Mathematical Modelling of Heat Transfer in Pipes with Turbulators in the Laminar Region and in the Transition to Turbulent Flow

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Abstract. The article presents an analysis of mathematical modeling of heat transfer in pipes with turbulators at low Reynolds numbers characteristic of laminar and transient flow modes of heat carriers. The solution of the heat exchange problem for semicircular cross-section flow turbulators based on multi-block computing technologies based on the solution of Reynolds equations closed using the Menter shear stress transfer model and the energy equation (on multi-scale intersecting structured grids) by the factorized finite-volume method is considered. Both local and averaged characteristics of the flow and heat transfer to a pipe with a system of turbulators were obtained, which made it possible to determine the levels of heat exchange intensification for these regime ranges.

1 Introduction

In the present study, mathematical modeling of heat transfer in pipes with turbulators at low Reynolds numbers is carried out, characteristic of laminar (Re=100…1500) and transient (Re=1600…10000) modes of heat carriers, which were previously studied mainly experimentally.

The solution of the heat exchange problem for semicircular cross-section flow turbulators based on multiblock computing technologies based on the solution of Reynolds equations, closed using the Menter shear stress transfer model, and energy equations (on multi-scale intersecting structured grids) by the factorized finite volume method (FFVM) is considered.

This method has previously been successfully applied and verified by experiment [1] for higher Reynolds numbers [2].

Theoretical results illustrating the following are presented:

- both local and averaged characteristics of the flow and heat transfer to a pipe with a system of turbulators for laminar and transient modes of coolant flows were obtained, which made it possible to determine the levels of heat exchange intensification for these regime ranges;
- external manifestations of interaction in the flow of artificial surface protrusions in spaces with transitive flows are revealed: a decrease in the values of the average time coefficient of heat transfer in the channel sections under the laminar flow regime;

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generation of the effect of heat transfer intensification in segments with underdeveloped structural turbulence of flows; a decrease in the critical Reynolds numbers;

the advantage of the method used in the work based on the control volume method over the existing ones is that the latter [3] were based on a number of approximations, for example: Galerkin approximations, linearization of equations, application of methods of variable directions with subsequent implementation of run-through methods.

2 Research methodology

A well-known and very well-tested method of vortex intensification of heat exchange is the application of periodic protrusions on the walls of the washed surfaces [1, 2] (Fig. 1).

![Fig. 1. Section of a straight round pipe with transverse surface-positioned turbulators of the flow of square (upper figure) and semicircular (lower figure) cross sections.](image1)

The study of the structure of the intensified flow is mainly carried out by experimental methods [1, 2], while modern computational works on this topic are relatively few, for example [3...6], and are only partially devoted directly to the structure of the intensified flow; some of the methods, for example [12], use only integral approaches to this problem.

Recently, multiblock computing technologies for solving problems of vortex aeromechanics and thermophysics based on intersecting structured grids have been intensively developing.

This work is directly devoted to the study of heat transfer at Reynolds numbers, characteristic of laminar and transient flow regimes in pipes intensified by surface periodically located turbulators of semicircular cross-section, since there are not yet sufficiently reliable theoretical calculation data in this range.

The theoretical study of local and averaged parameters of flow and heat exchange in pipes with turbulators seems to be the most promising in the direction of developing specialized paralleled packages based on multiblock computing technologies, the target directions of which can be characterized as follows.

3 Mathematical modeling
The original multiblock computing technologies [6...9], based on multi-scale intersecting structured grids, were developed for the exact solution of non-stationary two-dimensional and three-dimensional problems of convective heat transfer in straight round pipes with organized roughness in the form of protrusions in a homogeneous working medium in a fairly wide range of Reynolds and Prandtl numbers.

The difference from the previous variants [6...9] is that the methodology is supplemented by the use of periodic boundary conditions, which allows us to evaluate the asymptotic characteristics of pipes with discrete roughness.

The modification made it possible to increase the computational efficiency of modeling, to implement correction for the curvature of the current lines. For pipes with turbulators, the following are determined: surface distributions of local and integral power and thermal characteristics (pressure, friction, heat flows, hydraulic losses), profiles of velocity components, pressure, temperature and turbulence characteristics (turbulence energy, turbulent viscosity, components of the Reynolds stress tensor, generation, dissipation, etc.).

