Study on hydraulic pressures in the roll squeezing process

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Abstract. The results of the study of hydraulic phenomenon in the roll squeezing process. Mathematical models of regularities of changes in filtration rates are found. Based on the analysis of the graphs obtained, the hydraulic pressure distributions are approximated by empirical formulas. Formulas are derived to determine the maximum value of hydraulic pressure and the angle at which this maximum is reached. A formula for calculating the part of the external pressure acting on the liquid is defined.

1. Introduction

One of the most economical and versatile types of machines used in many industries, including construction, when performing various technological processes, are roller machines. Among these processes, the process of pressing the semi-finished leather product after tanning is of particular importance, since this process creates the moisture necessary for subsequent technological processes. Despite numerous studies of the problem of contact interaction and moisture filtration during squeezing, mathematical modeling of the distribution of hydraulic pressures in the process of squeezing the skin is poorly understood \cite{1-18}. The fact that the filtering properties of the semi-finished leather product after tanning are not used in solving hydraulic problems indicates that the hydraulic problems in the roller pressing of the skin are poorly understood.

The analysis showed that the two-roll modules of the squeezing machines of the leather industry are mainly symmetrical \cite{19}.

![Fig. 1. Scheme of a two-roll module of leather squeezing](image)

We consider a two-roll module, in which the rolls are covered with feld thickness \( H \) have a radius \( R \). The skin has a thickness of \( \delta \), and the gap between the rolls is equal to the rolls \( h \) (Fig. 1).

2. Resultative Methods

In the process of squeezing, the location of the capillaries at a certain angle relative to the surface of leather prevents the movement of moisture along the surface. Therefore, when hydraulic pressure occurs, the liquid moves in the direction of the capillaries, i.e. under the action of a hydraulic gradient, the fluid moves at an angle of \( \alpha \) relative to
the horizontal line (the \( Ox \)-axis). It was revealed that during roll pressing, when the rolls are covered with cloth, the liquid from the leather moves up and down to the cloth at an angle \( \theta \) (in direction \( r \)) to the \( Oy \)-axis [20].

![Diagram of process scheme leather squeezing]

It is known [21] that the hydraulic gradient in direction \( r \) determined by angle \( \theta \), is expressed by the following formula:

\[
\frac{\partial H}{\partial r} = \frac{\partial H}{\partial x} \sin \theta + \frac{\partial H}{\partial y} \cos \theta. \tag{1}
\]

The studies in [22, 23] are devoted to the deformation properties of semi-finished leather product after chromium tanning and the following dependence is given

\[
\frac{\partial H}{\partial x} = (a_1 \varepsilon + b_1) v_x, \quad \frac{\partial H}{\partial y} = (a_2 \varepsilon + b_2) v_y, \tag{2}
\]

where \( v_x, v_y \) are the rates of fluid filtration through the leather; \( \varepsilon \) is the relative deformation of the skin; \( a_1, a_2, b_1, b_2, c_1, c_2 \) are the coefficients.

Substituting expressions for hydraulic gradients from (2) into (1), we get:

\[
\frac{\partial H}{\partial \theta} = ((a_1 \varepsilon + b_1) v_x \sin \theta + (a_2 \varepsilon + b_2) v_y \cos \theta) r'. \tag{3}
\]

According to [20], the change in the fluid filtration rate in direction \( x \) has the following form:

\[
v_x = \frac{\delta - \delta(x)}{\delta(x)} V_k, \tag{4}
\]

where \( \delta \) is the thickness of the skin in section \( 2-2 \) with the filtration rate and hydraulic pressure equal to zero; \( \delta(x) \) is the skin thickness in section \( x-x \); \( V_k \) is the velocity of the skin feed.

Thicknesses \( \delta \) and \( \delta(x) \) are expressed by the following formulas

\[
\delta = \delta_i + 2R \cos \phi_1 - 2R_2 \cos \phi_2, \tag{5}
\]

\[
\delta(x) = \delta_i + 2R \cos \phi_1 - 2r(\theta) \cos \theta, \tag{6}
\]

where \( \delta_i \) is the thickness of the skin before entering the contact zone of the rolls; \( \phi_1 \) is the nip angle; \( r(\theta) \) is the roll radius determined by the angle \( \theta \); \( R_2, \phi_2 \) are the radius and the polar angle defining section \( 2-2 \).

\[
v_x(\theta) = \frac{2(r \cos \theta - R_2 \cos \phi_2)}{\delta_i + 2R \cos \phi_1 - 2r \cos \theta} V_k. \tag{7}
\]

It is necessary to select section \( 5-5 \) located to the right of the minimal section \( 3-3 \) by the same value of \( l \) as section \( 2-2 \). In this section, the thickness of the skin is \( \delta_2 = \delta_3 \), and, as follows from (7), \( v_{x2} = v_{x5} \); in the
area between sections 2 - 2 and 5 - 5 - \( v_x > V_k \) (\( \delta(x) < \delta \)), and in the area 1 - 1 and 2 - 2 - \( v_x < V_k \) (\( \delta(x) > \delta \)), where 1 - 1 is the section where the contact of the rolls and the skin begins.

