a standard program, we solve the problem of determining the reactions of the supports at the points of contact of the shaft gears with other gears, then we set the modulus of elasticity of the shaft material and for each different-sized section of the shaft, the length, outer and inner diameter (if the shaft in this area is hollow). The program automatically calculates the moment of inertia of the shaft sections, its cross section, center of gravity, deflection and, ultimately, the equivalent shaft diameter. Having determined the equivalent diameter of each shaft, we calculate the gearbox as a whole. The calculation of the gearbox is to determine the stiffness of the gearbox and its natural frequency.
2.7 Initial data

Initial data moments of inertia of shafts $J_5, J_4, J_3, J_2, J_1$ (kg $\times$ m $\times$ s$^2$), angular velocity $\omega_5$ (1/s), angular acceleration $\omega_5$ (1/s$^2$) and frequency $f$ (Hz).

Gear ratios $i_{54} = z_8/z_7$, $i_{43} = z_6/z_5$, $i_{32} = z_4/z_2$, $i_{21} = z_2/z_1$, $i_{11} = z_1/z_1$.

Gear ratios related to the input shaft $i_{51} = i_{54} \times i_{43} \times i_{32} \times i_{21} \times i_{11}$, $i_{41} = i_{51} / i_{54}$, $i_{31} = i_{41} / i_{43}$, $i_{21} = i_{31} / i_{32}$, $i_{11} = i_{21} / i_{11}$.

General gear ratio $i_{\text{tot}} = i_{54} \times i_{43} \times i_{32} \times i_{21} \times i_{11}$.

Gear ratio $i_{\text{red}} = i_{43} \times i_{32} \times i_{21} \times i_{11}$.

Moments of inertia referenced to input shaft $J_{51} = J_5 / i_{51}^2$, $J_{41} = J_4 / i_{41}^2$, $J_{31} = J_3 / i_{31}^2$, $J_{21} = J_2 / i_{21}^2$, $J_{11} = J_1 / i_{11}^2$.

Shaft accelerations $\omega_4 = \omega_5 \times i_{54}$, $\omega_3 = \omega_4 \times i_{43}$, $\omega_2 = \omega_3 \times i_{32}$, $\omega_1 = \omega_2 \times i_{21}$.

Moments on the shaft $M_5$, $M_4 = J_4 \times \omega_4 + M_5 / i_{54}$, $M_3 = J_3 \times \omega_3 + M_4 / i_{43}$, $M_2 = J_2 \times \omega_2 + M_3 / i_{32}$, $M_1 = J_1 \times \omega_1 + M_2 / i_{21}$.

2.8 Shaft load

$R_{\text{ocr}} = M_j / m \times z$, $R_{\text{rad}} = R_{\text{ocr}} \times \tan^2 \alpha$, where $m$ is the module, $z$ is the number of teeth and $M_j$ is the moment on the shaft.

Reactions of supports $R_1$ and $R_2$ are determined by methods of resistance of materials, taking the moment of inertia of the shaft section $J_{\text{shaft}} = 3.14 \times D_{\text{eq}}^4 / 64$ based on $D_{\text{eqv}}$ - equivalent constant diameter for bending.

Displacements and angles of rotation at the points of application of force and supports, respectively, are determined by BEAM program.

2.9 Deformation of the bearing from fit on the shaft and into the housing

$g_r = g_{r0} - \Delta d_1 - \Delta D_2$, where $g_{r0}$ is the initial clearance of the ball bearing, $\Delta d_1$ is the increase in the outer diameter of the inner ring from the fit on the shaft, $\Delta D_2$ is the decrease in the inner diameter of the outer ring from the fit into the housing are determined from the graphs [9], depending on the ratio of the stiffness of the mating parts:

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2.10 Bearing deformation under load

\[ \delta = \delta_1 + \delta_2 \]

\( \delta_1 \) is the deformation in contact of the most loaded rolling element with the raceway in the bearing, \( \delta_2 \) is the radial compliance in contact of the bearing rings with the shaft and housing mounting surfaces.

2.11 Radial compliance in contact of bearing rings with shaft and housing seating surfaces

\[ \delta_{r2} = \frac{4 \times R \times k}{\pi \times d \times B} \left( 1 + \frac{d_1}{D} \right) \]

2.12 Calculation of the rotation of the gear wheel caused by the elastic deflection of the shaft

\[ \phi_{defl} = \frac{2}{m \times Z} \times (w_{circ} + w_{rad} \times tg20^\circ) \]
2.13 Calculation of error due to shaft twist

\[ \phi_{tw} = \sum \left( \frac{M_i}{G \cdot J_p} \right) \times \text{radian} \]

where \( G \) are the shear modulus, \( J_p = \pi \times \left( \frac{D^4 - d^4}{32} \right) \) is the polar moment of inertia, \( l_i \) is the length of twisted section, \( M \) is the moment on the shaft.

2.14 Calculation of the error caused by the compliance of the ring gear

\[ \phi_{tooth} = \frac{M}{k \times d^2 \times b} \times \text{radian} \]

Here \( d \) is the pitch diameter of the gear rim, \( b \) is the working width of the gear rim and \( k = 368 \text{kg/mm}^2 \) is the experimental coefficient.

