Study on equations of curves of the squeezing machines’ contact of rolls

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Abstract. This study deals with the issues of mathematical modeling of curves of contact of rolls of squeezing machines. The paper deals with the issues of mathematical modeling of curves of contact of rolls of squeezing machines. Equations are obtained of curves contact of the rolls of the squeezing machine, in which the upper roll is mixed relative to the lower roll towards the movement of the processed material, the processed material is fed so that the line, which is a continuation of its front end, passes through the axis of rotation of the upper roll. It was been established that the contact curves of the rolls depend mainly on the deformation and geometric parameters of the processed material and both rolls, as well as the coefficients of friction between the contacting bodies.

1. Introduction

One of the technological operations that largely determines the quality of the finished product is spinning, which creates the moisture necessary for subsequent operations and is carried out squeezing in roller machines. In this regard, it is considered important to create and introduce into production highly efficient and energy-saving roller squeezing machines in order to ensure that the specified characteristics of the finished product are obtained.

Based on the analysis of the roller machine design used in textile, light and pulp, and paper industries [1-3] and the features of the roller squeezing of materials, we select the scheme of the studied two-roll module of squeezing

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machines (Fig. 1), in which the upper roll is shifted relative to the lower roll toward the movement of the material layer to distance \( \Delta \), determined by angle \( \beta_1 \). The material being processed is fed in such a way that the line, which is a continuation of its front end, passes through the axis of rotation of the upper roll, since no additional external forces are required to carry out the capture [4].

Works [5-19] are devoted to the creation of scientific foundations for the development of roller squeezing machines and the improvement of their parameters. At the same time, it is observed that in the listed works, mathematical models of curves of contact rolls are developed under static interaction conditions. In this regard, these models do not allow to give full explanations for the phenomenon of contact interaction in squeezing machines.

The purpose of this work is to simulate the curves of contact the rolls of the squeezing machine shown in Fig. 1.

2. Resultative Methods

The mathematical model of the shape of the roll contact curves, in polar coordinates with the pole at the center of the lower (or upper \( i = 2 \)) roll, can be expressed by equation \( r_{ij} = r_j(\theta_j) \), where \( j \) is the index indicating the number of sections of strain compression (or recovery). The equations \( r_{ij} = r_j(\theta_j) \) that determine the contact curves of the rolls primarily depend on the deformation of the contacting bodies of the squeezing machines.

There is a large amount of experimental data on the patterns of deformation of leather, fabrics, cotton, paper and other materials pressed in roller machines, as well as rubber, wool, industrial cloth and a number of others used to cover rollers. An analysis of these data showed that the deformations of the contacting bodies of squeezing machines are described mainly by empirical dependencies \( \sigma = A e^n \) [20-28], where \( A, n \) are the coefficients of strain and hardening.

Therefore, we can assume that

\[
\sigma_j = A_j e_j^n, \quad \sigma_{ij} = B_{ij} e_{ij}^n, \quad i = 1, 2, \quad j = 1, 2,
\]

where \( \sigma_j, e_j, B_j, m_j \) are the stresses, relative strains, coefficients of strain and hardening of the points of the processed material; \( \sigma_{ij}, e_{ij}, B_{ij}, m_{ij} \) are the stresses, relative strain, coefficients of strain and hardening of the points of the elastic coating.

An analysis of a number of studies [1, 27] showed that under static conditions of interaction, the maximum compression of the processed material occurs at the point (point \( A_3 \)) lying on the line of centers. Under dynamic conditions, due to the action of reactive forces directed opposite to their rotation, point \( A_3 \) is shifted from the line of centers toward the material entering the contact zone of the rolls [28]. It was found [29] that the angle, defining point \( A_3 \) can be taken as \( -\varphi_{13} = \frac{-\varphi_{11} + \varphi_{12}}{2} \).

Thus,

\[-\varphi_{11} - (-\varphi_{13}) \leq -\theta_{11} \leq -\varphi_{13}, \quad -\varphi_{13} \leq \theta_{12} \leq -\varphi_{13} + \varphi_{12}, \]

or

\[-\varphi_{11} \leq -\theta_{11} + \varphi_{13} \leq 0, \quad 0 \leq \theta_{12} + \varphi_{13} \leq \varphi_{12}, \]

where

\[\varphi_{13} = \frac{\varphi_{11} - \varphi_{12}}{2}.\]

Similarly, for the contact curve of the upper roll (Fig. 1), we have:

\[-\varphi_{21} \leq -\theta_{21} + \varphi_{23} \leq 0, \quad 0 \leq \theta_{22} + \varphi_{23} \leq \varphi_{22}. \]

Where

\[\varphi_{23} = \frac{\varphi_{21} - \varphi_{22}}{2}.\]
In the process of interaction with the roll, the angle of inclination of the material being processed changes depending on the contact angle of the upper roll: in the compression zone, it decreases from the value of \((-\varphi_21)\) to \((-\varphi_23)\), and in the strain recovery zone, it increases from the value of \((-\varphi_23)\) to \(\varphi_22\).

