Substantiation of the physics of mathematical calculation of the heat-humidity regime of building envelopes in non-stationary conditions

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Abstract. This article explores calculation methods and experimental studies of newly erected, restored, and reinforced, multi-layer enclosing structures, which most fully considers the specifics of the impact on them of the individual properties of materials, the climate of the construction area, and operating conditions. The calculation of the humidity regime of external enclosing structures in a non-stationary mode will make it possible to more accurately determine the moisture content of materials in the structure, primarily in the heat-insulating layer, and, depending on humidity, establish the actual coefficients of thermal conductivity of materials, which will help to more accurately determine the thermal resistance to heat transfer and predict the effectiveness of thermal protection of enclosing structures of buildings in operational conditions.

1. Introduction

The most effective way to save fuel and energy resources (FER) resources in capital construction is to reduce heat losses through the building envelope by increasing the level of thermal protection. Therefore, all the norms for the design of thermal protection, enclosing structures of buildings were gradually revised towards tightening; With the increase in regulatory requirements, civil engineering in recent years has been oriented towards the construction of buildings in multi-layer walls using effective heat-insulating materials with a thermal conductivity coefficient of up to 0.06 W/(m·°C). This is due to the desire of designers to obtain the required normalizing resistance to heat transfer $R_{eq}$ of enclosing structures, which is 3-4 times higher than the requirements of the old standards. Most currently used in the construction of building walls, materials can provide the required $R_{eq}$ in single-layer structures, but the thickness of the walls; in this case, it can go beyond reasonable limits [1-19].

In the works of V. N. Bogoslovsky [2,3], a heat-humidity calculation based on the humidity potential is proposed. Humidity potential V.N. Bogoslovsky is an isotherm potential, the gradient of which simultaneously reads the moisture conductivity and thermal moisture conductivity.

Application of the potential of V.N. Bogoslovsky allows reduce the differential equations describing the system to the form of writing the classical Fourier equation for non-stationary thermal conductivity, but there is a need to experimentally determine their moisture conductivity coefficients, which also depend on moisture content; and temperature.

In the theory of A.V. Lykov [4,5,6,7], the concept of humidity potential is also used for heat-humidity calculations. A.V. Lykov’s moisture potential is an experimental potential, the use of which causes the division of the heat and mass transfer equation into two, one of which describes moisture transfer due to the action of a gradient, Isotherm potential is the second due to the action of a temperature gradient.

So, for solving two and three-dimensional heat conduction problems, V.N.Bogoslovsky in [2] proposes the superposition method; consisting of the summation of simple solutions corresponding to particular components of the general problem.

A comprehensive (thermal imaging and full-scale) examination of the thermal condition of the external enclosing structures of operated buildings shows that there is almost always a discrepancy between the thermal characteristics

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and the values that were laid down in the projects. This discrepancy is caused not only by deviations from design solutions during construction but also due to changes in the thermal characteristics of the material layers of enclosing structures during operation under the influence of constantly changing ambient temperature and humidity. In addition, it should be noted that the current norms and rules for the design of thermal protection of buildings are based on stationary calculations of heat and moisture transfer in building envelopes and do not fully consider the climate features of the construction area, which also cannot guarantee the reliability of thermal protection during the operation of buildings. A new approach to the design of thermal protection, multi-layer building envelopes is required, based on multi-variant redesign calculations of their heat and moisture state, and assessment of their thermal efficiency in operational (non-stationary) conditions.

To implement this approach, a fairly simple engineering method of computer calculation is required, which allows modeling the processes of heat and moisture transfer occurring in the material layers of enclosing structures during operation, s; considering their individual properties and climate characteristics of the construction area. To provide the first indicator with external walls, it is necessary that the reduced resistance to heat transfer $R_0$, m²°C/W, be, according to [8], not less than the normalized value $R_{req}$, m²°C/W, i.e.

$R_0 \geq R_{req}$

The value of the normalized resistance to heat transfer is determined according to building regulations 23-02-2003 [8] depending on the degree-day of the heating period of the construction area $D_d$ °C, which in turn is determined; design air temperature inside the room $t_{ind}$, °C, design outdoor temperature (average for the coldest five-day period) $t_{out}$, °C, and duration of the heating period that days $R_{req}$.