The original system of partial differential equations, the Navier—Stokes and Reynolds equations, is closed using a modified model of shear stress transfer taking into account the curvature of the current lines, according to the Menter approach.

The initial information about the governing equations and acceptable boundary conditions is contained in [10].

Original pressure and mass average temperature correction procedures based on periodic boundary conditions are used. The methodology for solving the initial equations is a pressure correction procedure based on the concept of splitting by physical processes.

For problems with periodic boundary conditions, pressure gradient and average mass temperature correction procedures are applied.

The methodological basis of a promising computational tool is multiblock computing technologies based on the use of structured, intersecting multiscale grids associated with the capture of characteristic structural elements of the vortex flow and temperature field, which will provide acceptable accuracy and high efficiency comparable to the use of adaptive grids.

Here it is necessary to dwell in more detail on the specific features characteristic of periodic boundary conditions.

Periodic boundary conditions determine a more optimal construction of the pipe grid (Fig. 2). The pipe is divided into several sections with a turbulator located in the middle and input and output smooth sections.
The pipe grid consisting of several sections with a turbulator located in the middle, the input and output smooth sections.

In the periodic statement, only one section is considered, while in general it is necessary to use several sections, the number of sections reached 12; the same number of sections was used for verification.

To reduce the number of design nodes in the pipe, a more detailed wall area (blue grid) and a less detailed axial area (green) are allocated. At the same time, the degree of detail varies, both in the longitudinal and circumferential directions (when using a three-dimensional case). In addition, for three-dimensional calculation, a so-called "patch" is introduced in the axial region, eliminating unnecessary thickening of the grid near the axis.

The latter circumstance, all other things being equal, reduces the required number of calculation cells by about one and a half times (this circumstance becomes even more important with three-dimensional calculations).

You can also reduce the number of cells if you apply periodic conditions along the longitudinal axis, because the input and output sections are eliminated and one section is left.

From the point of view of hydrodynamics, the periodic problem is posed as a problem with the preservation of a given mass flow rate calculated for a unit velocity at the inlet. From the point of view of heat exchange, depending on the selected boundary conditions for temperature, two options are possible: for isothermal walls, the problem is solved under the assumption of the constancy of the average mass temperature in the input section; in the second case, the gradient of the average mass temperature is considered to be known, calculated from the value of the heat flow on the walls. Naturally, the input temperature is not fixed at the same time.

In addition to periodic complete recording of the current state of the task, the program provides for the possibility of performing selective recordings with their accumulation in a file at a given interval, which is especially important for use in solving non-stationary tasks.

The main attention is paid to the local and integral characteristics of convective heat exchange, including the components of velocity, hydraulic losses and the average heat transfer over the allocated area of the channel wall section, the results of calculation based on the turbulent characteristics of the equation terms for the energy of turbulent pulsations (generation, dissipation, convective and diffusion transfer). For the external flow of rectangular projections, a similar approach was applied, for example, in [11].

The main direction of this study can be briefly described as follows: calculations for relatively low Reynolds numbers characteristic of laminar and transient flow regimes in pipes with turbulators at various Prandtl numbers, where there are no reliable theoretical calculation data yet, since earlier calculations were carried out for higher Reynolds numbers; the main attention is paid to the specific aspects of the computational study of intensified heat transfer in the transition region, since the regions with higher Reynolds numbers have been studied earlier; analysis of the calculated data on heat exchange and hydraulic resistance in pipes with turbulators of semicircular cross-section for laminar and transition regions of Reynolds numbers Re=10^2...10^4.

4 Specifics of heat exchange intensification in laminar and
transition regions

In the area of laminar flow, the intensification of heat exchange is of little interest [1, 2]. However, even with large Reynolds numbers, the existence of a laminar boundary layer at a considerable distance from the entrance to the channel is possible, therefore, the nature of the interaction of the laminar boundary layer with artificial turbulators and the process of heat exchange under the conditions of viscous and viscous-gravitational modes of the coolant flow is of great importance.

For the region of the viscous regime, heat transfer in pipes with turbulators may be lower than for smooth pipes due to the thermal resistance of stagnant sedentary zones between turbulators [1, 2].