Therefore, we can assume [28] that

\[
\gamma = \gamma(-\varphi_21 + \varphi_23) = a_{11}(-\varphi_21 + \varphi_23) + b_{11}, \quad -\varphi_21 \leq -(\varphi_21 + \varphi_23) \leq 0,
\]

\[
\gamma = \gamma(\varphi_22 + \varphi_23) = a_{12}(\varphi_22 + \varphi_23) + b_{12}, \quad 0 \leq \varphi_22 + \varphi_23 \leq \varphi_22.
\]

Coefficients \(a_{11}, b_{11}, a_{12},\) and \(b_{12}\) are found from the initial and boundary conditions:

\[
\text{for } -\varphi_21 = -\varphi_21 + \varphi_23, \quad \gamma = -\varphi_21; \quad \text{for } \varphi_21 = \varphi_22 = 0, \quad \gamma = -\varphi_23; \quad \text{for } \varphi_22 = \varphi_23, \quad \gamma = 0.
\]

They have the following form: \(a_{11} = a_{12} = 1, \quad b_{11} = b_{12} = 0.\)

Then we obtain:

\[
\gamma = -(\varphi_21 + \varphi_23), \quad -\varphi_21 \leq -(\varphi_21 + \varphi_23) \leq 0, \quad \gamma = \varphi_22 + \varphi_23, \quad 0 \leq \varphi_22 + \varphi_23 \leq \varphi_22
\]

or

\[
\gamma = -\varphi_1(\varphi_1 + \varphi_1), \quad -\varphi_1 \leq -(\varphi_1 + \varphi_1) \leq 0, \quad \gamma = \varphi_2(\varphi_1 + \varphi_1), \quad 0 \leq \varphi_2 + \varphi_2 \leq \varphi_2.
\]

(4)

At each point of section \(A_2A_3\) condition \(\frac{\sigma_{11}}{\cos \psi_{11}} = \frac{\sigma_{11}^*}{\cos \psi_{11}}\) is fulfilled, where \(\psi_{11}\) – is the angle between the radius \(r_{11}\) and the normal \(n - n\). Hence, we have \(\sigma_{11} = \sigma_{11}^*\) or according to expressions (1)

\[
B_{11}^1 e_{11}^{m_{11}^1} = A_{11}^1 e_{11}^{*m_{11}^1}.
\]

(5)

Differentiating equalities (7) we find

\[
m_{11} B_{11} e_{11}^{m_{11}^1} d e_{11} = m_{11}^* A_{11}^* e_{11}^{*m_{11}^1} d e_{11}^*.
\]

Hence

\[
\frac{d e_{11}}{e_{11}} = \frac{m_{11}}{m_{11}^*} \frac{d e_{11}^*}{e_{11}^*}.
\]

(6)

We assume that

\[
\frac{d e_{11}}{d e_{11}^*} = \alpha_{11}.
\]

(7)

Where \(\alpha_{11}\) is the index that determines the ratio of the rate of strain compression of the coating of the lower roll to the rate of strain compression of the processed material.

From equality (6) and (7), we have

\[
\frac{e_{11}}{e_{11}^*} = \frac{m_{11}}{m_{11}^*} \alpha_{11}.
\]

(8)

From a Fig. 1 it follows that:
\[ e_{11} = \frac{R_1 - r_{11}}{H_1} \quad e_{11}^* = \frac{r_{11} - R_1}{\delta_1} \cos(-\varphi_{11} + \gamma_1) \cos(-\theta_{11} + \varphi_{13} + \gamma) \],

where \( \gamma_1 = -c_1 \varphi_{11} \).

From equality (9) considering expressions (4), (6), and (7), we obtain

\[ R_1 - r_{11} = c_{11} \alpha_{11} \left( r_{11} - R_1 \right) \cos \frac{\zeta_{11}}{\cos \theta_{11}}, \]

where \( c_{11} = \frac{m_{11} H_1}{m_1 \delta_1}, \quad \theta_{11} = \theta_{11} + \varphi_{13} - \gamma, \quad \zeta_{11} = \varphi_{11} - \gamma_1. \)

Solving equality (10) with respect to \( r_{11} \), we find the equation of the contact curve of section \( A_1 A_3 \)

\[ r_{11} = \frac{R_1}{1 + c_{11} \alpha_{11}} \left( 1 + c_{11} \beta_1 \cos \frac{\zeta_{11}}{\cos \theta_{11}} \right), \quad -\zeta_{11} \leq -\delta_{11} \leq 0. \]