We find the derivative of expression (4)

\[
v'_x = 2(r' \cos \theta - r \sin \theta)(\delta_1 + 2R \cos \varphi_1 - 2R_2 \cos \varphi_2) \frac{v'}{V_k}.
\]

(8)

The fluid filtration rate in direction \( y \) is found from the flow continuity condition [21]:

\[
v_x' + v_y' = 0
\]

or

\[
v_y' = -v_x' \cdot \frac{y'}{x'},
\]

(9)

where \( y = r \cos \theta, \ x = r \sin \theta \). Hence \( y' = r' \cos \theta - r \sin \theta, \ x' = r' \sin \theta + r \cos \theta \).

Substituting derivatives \( y', \ x' \) and \( v'_y \) in (9), we get:

\[
v'_y = \frac{r \sin \theta - r' \cos \theta}{r \cos \theta + r' \sin \theta} \cdot \frac{2(r' \cos \theta - r \sin \theta)(\delta_1 + 2R \cos \varphi_1 - 2R_2 \cos \varphi_2)}{(\delta_1 + 2R \cos \varphi_1 - 2r \cos \theta)^2} V_k \cdot
\]

(10)

Integrating (10), we find \( V_y \) in the area between sections 1 - 1 and 3 - 3:

\[
V_y(\theta) = V_k(\delta_1 + 2R \cos \varphi_1 - 2R_2 \cos \varphi_2) \int_{-\varphi_1}^{\theta} \frac{r' \cos \theta - r \sin \theta}{r' \sin \theta + r \cos \theta} \cdot \frac{2(r' \cos \theta - r \sin \theta)}{(\delta_1 + 2R \cos \varphi_1 - 2r \cos \theta)^2} d\theta.
\]

We calculate the integral using the second mean value theorem:

\[
\int_{a}^{b} u(x) \cdot \vartheta(x) dx = \frac{1}{b-a} \int_{a}^{b} u(x) dx \cdot \int_{a}^{b} \vartheta(x) dx.
\]

After integration and substitution of limits, we have:

\[
V_y(\theta) = \frac{1}{\theta + \varphi_1} \cdot V_k(\delta_1 + 2R \cos \varphi_1 - 2R_2 \cos \varphi_2) \times
\]

\[
\left[ \frac{2(r \cos \theta - R \cos \varphi_1)}{(\delta_1 + 2R_1 - 2R \cos \varphi_1)(\delta_1 + 2R_1 - 2r \cos \theta)} \int_{-\varphi_1}^{\theta} \frac{r' \cos \theta - r \sin \theta}{r' \sin \theta + r \cos \theta} d\theta, - \varphi_1 \leq \theta \leq 0 \right].
\]

(11)

After passing section 3 - 3, the skin begins to recover. Therefore, in section 4 - 4, which is located to the right of section 3 - 3, the liquid cease to move in the vertical direction, that is, in section 4 - 4, \( v_y = 0 \). With this in mind, similarly to (11), we find the pattern of change in \( V_y \) in the area between sections 3 - 3 and 4 - 4:

\[
V_y(\theta) = \frac{1}{\varphi_2 - \theta} \cdot V_k(\delta_0 + 2R_0 - 2R_2 \cos \varphi_2) \times
\]

\[
\left[ \frac{2(R_3 \cos \varphi_3 - r \cos \theta)}{(\delta_0 + 2R - 2R \cos \varphi_3)(\delta_0 + 2R_0 - 2r \cos \theta)} \int_{-\varphi_1}^{\theta} \frac{r' \cos \theta - r \sin \theta}{r' \sin \theta + r \cos \theta} d\theta, 0 \leq \theta \leq \varphi_3. \right]
\]

(12)

where \( R_3, \varphi_3 \) - is the radius of the roll and the polar angle in section 4 - 4. It is known [7] that

\[
\varepsilon = \frac{2(r \cos \theta - R \cos \varphi_1)}{\delta_1}, \quad - \varphi_1 \leq \theta \leq 0; \quad \varepsilon = \frac{2(r \cos \theta - R \cos \varphi_2)}{\delta_2}, \quad 0 \leq \theta \leq \varphi_4.
\]