### 2. Materials and Methods

Determination of resistance to heat transfer of a multi-layer building envelope in non-stationary conditions. The heat-shielding properties of the building envelope layer are determined by the thickness ratio. Layer (d) to its coefficient of thermal conductivity ($\gamma$), $d/\gamma$ which is called heat transfer resistance. The heat transfer resistance of a multi-layer building envelope is determined by the formula [9]:

$$R = \frac{d_1}{\gamma_1} + \frac{d_2}{\gamma_2} + \cdots + \frac{d_n}{\gamma_n} = R_1 + R_2 + \cdots + R_n,$$

(1)

Where $d_1, d_2, d_n$ - the thickness of the layers of enclosing structures, $\gamma_1, \gamma_2, \gamma_n$ - coefficient of thermal conductivity of the material layers of the building envelope; $R_1, R_2, R_n$ - resistance to heat transfer of the layers of the building envelope.

The streams of air constantly in motion (due to convection inside the room and wind influences from the outside) are decelerated in front of the surface of the structures; Slow-moving air has a higher heat-insulating capacity than fast-moving air, therefore, zones of slower air movement directly in front of the surface, structures act as additional heat-shielding layers [9], for the characteristics of which the concept of resistance to heat transfer has been introduced as $a_{in}$ near the inner surface of the enclosure, and sensing resistance defined as $a_{out}$ outside. The total heat transfer resistance of the fence $R_0$, considering heat transfer and heat absorption, the s determined, from expression

$$R_0 = \frac{1}{a_{in}} + \sum_{i=1}^{n} \frac{d_i}{\gamma_i} + \frac{1}{a_{out}}$$

(2)

In expressions (1) and (2), the thermal conductivity coefficient $\gamma$ characterizes the property of material layers to conduct heat. Heat transfer occurs in several ways in the thickness of wet building material. Through, a solid skeleton, as well as a film of liquid moisture, heat is transferred through thermal conductivity, and in pores filled with moist air, in addition to thermal conductivity, - convection, and radiation. During moisture exchange, heat can be transferred by liquid and vaporous moisture. Therefore, the coefficients of thermal conductivity of certain types of materials depend on the chemical and mineralogical composition of the forming substances, density, temperature, and humidity of the material. However, if materials' density and mineralogical composition are relatively constant over time, the moisture content and temperature of materials during operation are constantly changing. Various researchers give empirical formulas for the dependence of the thermal conductivity coefficient on the humidity for individual materials; to determine the coefficient of thermal conductivity of a wet material $\gamma_w$ in [9], the following dependence is given:

$$\gamma_w = \gamma \cdot \left(1 + W \cdot \frac{Z}{100}\right)$$

(3)

where $\gamma$ is the thermal conductivity of dry material; $W$ - weight moisture content of the material (moisture content), %; $Z$ is the coefficient of thermal conductivity increment per 1% humidity.
E. Schild in [9] grouped the materials most commonly used in enclosing structures by density and structure and gives the numerical values of Z shown in Table 1.

<table>
<thead>
<tr>
<th>N.</th>
<th>Name of building materials</th>
<th>Z gain per 1% humidity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Brick-full</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>- bodiedhollow</td>
<td>12.5</td>
</tr>
<tr>
<td>2</td>
<td>Concrete; silicate brick; aerated concrete; slag concrete; pumice concrete; wood-concrete</td>
<td>12.5</td>
</tr>
<tr>
<td>3</td>
<td>Gypsum boards</td>
<td>12.5</td>
</tr>
<tr>
<td>4</td>
<td>Particle boards; fibre board; light boards made of wood wool; straw or reed hollow slabs. Other organic fiber insulation</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>Cork slabs</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>Inorganic fiber insulation</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>Foamed artificial materials</td>
<td>2</td>
</tr>
</tbody>
</table>

However, for new-generation heat-insulating materials, such as urea-formaldehyde foam, new types of expanded polystyrene, etc., dependence (3) and Z values need to be refined. The value of the Z coefficient is determined experimentally individually for each material, however, there is no standard methodology in Interstate standard for building materials for determining the dependence of the thermal conductivity coefficient on the degree of moisture content of the material. Due to the limited nature of these data on new-generation heat-insulating materials in the literature and their absence in regulatory documents, dependence (3) for urea-formaldehyde foam and expanded polystyrene.

Samples of heat-insulating material (25x25x5) cm in size, dried at a temperature of (100 ± 5) °C and preliminary weighed, were kept in a tightly closed chamber with a mesh stand above a layer of water (Fig. 1) for 6 hours, then removed, weighed (to determine the current absorption humidity) and immediately tested on the ISK-U building materials thermal resistance (thermal conductivity) meter. Further, the tests were repeated by keeping the samples above water for 12, 18, 24, 30, 36, 42, 48, etc. hours until the samples reach the maximum absorption moisture content.