Determining the equation of the contact curve of the restoration section \( A_2 A_2 \), and generalizing with equation (11), we find a system of equations that describes the contact curve of the lower roll

\[ \begin{align*}
    r_{11} &= \frac{R_1}{1 + c_{11} \alpha_{11}} \left( 1 + c_{11} \beta_1 \cos \frac{\zeta_{11}}{\cos \theta_{11}} \right), \quad -\zeta_{11} \leq -\delta_{11} \leq 0, \\
    r_{12} &= \frac{R_1}{1 + c_{12} \alpha_{12}} \left( 1 + c_{12} \beta_2 \cos \frac{\zeta_{12}}{\cos \theta_{12}} \right), \quad 0 \leq \delta_{12} \leq \zeta_{12}.
\end{align*} \]

where

\[ c_{12} = \frac{m_{12} H_1}{m_2 \delta_2}, \quad \alpha_{12} = \frac{d \delta_{12}}{d \varphi_{12}}, \quad \gamma_2 = c_2 \varphi_{22}, \quad \delta_{12} = \theta_{12} + \varphi_{13} + \gamma, \quad \zeta_{12} = \varphi_{12} + \gamma_2. \]

In a similar way, we find a system of equations describing the curve contact of the upper roll

\[ \begin{align*}
    r_{21} &= \frac{R_2}{1 + c_{21} \alpha_{21}} \left( 1 + c_{21} \beta_{21} \cos \frac{\zeta_{21}}{\cos \theta_{21}} \right), \quad -\zeta_{21} \leq -\delta_{21} \leq 0, \\
    r_{22} &= \frac{R_2}{1 + c_{22} \alpha_{22}} \left( 1 + c_{22} \beta_{22} \cos \frac{\zeta_{22}}{\cos \theta_{22}} \right), \quad 0 \leq \delta_{22} \leq \zeta_{22},
\end{align*} \]

where

\[ c_{21} = \frac{m_{21} H_2}{m_1 \delta_1}, \quad \alpha_{21} = \frac{d \delta_{21}}{d \varphi_{21}}, \quad \delta_{21} = \theta_{21} + \varphi_{23}, \quad \zeta_{22} = \varphi_{22}. \]

It is known [20] that the ratio of the strain rates of the roll coating and the material being processed has the form

\[ \lambda_{ij} = \frac{A_j m_{ij} (\Delta l_{ij})_{cp} - (A_j (1 - m_{ij}) - A_j (1 - m_{ij}^*) \delta_j)}{A_j m_{ij} (\Delta l_{ij})_{cp} + (A_j (1 - m_{ij}) - A_j (1 - m_{ij}^*)) H_i}, \]

where
\[
\Delta l_{1p} = R_1 \left( 1 - \frac{\sin 2(\phi_{11} - \gamma_1)}{2\phi_{11} - \gamma_1} \right), \quad \Delta l_{12} = R_1 \left( 1 - \frac{\sin 2(\phi_{12} + \gamma_2)}{2\phi_{12} + \gamma_2} \right), \\
\Delta l_{21} = R_1 \left( 1 - \frac{\sin 2\phi_{21}}{2\phi_{21}} \right), \quad \Delta l_{22} = R_1 \left( 1 - \frac{\sin 2\phi_{22}}{2\phi_{22}} \right).
\]

The systems of equations (12) and (13) describe the contact lines of the rolls in the squeezing machine under consideration.

3. Conclusions
1. Mathematical models of the contact curves of the rolls of the squeezing machine are obtained, in which the upper roll is mixed relative to the lower roll towards the movement of the material layer, the processed material is fed so that the line, which is a continuation of its front end, passes through the axis of rotation of the upper roll.
2. The obtained models show that the main indicators of mathematical models of the shape of the roll contact curves are the values \( \phi_{ij} \), \( c_{ij} \) and \( \alpha_{ij} \), which in turn depend on the deformation and geometric parameters of the material being processed and both rolls, as well as the coefficients of friction between the contacting bodies.
3. If, in a two-roll module, the rolls have a non-deformable coating, then \( c_{ij} = 0 \) and \( \alpha_{ij} = 0 \), therefore \( r_{ij} = R_i \).
4. If, in a two-roll module, the material layer is non-deformable, then \( c_{ij} = \infty \) and \( \alpha_{ij} = \infty \), therefore \( r_{ij} = R_i \cos \theta_j \).
5. It was established that the contact curves of the rolls depend mainly on the deformation and geometric parameters of the processed material and both rolls, as well as on the coefficients of friction between the contacting bodies.
6. It was revealed that any graph of the roll contact line is located between the graphs of the lines \( r_i = R_i \cos \theta_i \) and \( r_{ij} = R_i \).

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