Calculation of reliability of a centrally compressed column with a cross-corrugated wall

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Abstract. Issues of safe operation of buildings and structures in the Russian Federation are regulated by various legal and regulatory and technical documents, including Federal Law No. 384 "Technical Regulations on the safety of buildings and structures", to which is attached a list of documents with the status of mandatory, including GOST 27751-2014 "Reliability of building structures and foundations. The main provisions". In this normative document, it is proposed to calculate the reliability of building structures using probabilistic and statistical methods that can be used only in cases where there is complete, homogeneous and statistically independent information about the controlled parameters, while there are no instructions on how to calculate the reliability of building structures if it is impossible to obtain statistical information as a result of the survey and monitoring of building structures. information on controlled parameters, allowing to reliably identify the distribution law and its parameters. The article proposes a particular method for calculating the reliability of a centrally compressed column made of an I-beam with a cross-corrugated wall according to the stability criterion, in which part of the controlled parameters are considered as random variables with complete statistical information, and the other part of the parameters having incomplete statistical information is considered as fuzzy variables. The combination of random variables and fuzzy variables in calculating the reliability of the column made it possible to more fully take into account statistical information about the controlled parameters.

1 Introduction

Collapses of buildings and structures, including individual building structures, are serious man-made emergencies, as a result of which not only material damage can be caused, which can amount to significant sums of money, but also damage caused to human life and health, and in the most unfavorable case, their death is possible. At the same time, one should not forget about one more important point: accidents of building structures can cause such emergencies as fires, explosions, etc., and as a result: the damage caused can be increased many times, and the consequences will be even more tragic. To date, both in Russia and in the world, a large number of emergencies have already occurred and continue to occur,

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related to accidents of building structures leading to the collapse of buildings and structures. Examples of such collapses include the following: the collapse of the Sampoong Shopping center (1995, South Korea); the collapse of the roof of the International Fair Trade and Exhibition Complex (2006, Katowice, Poland); the collapse of the Pavlov House (2010, St. Petersburg, Russia); the collapse of the Rana Plaza complex (2013, Savar-Upazile, Bangladesh); the collapse of the roof of the Maxima shopping center (2013, Riga, Latvia); the collapse of the building of the Airborne Training center (2015, Omsk, Russia); the collapse of two houses No. 63, No. 65 (2018, Marseille, France), etc. In order to exclude and prevent the occurrence of such emergencies, improve the quality and safety of construction products, including buildings and structures, protect the life and health of citizens of the Russian Federation, property, etc., Federal Law No. 384 "Technical Regulations on the Safety of Buildings and Structures" was adopted on the territory of the Russian Federation in 2009, in accordance with with which the technical regulation of the construction industry is carried out. Two lists of documents on the basis of mandatory and voluntary use are attached to this federal law. The updated list of documents with the status of mandatory for use was approved on May 20, 2022 by the Decree of the Government of the Russian Federation No. 914 and entered into force on September 1, 2022. According to this decree, there are only five regulatory and technical documents in the construction industry that are mandatory for use, including GOST 27751-2014 "Reliability of building structures and foundations. The main provisions". In this normative and technical document, to determine the reliability of building structures, it is recommended to use probabilistic and statistical methods, an important condition for the use of which is the mandatory availability of complete statistical information about the controlled parameters, i.e. the information must be homogeneous, statistically independent, and sufficient to conduct its statistical analysis, identify the distribution law of a random variable and the parameters of this distributions. The use of mathematical analysis methods to assess the reliability of building structures began quite a long time ago and was presented in the scientific works of Russian and foreign scientists. The earliest works in which attempts were made to use probability theory and, which, later, served as the beginning of the development of the theory of calculating the reliability of building structures, are the developments of the German scientist M. Mayer, the Russian engineer N. F. Khotsialov, the Soviet scientist N.S. Streletsky. Further development of the theory of calculating the reliability of building structures based on probabilistic and statistical methods is presented in the works of V.M. Keldysh, I.I. Goldenblat, A.A. Gvozdev, A. R. Rzhanitsyn, V. V. Bolotin, A. Ya. Driving, A.S. Lychev, A.G. Roitman, G. Augusti, A. Baratt, F. Kashiati, G.Shpete, V.D. Raiser, A.G. Tamrazyan, B.A. Garagash, S.G. Shulman and others [1-8]. In practice, it is not always possible to obtain the necessary amount of complete and reliable information about the controlled parameters of limit state models, allowing for its statistical analysis, and in this situation it is not recommended to use probabilistic statistical methods to calculate reliability, because in this case their application may lead to incorrect results. How to calculate the reliability of building structures when it is impossible to obtain complete statistical information about the controlled parameters of limit state models, there is no information in existing regulatory and technical documents. In order to describe the incompleteness of information on controlled parameters obtained as a result of a survey of a building structure, the mathematical apparatus of the theory of possibilities, the theory of fuzzy sets, the theory of evidence, etc. can be used when performing reliability calculations[9 – 14] So, for example, when using the probabilistic method (based on the theory of possibilities) to calculate the reliability of building structures, all the values of the mathematical model of limit states are assumed to be fuzzy variables with a certain distribution function of possibilities [10], in this case, some useful statistical information may be lost about some of the controlled parameters, according to which as a result of the survey of the building structure, it was possible to obtain complete statistical information that allows