![Fig. 1. Chamber for sorption humidification of samples: 1 - a chamber with a tight-fitting lid; 2 - test sample; 3 - mesh stand; 4 - water.](image)

As a result of the approximation of the results obtained (the values of absorption moisture and the corresponding values of the thermal conductivity coefficients of the tested material), an empirical dependence of the thermal conductivity coefficient of thermal insulation materials on the weight humidity was obtained, which has the form.

\[ y_w = y \cdot (1 + kW) \]  

Where \( y \) and \( W \) are the same as in (3), \( k \) is the coefficient of the variable, which is determined for each heat-insulating material. For example, for materials such as urea-formaldehyde foam and expanded polystyrene, (0.046 for urea-formaldehyde foam and 0.033 for expanded polystyrene, Fig. 2).
For the outer (decorative) layer of multi-layer enclosing structures, the dependence of thermal conductivity on temperature is no less important, since at negative temperatures the thermal conductivity of materials at a similar humidity has higher values. By approximating the results of studies of changes in the thermal conductivity of building materials depending on temperature and humidity, A.U. Franchuk, given in [10], the dependence of the form (4) of the thermal conductivity coefficient on weight humidity at positive and negative temperatures for silicate brick with a density of 1700 kg/m³ were obtained (Fig. 3).

The moisture content of material W changes with time depending on the change in the parameters of the internal and external air, as well as the absorption properties of the material layers of the structure. All building materials have absorption properties to a greater or lesser extent. For each building material, except for absolutely dense materials (glass, metal), there is an “own” dependence of its humidity on the relative humidity of the air at a constant temperature — the adsorption isotherm, which is determined by experimental methods according to the standard method interstate standard 24816-81 [11]. On Figure 4 shows the absorption isotherms of building materials most commonly used in building envelopes, constructed based on the results of laboratory tests according to the method [11].
Fig. 2. Dependence of the thermal conductivity coefficient of heat-insulating materials on sorption moisture

For the outer (decorative) layer of multi-layer enclosing structures, the dependence of thermal conductivity on temperature is no less important, since at negative temperatures the thermal conductivity of materials at a similar humidity has higher values. By approximating the results of studies of changes in the thermal conductivity of building materials depending on temperature and humidity, A.U. Franchuk [10], given in the form (4) of the thermal conductivity coefficient on weight humidity at positive and negative temperatures for silicate brick with a density of 1700 kg/m³ were obtained (Fig. 3).

Fig. 3. Dependence of the thermal conductivity coefficient of silicate brick with a density of 1700 kg/m³ on weight humidity at positive and negative temperatures

The moisture content of material W changes with time depending on the change in the parameters of the internal and external air, as well as the absorption properties of the material layers of the structure. All building materials have absorption properties to a greater or lesser extent. For each building material, except for absolutely dense materials (glass, metal), there is an "own" dependence of its humidity on the relative humidity of the air at a constant temperature—the adsorption isotherm, which is determined by experimental methods according to the standard method interstate standard 24816-81 [11]. On Figure 4 shows the absorption isotherms of building materials most commonly used in building envelopes, constructed based on the results of laboratory tests according to the method [11].

Figure 4 shows that the absorption properties of building materials, characterized by absorption isotherms, are indeed individual for each material, but the same character of their distribution is observed depending on the change in the moisture potential. As a result of a graphical study of the absorption isotherms shown in Figs. 4, in the Excel program, by selecting trend lines, it was found that they are all approximated by similar functions—polynomials: of the third degree with correlation coefficients $R^2 = 0.9994 – 0.9999$ of the form.

$$w_s = A_1\varphi + A_2\varphi^2 + A_3\varphi^3$$  \hspace{1cm} (5)

where $w_s$ is the sorption moisture content of the material, %; $\varphi$-relative, air humidity, rel. units; $A_1$,$A_2$,$A_3$—variable coefficients;

The numerical values of the coefficients of the variable $A_1$,$A_2$,$A_3$ for materials whose sorption isotherms are shown in Figure 4, defined by the author and given in the table 2.