In the transient flow mode, artificial turbulators affect the flow pattern in the channel in two ways. On the one hand, they are perturbation generators, generating additional turbulent perturbations to the natural turbulent perturbations already existing in the flow. On the other hand, turbulators, reaching certain relative heights, interact with turbulent sections of intermittent flow and contribute to the rapid development of turbulent disturbances that have developed to the size of the passage section of the channel, the so-called "turbulent jams".

The alternation of sections of the channel having both laminar and turbulent structures, so, the intermittency of the flow in the transition region, causes a change in the conditions of heat exchange in any fixed section of the channel, therefore there are fluctuations in the heat transfer coefficients.

Under boundary conditions of the second kind, so, with regulated heat supply, fluctuations in the heat transfer coefficient manifest themselves in fluctuations in the temperature of the wall with an amplitude that depends on several factors: the limiting values of the heat transfer coefficients that correspond to the turbulent and laminar flow regimes in the channel with a fixed Reynolds number; the Struhal number, so, the oscillation frequency; the magnitude of the thermal load; the method of heat supply; from the values and ratios of the heat capacities of the wall and the liquid.

The external manifestations of interaction in the coolant flow of artificial flow turbulators in the transient flow region include the following: a decrease in the values of time-averaged heat transfer coefficients in sections of the pipe with a laminar flow regime; a much earlier generation of flow intermittency with a simultaneous narrowing of the range of Reynolds numbers of its existence; the emergence of the effect of heat exchange intensification in areas with an underdeveloped flow turbulence structure; a decrease in the critical Reynolds number Recr.

The determination of the critical Reynolds number Recr was carried out experimentally in the works [1, 2] by three independent methods that gave similar values: both by changing the laws of time-averaged local heat transfer at the end of the heated section of the channel, and by the Reynolds number corresponding to the maximum pulsation characteristic \( \Delta T_{\text{max}}/\Delta T_{\text{min}} = N_{\text{max}}/N_{\text{min}} \) for the section at the end of the heated section, as well as by changing the law of resistance in isothermal modes.

For flows with underdeveloped turbulence, the effect of artificial turbulence of the flow is as follows.

The turbulent flow at low Reynolds numbers has a low-filled velocity profile, in contrast to the developed turbulent flow. Consequently, the temperature profile will be less filled, and the main thermal resistance in the flow with underdeveloped turbulence will not be localized in a narrow wall layer, as for a developed turbulent flow, but distributed in much more extended wall layers. Therefore, in order to achieve effective turbulization of the flow, it is necessary to use turbulators of relatively high heights, commensurate with the thickness of the wall layer, where almost the entire temperature pressure is triggered.
The theoretical study of intensified heat transfer in the region of underdeveloped turbulence and the transition region was carried out to a much lesser extent than for the region of developed turbulent flow. Within the framework of this study, specific attention is paid to this aspect.

In [1, 2] it is indicated that there are experimental data on a high intensification of heat exchange in areas with lower Reynolds numbers: \( \text{Re}=2\cdot10^3\ldots10^4 \), which provides empirical data on intensified heat exchange in this area for various Prandtl numbers: \( \text{Pr}=2\ldots50 \).

The analysis of the experimental data of various researchers given in [1, 2] shows that there is a tendency to increase the effect of heat transfer intensification, with an increase in the relative height of the turbulators and an increase in the relative step between the turbulators in the transition region, since the above reduces the values of the critical Reynolds number \( \text{Re}_{cr} \), while with a developed turbulent flow of a droplet liquid, it is more appropriate use turbulators of relatively low altitude and with relatively small steps.

Consequently, the intensification of heat exchange in pipes in the transition region of the flow is promising and may be higher than the intensification of heat exchange for a droplet liquid in a turbulent region.

5 Analysis of local and averaged results of numerical studies

The above conditions the relevance of mathematical modeling of intensified heat transfer in pipes in areas with underdeveloped turbulence and in transition flow regions for gas and liquid heat carriers.

In addition to the experimental study, the intensification of heat transfer in the transient range of flows was studied theoretically for a protrusion with a transverse profile in the form of a semicircle based on a multiblock numerical technology based on calculations of factorized finite-volume technologies (FFVM) Reynolds equations (closed by models of transfer Menter shear stresses) and energy equations (on an unequal-scale structured intersecting grid) [6].

Numerical calculation has shown that the intensification of heat transfer will take place from certain Reynolds numbers, and for small Reynolds numbers it is insignificant.