us to identify the law of distribution, the parameters of this distribution and this parameter could be considered as a random variable (in terms of probability theory), which as a result will lead to a wider reliability interval. This article proposes a particular method for calculating the reliability of a centrally compressed column made of an I-beam with a cross-corrugated wall based on a modernized method that allows combining fuzzy variables and random variables in order to increase the informativeness of the resulting calculation result.

2 Methods and materials

To calculate reliability based on the upgraded method, it is necessary to determine the probability of an event ($X \leq Y$) based on the initial information obtained as a result of a survey of a building structure. In this paper, we consider the case in which: parameter X (generalized load) has complete statistical information (represented as a random variable), and parameter Y (generalized strength) has incomplete statistical information (represented as a fuzzy variable).

The random variable X will be characterized by a normal Gaussian distribution with a probability distribution density:

$$\rho_X(x)=\frac{1}{\sqrt{2\pi}S_x}e^{-\frac{(y-m_X)^2}{2S_x^2}} \quad (1)$$

there m_x - mathematical expectation; S_x - standard deviation.
The fuzzy variable Y will be characterized by a distribution function of the form:

$$\pi_Y(y)=e^{-[(y-a_y)/b_y]^2} \quad (2)$$

there $a_y=(Y_{max}+Y_{min})/2$, $b_y=(Y_{max}-Y_{min})/(2\sqrt{-\ln\alpha})$, $\alpha\in[0;1]$, α – the level of risk.

The fuzzy variable Y can also be represented graphically by probability distribution functions $\underline{P}_Y(y)$ и $\overline{P}_Y(y)$. Figure 1 shows these functions and their relation to $\pi_Y(y)$ according to Eg. (2).

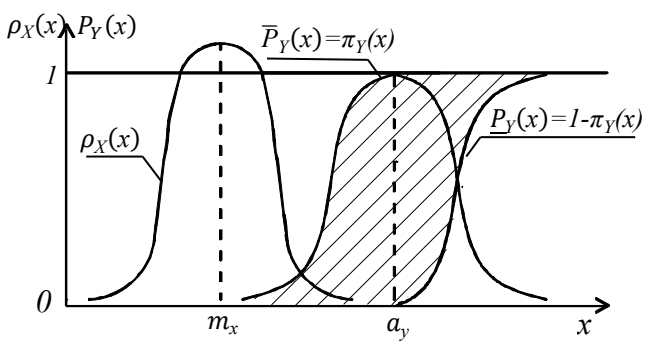

Fig. 1. Distribution function $\underline{P}_Y(x)$, $\overline{P}_Y(x)$, ($\underline{P}_Y(x)\leq P_Y(x)\leq\overline{P}_Y$) , probability density function $\rho_X(x)$

Figure 1 conditionally shows the function according to Eq. (2) (conditionally, since they have different units of ordinates, the notation of the argument x and $y = x$ are assumed to be the same in the further calculation).

In accordance with the upgraded reliability calculation method and with Figure 1. With a combination of random and fuzzy parameters, the formulas for determining the values (lower and upper) of uptime are presented as:

$$\left.\begin{aligned} \underline{P} &= \inf_{\underline{P}_Y(x)\le P_Y(x)\le \overline{P}_Y(x),\ \forall x\ge 0} \left[1-\int_0^\infty \rho_X(x)P_Y(x)dx\right] = 1-\int_0^\infty \rho_X(x)\overline{P}_Y(x)dx \\ \overline{P} &= \sup_{\underline{P}_Y(x)\le P_Y(x)\le \overline{P}_Y(x),\ \forall x\ge 0} \left[1-\int_0^\infty \rho_X(x)P_Y(x)dx\right] = 1-\int_0^\infty \rho_X(x)\underline{P}_Y(x)dx \end{aligned}\right\} \quad (3)$$

there

$$\left.\begin{aligned} \underline{P}_Y(x) &= \begin{cases} 0, & \text{if } x\le a_y \\ 1-\pi_Y(x), & \text{if } x\ge a_y \end{cases} \\ \overline{P}_Y(x) &= \begin{cases} \pi_Y(x), & \text{if } x\le a_y \\ 1, & \text{if } x\ge a_y \end{cases} \end{aligned}\right\} \quad (4)$$