Table 2. Coefficients of the variable in the empirical dependences of the equilibrium moisture content of materials on the moisture potential

<table>
<thead>
<tr>
<th>Material</th>
<th>Density, g/sm³</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$A_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Urea-formaldehydefoam</td>
<td>17</td>
<td>0.0125</td>
<td>-0.0175</td>
<td>0.0140</td>
</tr>
<tr>
<td>Styrofoam</td>
<td>25</td>
<td>0.0250</td>
<td>-0.0325</td>
<td>0.0270</td>
</tr>
<tr>
<td>Mineralwool</td>
<td>50</td>
<td>0.0075</td>
<td>-0.0105</td>
<td>0.0209</td>
</tr>
<tr>
<td>silicatebrick</td>
<td>1700</td>
<td>0.0125</td>
<td>-0.0200</td>
<td>0.0120</td>
</tr>
</tbody>
</table>

Figure 5. Dependence of the change in the partial pressure of vapor in saturated air on temperature
The absorption humidity of the material layers of the building envelope depends on the change in the relative humidity on the layer, which is determined by the value of the partial air pressure in the pores of the material. The value of the partial pressure (elasticity) of water vapor in saturated air depends on the temperature $T$ of the air — it increases with increasing temperature and is determined either by constructing I-diagrams [2,12,13] or from tables compiled based on the results of experimental studies [14]. When performing calculations on a computer in a non-stationary mode, the use of tabular and diagrammatic data is practically unacceptable — a functional dependence is necessary. In a graphical study of these data, it was found that the dependence of the change in the partial pressure of water vapor on temperature is non-linear (see Fig. 5) and quite accurately, with a correlation coefficient $R^2 = 0.9996$, is approximated by a polynomial of the third degree of the form:

$$ p_u = 0.0213T^3 + 1.69T^2 + 48.02T + 611 $$

(6)

The humidity state of the air is characterized by relative humidity($\%$), which, according to the Boyle-Mariotte law, is determined by the ratio [15]

$$ \varphi = \frac{p_n}{p_u} $$

(7)

where $p_n$ — partial air pressure, Pa.

For saturated air $\varphi = 1$, or 100%, and unsaturated moist air $\varphi < 1$.

Thus, to determine the resistance to heat transfer of the building envelope, considering the actual moisture content of the material layers, it is necessary to obtain mathematical models for the non-stationary calculation of the following parameters:

- temperature distribution (temperature field) c. section and on the inner surface of the enclosing structures;
- partial pressure in the cross-section of the material layers of the structure;
- relative humidity in the cross-section of the material layers of the building envelope.

### 3. Results and Discussion

As a result, we can determine that:

- the temperature of the inner surface of the enclosing structure is at least 19.4, maximum - 20.5, which is higher than the standard dew point temperature;
- temperature difference between the temperatures of the internal air and the internal surface; fencing in the winter months is 1.6°C, which does not exceed the maximum allowable standard value of 4°C;
- the temperature in the section of the carrier layer (1,2,3 conditional layers) has an amplitude of daily fluctuations of 1-2°C and has values higher than the standard dew point temperature, with standard microclimate parameters; indoors) during the annual cycle of operation; therefore, it can be assumed that the formation and accumulation of condensation moisture in the bearing layer of the building envelope will not occur;
- the temperature in the section of the heat-insulating layer (conditional layer 4) has an amplitude of daily fluctuations of about 10°C and reaches a value above the standard dew point temperature, the rest of the time it has negative values and values below the dew point temperature, therefore, precipitation and accumulation of condensation moisture is possible in the layer during 8 months of the annual cycle of operation;
- the temperature in the cross-section of the decorative layer (conditional layer 5) has an amplitude of daily fluctuations of 15°C on average and reaches a value above the standard dew point temperature in the period from June 15 to August 15, in the rest of the experiment it has negative values and values below the dew point temperature, therefore; precipitation and accumulation of condensation moisture are possible in the layer during 10 months of the annual cycle of operation.

Analyzing Figure 6, it can be assumed that when maintaining the regulatory parameters of the climate in the room, in the load-bearing layer during the cycle of operation of the enclosing structure, the formation of condensation moisture — "waterlogging", does not occur (Partial pressure curves on the layer and pressure of saturated vapors do not intersect and do not coincide), in the thermal insulation layer the period of "waterlogging" (the coincidence of curves) is observed from December to February to February, and in the third decorative layer - from November to April. The coincidence of the partial pressure and saturated vapor pressure curves indicates the achievement of the relative air humidity on the layer of the maximum value equal to 1.0, and the beginning of the formation of condensation (drop) moisture, which is confirmed by the results of calculating the relative humidity in the structure section, shown in Fig. 7.
As a result, we can determine that:

- The material layers, it is necessary to obtain mathematical models for the non-saturated air.