Current lines were also calculated for transient flow conditions, which differ significantly with an increase in the Reynolds number \( \text{Re}=2\cdot10^3\ldots10^4 \), which justifies a qualitative increase in the intensification of heat exchange [6].

Numerical studies were also carried out for higher Reynolds numbers for pipes with turbulators: \( \text{Re} =10^4\ldots10^6 \), and then for \( \text{Re} =10^6\ldots10^{10} \) [6, 13].

Successful mathematical modeling for turbulent and transient modes of coolant flow justifies the use of this method for lower Reynolds numbers, so for the laminar region, which was experimentally studied for transformer oil [6, 13].

Similar flows and heat transfer for non-Newtonian fluids have also been studied [5], the level of heat transfer intensification, which may exceed Newtonian ones.

For some flow conditions, calculated current lines between protrusions with semicircular transverse profiles are calculated based on low-Reynolds Menter models implemented in the work (for transitive ranges), characteristic of the transient (\( \text{Re} =2\cdot10^3\ldots10^4; \frac{d}{D}=0.875\ldots0.983; \frac{t}{D}=0.486\ldots1.987; \text{Pr}=0.72\ldots50 \)) and laminar and flow modes (\( \text{Re} =10^2\ldots1.5\cdot10^3; \frac{d}{D}=0.80\ldots0.92; \frac{t}{D}=0.33\ldots1.94; \text{Pr}=170\ldots320 \)). The value of the temperature factor (the ratio of the wall temperature to the average mass temperature of the oil): 1.07…1.15 [5].

This area was experimentally studied in [5], where it was found that at \( \text{Re} \approx 1600 \) the flow regime becomes transient, since the nature of the change in hydraulic resistance changes qualitatively [5].

Mathematical simulations of the regime after \( \text{Re}>1600 \) (\( \text{Re} =1.6\cdot10^3\ldots2\cdot10^3 \)) and further up to \( \text{Re} =2.4\cdot10^3 \) were carried out both for turbulent flows by the method tested in [3, 4].
Calculated data on intensified heat exchange and hydroresistance were obtained for the conditions under consideration \((\text{Re} = 10^2 \div 2.4 \times 10^3; \ d/D=0.80 \div 0.92; \ t/D=0.33 \div 1.22; \ Pr=170 \div 320)\). The maximum value of the relative heat transfer was \(\text{Nu}/\text{Nusm}=2.5\) at \(\text{Re}=2.4 \times 10^3; \ d/D=0.80; \ t/D=0.66; \ Pr=250\) and the relative hydro resistance was greatest at \(\xi/\xi_{\text{sm}}=2.5\) at \(\text{Re}=2.4 \times 10^3; \ d/D=0.80; \ t/D=0.33; \ Pr=250\).

For lower tubulizers \(d/D=0.86\), the above values are lower: \(\text{Nu}/\text{Nusm}=2.3\) at \(\xi/\xi_{\text{sm}}=2.3\) under the same identity conditions.

When the height of the turbulator is reduced to the parameter \(d/D = 0.92\), the above relative parameters will be even smaller.

The minimum values of relative heat transfer took place in the laminar flow region at \(\text{Re} = 10^2\): \(\text{Nu}/\text{Nusm}=0.90 \div 0.95\) with relative hydraulic resistances \(\xi/\xi_{\text{sm}}=1.15 \div 1.75\). The intensification of heat transfer is manifested in the laminar region at \(\text{Re} = 10^3\), when the values of the relative intensified hydro resistance are \(\xi/\xi_{\text{sm}}=1.35 \div 2.25\).

The calculated data obtained in the article correlate well with similar data previously obtained by the authors [16].

The correlation of the calculated data on intensified heat exchange and hydro resistance in the laminar and transition regions with experimental data presented in this paper [5, 13, 14, 15] shows that the calculation of hydraulic resistance is in satisfactory agreement with the experiment [5, 13, 14, 15], and for heat exchange, the calculated data qualitatively correspond to experimental data [5, 13, 14, 15] (the maximum of relative heat transfer both in the experiment and in the calculation is realized at \(t/D = 0.66\)). But the quantitative correlation is complicated by the fact that in the works [5, 13, 14, 15] there is not enough data to verify the relative correspondence method implemented in the study [5, 13, 14, 15], therefore, experimental data give overestimated results regarding both the calculations performed in the article and the works [1, 3, 16].