Then, taking into account Figure 1, Eq. (1) - (4), the calculation formulas can be represented as:

$$\left.\begin{aligned} \underline{P} &= 1-\int_0^{a_y} \frac{1}{\sqrt{2\pi}S_x} e^{-\frac{(x-m_x)^2}{2S_x^2}} \cdot e^{-\left[(x-a_y)/b_y\right]^2} dx - \int_{a_y}^\infty \frac{1}{\sqrt{2\pi}S_x} e^{-\frac{(x-m_x)^2}{2S_x^2}} dx \\ \overline{P} &= 1-\int_{a_y}^\infty \frac{1}{\sqrt{2\pi}S_x} e^{-\frac{(x-m_x)^2}{2S_x^2}} \cdot (1-e^{-\left[(x-a_y)/b_y\right]^2})\, dx \end{aligned}\right\} \quad (5)$$

In accordance with Eq. (5), the interval of uptime values is found $[\underline{P}\ ;\ \overline{P}]$ inside which the true reliability value will be located.

3 Results and Discussion

As an example, consider the calculation of the reliability of a centrally compressed column made of an I-beam with a cross-corrugated wall. The mathematical model of the limiting state of the column under consideration according to the stability criterion (in a deterministic formulation) has the form:

$$\frac{N}{\varphi A R_y \gamma_c} \le 1, \quad (6)$$

there N – the force compressing the column; φ – the coefficient of stability under central compression (taken depending on the conditional flexibility of the element); A – gross area of two I-beam belts with a cross-corrugated wall ($A=A_{f_1}+A_{f_2}$, there A_{f_1} - gross area of the upper I-beam belt, A_{f_2} – gross area of the lower I-beam belt); R_y – the yield strength of steel (for the limiting state in the example under consideration, instead of Ry, we take the yield strength σ_T); γ_c – coefficient of working conditions.

Transform equation (6) and write it taking into account the variability of individual parameters, which we denote by the symbol " ~ ":

$$\widetilde{N} \le \widetilde{N}_{ul} = A\varphi\gamma_c\tilde{\sigma}_T, \quad (7)$$

In equation (7), we take the parameters A, φ, γ_c us as deterministic quantities. The operational load $\widetilde{N}$ is assumed to be a random variable, its values can be determined by the results of monitoring the load on the column and calculations to determine the forces arising in the

column in question. Consider the case in which statistical information about the parameter $\tilde{\sigma}_T$ at the stage of operation is obtained by testing samples from the metal structure. Because the number of samples studied may be insufficient to obtain complete statistical information that allows to identify the distribution law $\tilde{\sigma}_T$ and, accordingly, to determine the parameters of this distribution most reliably, then statistical information about the parameter $\tilde{\sigma}_T$ will be incomplete. Thus, $\tilde{\sigma}_T$, and correspondingly $\widetilde{N}_{ul}$, we take a fuzzy variable.

Let us calculate the reliability of the column under consideration according to criterion (7) for the case in which:

- $\tilde{N}$ - a random variable that varies according to the normal (Gaussian) law with the probability density of the distribution:

$$\rho_N(N)=\frac{1}{\sqrt{2\pi}S_N}e^{-\frac{(N-m_N)^2}{2S_N^2}}, \quad (8)$$

there m_N - mathematical expectation; S_N - standard deviation.

– $\widetilde{N}_{ul}$– a fuzzy variable characterized by an opportunity distribution function of the form:

$$\pi_{N_{ul}}(N_{ul})=e^{-\left[\left(N_{ul}-a_{N_{ul}}\right)/b_{N_{ul}}\right]^2}, \quad (9)$$

there $a_{N_{ul}}=\left(N_{ul,\,max}+N_{ul,\,min}\right)/2$, $b_{N_{np}}=\left(N_{ul,\,max}-N_{ul,\,min}\right)/(2\sqrt{-\ln\alpha})$, $\alpha\in[0;1]$.