The humidity state of the air is characterized by relative humidity $\phi$ and the value of the partial air pressure in the pores of the material is approximated by a polynomial of the third degree of the temperature $T$ of the air.

The humidity of the material layers of the building envelope depends on the change in the relative humidity of the material, which is determined by the value of the partial air pressure in the pores of the material.

Analyzing Figure 6, it can be assumed that when maintaining the temperature of the inner surface of the enclosing structure as a constant value, the period of "waterlogging" (the coincidence of the partial pressure and the saturated vapor pressure curves in the layer) is observed from December to February and in the third decorative layer from June 15 to August 15, in the rest of the experiment, it has negative values of fluctuations of 1 $^\circ$C on average and reaches a value above the standard dew point temperature in the period of February to March. The value of the temperature difference between the temperatures of the internal air and the internal surface; fencing in the layer of the building envelope for the period of winter months is 1.6 $^\circ$C, which does not exceed the maximum value of 2 $^\circ$C and has values higher than the standard dew point temperature, with standard fluctuations of 1 $^\circ$C.

Therefore, precipitation and accumulation of condensation moisture occur; fencing in the bore layer of the building envelope will not be possible in the layer during 8 months of the annual cycle of operation; therefore, it can be assumed that the temperature difference between the temperatures of the internal air and the internal surface; fencing in the layer of the building envelope for the period of winter months is 1.6 $^\circ$C, which does not exceed the maximum value of 2 $^\circ$C and has values higher than the standard dew point temperature, with standard fluctuations of 1 $^\circ$C.

Fig. 7. Estimated relative humidity of air in the cross-section of the layers of the building envelope in the annual cycle of operation.
An increase in the values of the coefficient of thermal conductivity of the material layers entails a decrease in the value of the reduced resistance to heat transfer of the enclosing structure. The results of calculating the reduced resistance to heat transfer of the building envelope in the annual cycle of operation are shown in Fig.8.

![Fig. 8. The reduced resistance to heat transfer of the building envelope in the annual cycle](image)

According to Fig.8, it can be seen that the initial value of the design resistance to heat transfer at the standard (by parameter A) values of the thermal performance of the material layers of the building envelope is 4.46 m²°C/W, which is higher than the normalized (4.14 m²- °C/W) and the minimum allowable (1.7 m²- °C/W) and meets the requirements of building regulations 2.01.04-2018 [16] for the mandatory indicator "a", but during operation, due to the moistening of the material layers of the structure, it decreases to 4.08 m²-°C/W, which is 1.4% lower than the standard value.

4. Conclusion

- As a result of the calculation, it was found that during operation the material layers of the building envelope are moistened, which causes an increase in their thermal conductivity coefficients. Moreover, the coefficient of thermal conductivity of the carrier; layer does not exceed the values recommended by building regulations 2.01.04-2018 [16] for parameter A, and the thermal conductivity of the decorative (outer) layer exceeds the recommended values by 15% and reaches the values recommended for parameter B and above.
- The coefficient of thermal conductivity of heat-insulating materials (the fourth heat-insulating layer) also increases during the operation of the structure and exceeds the values recommended by building regulations2.01.04-2018 [16] for parameter A, by 7÷21%, depending on the type of heat-insulating material.
- Locally produced expanded polystyrene and urea-formaldehyde foam satisfy both conditions and provide economic feasibility of use as thermal protection. At the same time, priority should be given to urea-formaldehyde foam, as the material with the lowest value of cmγm and providing the maximum amount of net, discounted income under these conditions.
- The functional dependence of the partial air pressure, thermal conductivity, absorption moisture, and moisture conductivity of building materials on temperature and humidity, necessary for the physical and mathematical model of non-stationary heat and moisture calculation, are obtained.
- According to the proposed method, the calculation of the effectiveness of thermal protection of three-layer building envelopes with a heat-insulating middle layer, made from local materials, was carried out in the operating conditions of the climate of Jizzakh. As a result of the calculation, it was found that three-layer enclosing structures with expanded polystyrene and urea-formaldehyde foam as the middle layer during operation in the climate of Jizzakh have a higher efficiency compared to structures with mineral wool as the middle layer.