2.15 Calculation of error due to keyed connection

\[ \phi_{shp} = \frac{M}{k_{shp} \times d^2 \times b \times l \times h \times z} \times \text{radian} \]

where \( d_b \) is the shaft diameter, \( l \) is the working length of the key, \( h \) is the height of the key, \( z \) is the number of keys, \( k_{shp} = 15 \text{kg/mm}^3 \) for parallel keys and \( k_{shp} = 25 \text{kg/mm}^3 \) for segmented keys.

2.16 Error due to pinning deformation

\[ \phi_{sht} = k_{sht} \times M \times \text{radian} \]

where \( k_{sht} \) is the coefficient of proportionality, \( M \) is the moment on the shaft [10].

2.17 Total gear stiffness

\[ C = \left\{ \sum_{j=1}^{n} \left[ \frac{\phi_{defl} + \phi_{tw} + \phi_{tooth} + \phi_{shp} + \phi_{sht}}{M_j \times i_j^2} \right] \right\}^{-1} \times \text{kg} \times \text{m} \times \text{rad} \]

2.18 Natural frequency of the design

\[ f = \frac{1}{\pi} \times \sqrt{\frac{C \times i_{total}^2}{i_5}}, \text{Hz} \]

The resulting rigidity of thrust bearings, gearboxes and gear rims make it possible to replace them with a rod system identical in stiffness to the calculated parameters: gear rim at the connection of the fork and base, thrust bearings at the junction of the pitch ring with the course fork and gear wheel at the junction of the roll ring in the pitch channel. The reducers connect the motors with the movement elements of the bench channels.
Tables 1, 2 and 3 show the characteristics of the materials used: SAN Foam, carbon fiber and magnesium alloy.

**Table 1.** Physical and mechanical characteristics of SAN Foam

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
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<tbody>
<tr>
<td>Modulus of elasticity, MPa</td>
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<tr>
<td>Poisson's ratio</td>
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<tr>
<td>Shear Modulus, MPa</td>
<td>23</td>
</tr>
<tr>
<td>Tensile strength, MPa</td>
<td>1.1</td>
</tr>
<tr>
<td>Shear strength, MPa</td>
<td>0.8</td>
</tr>
<tr>
<td>Shear 0z, MPa</td>
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</tr>
<tr>
<td>Density, kg/m³</td>
<td>81</td>
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</tbody>
</table>

**Table 2.** Physical and mechanical characteristics of carbon fiber

<table>
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<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Modulus of elasticity, MPa</td>
<td>$E = 1.233 \times 10^5$</td>
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<tr>
<td>Poisson's ratio</td>
<td>$\nu = 0.27$</td>
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<tr>
<td>Shear modulus, MPa</td>
<td>$G = 0.5 \times 10^4$</td>
</tr>
<tr>
<td>Tensile strength, MPa</td>
<td>$\sigma_s = 1632$</td>
</tr>
<tr>
<td>Ultimate compressive strength, MPa</td>
<td>$\sigma_u = 704$</td>
</tr>
<tr>
<td>Shear strength, MPa</td>
<td>$\tau_s = 80$</td>
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<tr>
<td>Density, kg/m³</td>
<td>$1518$</td>
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**Table 3.** Physical and mechanical characteristics of magnesium alloy

<table>
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<th>Property</th>
<th>Value</th>
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<tr>
<td>Modulus of elasticity, MPa</td>
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<tr>
<td>Poisson's ratio</td>
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<td>Density, kg/m³</td>
<td>1740</td>
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<tr>
<td>Tensile strength, compression, MPa</td>
<td>250</td>
</tr>
<tr>
<td>Tensile strength, compression, MPa</td>
<td>190</td>
</tr>
</tbody>
</table>

**3 Results and Discussion**
The stand was modeled in ANSYS computer-aided design system, and the bench was approximated by 420510 finite elements [14, 15]. Let us consider the procedure for modeling a stand of a three-layer structure made of a composite material.

- Modeling of the bench in CATIA, assigning material characteristics for the structure, which is a lightweight porous material such as foam. This material serves as a light intermediate layer in a three-layer structure between the carrier layers, preventing the bearing layers from approaching and absorbing mainly shear stresses.
- Next, we create surface elements for the entire structure, to which we further assign the characteristics of a multilayer composite package of five unidirectional layers with a given orientation: 0/45/0/-45/0 degrees [16-26].

![Finite element approximation of the stand model and the direction of the stand movement from the angular operating speed of 500 deg/s applied to the course fork](image)

Figure 4. Finite element approximation of the stand model and the direction of the stand movement from the angular operating speed of 500 deg/s applied to the course fork.

Figure 5 shows the stresses in the stand at an angular velocity of 500 deg/s calculated...
Fig. 5. Stresses in the stand at an angular velocity of 500 deg/s
Stresses in the 3rd and 4th layers of the composite material

Stresses in the 5th layer of the composite material

Fig. 6. The stresses of a composite package of five unidirectional layers with a given orientation: 0/45/0 degrees layer by layer.

On figure 7 shows the maximum stresses in a five-layer package. Here it is indicated: e - deformation, 2 - direction, e - tension, the number in brackets means the number of the material.

Fig. 6. 所示的复合材料包层的应力分布。图中显示了0/45/0度的层间应力分布，以及包含在复合材料包层中的五层不同材料的应力情况。
4 Conclusions
5 Acknowledgements

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