For the problem under consideration with a random variable $\tilde{N}$ a fuzzy parameter $\widetilde{N}_{ul}$, we use the equations (5), (8), (9) to determine the values of the probabilities of failure-free operation of the column:

$$\left.\begin{aligned}\underline{P}&=1-\int_0^{a_{N_{ul}}}\frac{1}{\sqrt{2\pi}S_N}e^{-\frac{(N-m_N)^2}{2S_N^2}}\cdot e^{-\left[\left(N-a_{N_{ul}}\right)/b_{N_{ul}}\right]^2}dN-\int_{a_{N_{ul}}}^{\infty}\frac{1}{\sqrt{2\pi}S_N}e^{-\frac{(N-m_N)^2}{2S_N^2}}dN\\ \overline{P}&=1-\int_{a_{N_{ul}}}^{\infty}\frac{1}{\sqrt{2\pi}S_N}e^{-\frac{(N-m_N)^2}{2S_N^2}}\cdot\left(1-e^{-\left[\left(N-a_{N_{ul}}\right)/b_{N_{ul}}\right]^2}\right)dN\end{aligned}\right\} \quad (10)$$

The reliability of a centrally compressed column made of an I-beam with a cross-corrugated wall according to the stability criterion (7) will be characterized by an interval of values of the probability of trouble-free operation [$\underline{P}$; $\overline{P}$], the true value of reliability will be within this interval.

Let's consider a numerical example of calculating the reliability of a column made of an I-beam with a cross-corrugated wall according to criterion (7). Let's assume that statistical information was obtained during the survey of the column, as a result of the analysis of which it was found that: 1. information about the parameter $\tilde{N}$ enough, it is complete and statistically homogeneous. We will consider this parameter as a random variable varying according to the normal distribution law with the probability density of the distribution (8), determined by the parameters of this distribution:: m_N = 1891 kN, S_N = 180 kN; 2. the information about the parameter $\widetilde{N}_{ul}$ was not complete. We will consider this parameter as a fuzzy variable with a distribution function of the form (9). According to the statistical information obtained, the parameters of the function (9) were determined: the information about the parameter $\widetilde{N}_{ul}$ was not complete. We will consider this parameter as a fuzzy variable with a distribution function of the form (9). According to the statistical information obtained, the parameters of the function (9) were determined: $a_{N_{ul}}$= 2367 kN, $b_{N_{ul}}$= 230 kN. To determine the reliability of the column according to criterion (7) based on the analysis of the statistical information obtained about the controlled parameters, we use equations (10) and as a result of the calculation we obtain the reliability interval, in the form of upper and lower values of the

probabilities of uptime: [0.902; 0.999]. The true value of the reliability of the column under consideration according to criterion (7) will be inside the resulting interval.
We introduce an assumption and assume that both parameters change according to the normal distribution law and solve the proposed problem by the probabilistic-statistical method. We will determine the reliability of the column by the stability criterion using the safety characteristic $\gamma = \frac{m_{N_{\text{пр}}} - m_N}{\sqrt{S_{N_{\text{пр}}}^2 + S_N^2}}$. In this case, the value of the probability of failure-free operation is determined using tables of normal distribution functions (Laplace) $P(N_{ul} \geq N) = \Phi(\gamma) = \Phi\left(\frac{m_{N_{ul}} - m_N}{\sqrt{S_{N_{ul}}^2 + S_N^2}}\right)$ According to the initial data of the example under consideration m_N = 1891 kN, S_N =180 kN, let's assume that $m_{N_{ul}} = a_{N_{ul}}$= 2367 kN, $S_{N_{ul}} = b_{N_{ul}}$=230 kN, then the probability of uptime $P(N_{ul} \geq N) = \Phi\left(\frac{m_{N_{ul}} - m_N}{\sqrt{S_{N_{ul}}^2 + S_N^2}}\right) = \Phi\left(\frac{2367-1891}{\sqrt{230^2+180^2}}\right) = \Phi\ (1{,}63) = 0.9484$.

Let's consider another option by presenting both parameters: $\widetilde{N}$ and $\widetilde{N}_{ul}$– fuzzy variables with distribution functions of the form (2) and the parameters of these functions: a_N= 1891 kN, b_N= 180 kN, $a_{N_{ul}}$= 2367 kN, $b_{N_{ul}}$= 230 kN. We use a possibilistic method for calculating reliability. As a_N= 1891 kN < $a_{N_{ul}}$= 2367 kN, the possibility of trouble-free operation *R=1*. To find the value of the possibility of failure, it is necessary to determine the value of N_*, i.e. the abscissa of the intersection point of the distribution functions of opportunities $\pi_N(N)$ и $\pi_{N_{ul}}(N_{ul})$, Figure 2.