As an illustration, in Fig. 3…10 for some conditions for the flow are shown the calculated line currents between the ledges with a semicircular transverse profiles calculated on the basis low - Reynolds Menter models (for transitive ranges), characteristic for the transition \((\text{Re} = 2 \times 10^3 \div 10^4; \ d/D=0.875 \div 0.983; \ t/D=0.486 \div 1.987; \ Pr=0.72 \div 50)\) (Fig. 3…6) and laminar (Fig. 7…10) and the flow regime \((\text{Re} = 10^2 \div 1.5 \times 10^3; \ d/D=0.80 \div 0.92; \ t/D=0.33 \div 1.94; \ Pr=170 \div 320)\).

Fig. 3. Current lines for pipes with turbulators of semicircular transverse profiles at \(t/D=0.496, \ d/D=0.875, \ Pr=0.72, \ \text{Re} = 10^3\).

Fig. 4. Current lines for pipes with turbulators of semicircular transverse profiles at \(t/D=0.5, \ d/D=0.912, \ Pr=0.72, \ \text{Re} = 2 \times 10^3\).
The presented data on current lines in channels with protrusions are fully consistent with the general physical concepts of the physical process occurring in the channel [1, 2, 5].
The gaps of detached and attached currents are clearly visible, as well as the main vortex for closed depressions.

The current lines also show the generation of vortices depending on the flow modes and the geometry of the ledge.

6 Conclusion

1. In the work, mathematical simulations of heat transfer in channels with turbulators with semicircular transverse outlines were performed under the Reynolds criteria characteristic of laminar \( R_e = 10^2 \ldots 2 \cdot 10^3 \) and transient \( R_e = 2 \cdot 10^3 \ldots 10^4 \) hydraulic flow modes, based on a multiblock numerical technology formed on solutions by finite-volume factorized the methods of the Reynolds equation and the energy equation, and revealed the intensification of heat transfer for relatively small Reynolds numbers \( R_e = 2 \cdot 10^3 \ldots 10^4 \) and a large range of Prandtl numbers, which may be relevant in channels.

2. As a result of the calculations carried out, the influence of the geometric parameters of the channel and the flow mode of the coolant on the intensified heat exchange during the transient flow mode was revealed. It is established that for turbulators of relatively medium and high altitudes in the transient mode, a large intensification of heat exchange takes place at large Prandtl numbers, and for relatively small heights of turbulators, the intensification of heat exchange decreases with an increase in the Prandtl number.

3. The advantage of the method used in the work based on the control volume method over the existing ones is that the latter [5] were based on a number of approximations, for example: Galerkin approximations, linearization of equations, application of methods of variable directions with subsequent implementation of run-through methods, application of the method of variable equations with subsequent implementation based on run-through methods, etc.

4. Mathematical modelling of heat transfer in pipes with turbulators of semicircular cross-section at Reynolds numbers characteristic of laminar \( R_e = 10^2 \ldots 2 \cdot 10^3 \) and transient \( R_e = 2 \cdot 10^3 \ldots 10^4 \) flow modes for viscous heat carriers \( \Pr = 170 \ldots 320 \) was carried out on the basis of multi-block computing technologies based on the solution of the Reynolds equations and the energy equation by the factorized finite-volume method and the intensification of heat exchange for relatively small Reynolds numbers \( R_e = 10^2 \ldots 2 \cdot 10^3 \ldots 10^4 \) in a wide range of Prandtl numbers is obtained, which may be relevant in the channels of heat exchangers.

5. The calculated data obtained in the laminar and transition regions on hydraulic resistance is in satisfactory agreement with the experiment [5, 13, 14, 15]; according to the heat exchange, the calculated data qualitatively correspond to the experimental data [5, 13, 14, 15], but quantitatively, experimental data give overestimated results with respect to both the calculations performed in the article and regarding the results of the work [1, 3, 16].

6. Using the FFVM method, both local and averaged characteristics of flow and heat exchange in pipes with turbulators for laminar and transient modes of coolant flow were obtained in the work, which made it possible to determine for these modes the levels of heat exchange intensification that satisfactorily correlate with the existing experiment.

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