(13)

Substituting \( V_x, V_y \) and \( \varepsilon \) from (7), (11), (12), and (13) into (2), after some transformations, we obtain:

\[
H(\theta) = \int_{-\varphi_1}^{\theta} \left( a_1 \frac{2(r \cos \theta - R \cos \varphi_1)}{\delta_1} + b_1 \right) \frac{2(r \cos \theta - R_2 \cos \varphi_2)}{\delta_1 + 2R \cos \varphi_1 - 2r \cos \theta} V_k r' \sin \vartheta d\theta
\]
We perform numerical analysis $H(\theta)$ using expression [11]

We approximate expression (14) by the following formulas

The values of $\varphi_5$ and $H_{\text{max}}$ depend on the equation of the roll contact curve $r = r(\theta)$. Therefore, first, $r(\theta)$ is given. Then $\varphi_5$ is determined from equality (4) by applying the necessary condition for the existence of an extremum, i.e. considering $H'(\varphi_5) = 0$.

Then by integrating the following expression

and substituting $\theta = -\varphi_5$, we determine $H_{\text{max}}$.
Then by integrating the following expression extremum, i.e. considering given. Then

In order to simplify expression (14), we approximate it by the following formulas

\[ \varphi_1 = \frac{2a_1 (R_0 - R \cos \varphi_1) + b_1 \delta_1}{\delta_1 (B_1 + \cos \varphi_1) (a_1 (R_0 - R \cos \varphi_1) + \delta_1 b_1)}; \]

\[ H_{\text{max}} = \frac{2b_1 (R_0 \cos \varphi_1 - R \cos \varphi_2)}{\delta_1 (B_1 + \cos \varphi_1)} \cdot (b_2 (\delta_1 + 2R \cos \varphi_1 - 2R_2 \cos \varphi_2) - (B_1 + \cos \varphi_1) b_1 \delta_1). \]

A comparison of the graph of expression (14) with the graphs of expressions (15) and (16) shows that they almost coincide. Therefore, formulas (15) and (16) can be used to calculate the residual skin moisture and the pressure required to remove moisture from the skin.

During the pressing process, under the external pressure applied to the rollers, the skin is deformed or compacted. Compaction consists of the rearrangement of solid particles, a decrease in the volume of pores between them, and is accompanied by squeezing out the liquid that fills these pores. Therefore, under squeezing, part of the applied pressure is perceived by the solid phase, and part - by the liquid phase.

With expressions (15) and (16), the part of the external pressure perceived by the liquid has the following form

\[ G = \frac{H_{\text{max}}}{2} \int_{-\varphi_2}^\varphi \left( 1 + \cos \left( \frac{\varphi_2 + \theta}{\varphi_1 - \varphi_2} \right) \right) d\theta + \frac{H_{\text{max}}}{4} \int_{-\varphi_2}^\varphi \left( 1 + \cos \left( \frac{\varphi_5}{\varphi_1 - \varphi_5} \right) \right) \cdot \int_0^{\varphi_5} \frac{1 + \cos \left( \frac{\theta}{\varphi_5} \right) d\theta \right). \]

or after integration and substitution of limits:

\[ G = \frac{H_{\text{max}}}{4} \left( 2\varphi_1 + 2 \frac{\varphi_1 - \varphi_5}{\pi} \sin \left( \frac{\varphi_3}{\varphi_1 - \varphi_5} \pi \right) + \left( 1 + \cos \left( \frac{\varphi_5}{\varphi_1 - \varphi_5} \right) \right) \varphi_2 \right). \] (17)

3. Conclusions

a) Mathematical models of regularities of changes in filtration rates are found.

b) Based on the analysis of the graphs obtained, the hydraulic pressure distributions are approximated by empirical formulas. Formulas are found to determine the maximum value of hydraulic pressure and the angle at which this maximum is reached. A formula is defined for calculating the part of the external pressure perceived by the liquid.

c) Analysis of graphs \( H(\theta) \) showed that the hydraulic pressure at the beginning of the contact zone, that is, in the section, determined by the angle \( -\varphi_2 \), is zero. From this section, it grows and, in the section, determined by the angle \( -\varphi_2 \), it reaches its maximum value \( H_{\text{max}} \). Then it decreases and, in the section, determined by the angle \( \varphi_2 \), it equals zero.

References


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