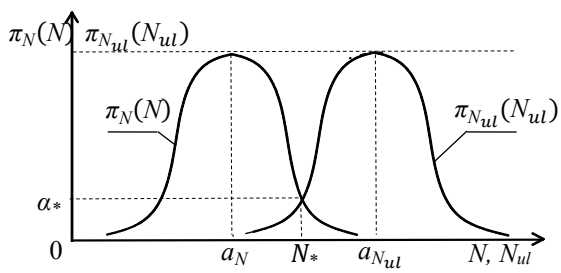

Fig. 2. Opportunity distribution functions $\pi_N(N)$ и $\pi_{N_{ul}}(N_{ul})$

Therefore $|(N_*-1891)/180|=|(N_*-2367)/230|$. We find the value N_*, which will be in the interval $[a_N\ ;\ a_{N_{ul}}]$, N_*=2099.97 kN. Then the possibility of failure $Q = \alpha_* = \pi_N(N_*) = e^{-[(2099.97-1891)/180]^2} = 0.26$, the need for trouble-free operation $N=1-Q$=1-0.26=0.74. The reliability of the column under consideration according to criterion (6) will be characterized by the interval [0.74; 1].

We will carry out reliability calculations using the methods presented above for several more variants of the initial data and we will summarize all the results obtained in Tables 1, Table 2, Table 3.

Table 1. Calculation of reliability of a centrally compressed column made of an I-beam with a cross-corrugated wall according to criterion (6) by the modernized method

Initial data				The value of the reliability indicator
m_N	S_N	$a_{N_{ul}}$	$b_{N_{ul}}$	$[\underline{P};\overline{P}]$
1911	189	2531	250	[0.961;0.999]
2087	197	2689	259	[0.944;0.999]
1931	193	2449	241	[0.912;0.999]
1891	180	2367	230	[0.902;0.999]
2128	198	2459	240	[0.702; 0.993]
2153	200	2484	241	[0.699; 0.992]
2041	193	2351	233	[0.683; 0.992]
1987	183	2265	228	[0.645; 0.990]

Table 2. Calculation of reliability of a centrally compressed column made of an I-beam with a cross-corrugated wall according to criterion (6) by probabilistic and statistical method

Initial data				The value of the reliability indicator
m_N	S_N	$m_{N_{ul}}$	$S_{N_{ul}}$	P
1911	189	2531	250	0.976
2087	197	2689	259	0.968
1931	193	2449	241	0.953
1891	180	2367	230	0.948
2128	198	2459	240	0.856
2153	200	2484	241	0.854
2041	193	2351	233	0.847
1987	183	2265	228	0.829

Table 3. Calculation of the reliability of a centrally compressed column made of an I-beam with a cross-corrugated wall according to criterion (6) by the possibility method

Initial data				The value of the reliability indicator
a_N	b_N	$a_{N_{ul}}$	$b_{N_{ul}}$	$[N;R]$
1911	189	2531	250	[0.864;1]
2087	197	2689	259	[0.825;1]
1931	193	2449	241	[0.759;1]
1891	180	2367	230	[0.740;1]
2128	198	2459	240	[0.435;1]
2153	200	2484	241	[0.431;1]
2041	193	2351	233	[0.411;1]
1987	183	2265	228	[0.367;1]

From the presented results of reliability calculations in Tables 1, Table 2, Table 3 by various methods, it can be concluded that the upgraded calculation method allows the fullest use of

statistical information about controlled parameters obtained as a result of inspection and monitoring of building structures at the operational stage by combining random variables and fuzzy variables, which allows you to increase the information content of the results of reliability calculations. When using the probabilistic method in the reliability calculations in this case, useful information about the controlled parameters with complete statistical information may be lost, since the parameters are presented in the form of fuzzy variables, and as a result, the result of calculating the reliability interval is wider and less informative compared to the calculation results of the modernized method (Table 1, Table 3). Table 2 presents the results of the reliability calculation using the probabilistic-statistical method, while some assumptions were introduced. These results are more attractive from the point of view of making decisions on them, because they have an unambiguous result, however, they are not sufficiently substantiated, since assumptions about the law and distribution parameters may be erroneous, which may lead to incorrect results. This point must be taken into account when carrying out practical calculations.

4 Conclusion

The article proposes a particular method for calculating the reliability of a centrally compressed column made of an I-beam with a cross-corrugated wall according to the stability criterion. The results of reliability calculations by various methods are presented, as a result of the analysis of which it is established that when performing calculations for the reliability of building structures, the combination of random variables and fuzzy variables allows for more complete and qualitative consideration of all the statistical information obtained about the controlled parameters, which allows for more reliable and informative results of reliability calculations.

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