Analysis Of Local Stability Of The Model On COVID-19 Spread In DKI Jakarta Province

Rizki Chika Audita Ariyani, Widowati Kartono, R. Heru Tjahjana, and R. Heri Soelistyo Utomo

Abstract. The province of DKI Jakarta in Indonesia has an advanced amount of COVID-19 incidents. Hence its dispersion must be restrained. The SEAIQHRD (Susceptible, Exposed, Asymptomatic, Infected, Quarantined, Hospitalized, Recovery, Deceased) model for the dispersion of COVID-19 was evolved in this article. Next, using NGM method to compute basic reproduction number and employing Routh-Hurwitz criterion method to analyze its local stability. Further, two equilibrium points, namely: endemic and disease-free equilibrium points, are obtained. The value of basic reproduction number is used to determine stability analysis. If basic reproduction number less than one, then the endemic equilibrium point is considered asymptotically stable. Based on the sensitivity analysis, the recovery rate of those who are symptomatic subpopulations can help stop the propagation of COVID-19 illness. This article employs data from the DKI Jakarta Province in numerical simulations to depict the dynamics of the COVID-19 dispersion model. According to the analysis's findings, the COVID-19 dispersion model is asymptotically stable at the endemic equilibrium point with $\mathbb{R}_0 = 2.1966$. This indicates that the average of each infected individual can infect two susceptible persons so that the number of infected persons will increase over time and cause an outbreak, which means that COVID-19 will remain in the community.

1 Introduction

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In mathematical modeling, various earlier research has examined the COVID-19 spread model, some of which include the SEIR model proposed by Suwardi Annas et al. [9] by including vaccination and self-isolation variables as model parameters, the SEIR model was proposed by Uttam Ghosh et al. [10–12] by considering hospitalization, quarantine, and the effects of panic, anxiety, and tension as model parameters, the Wintachai, et al. [13] SEIR model by taking into account the efficacy of vaccination in susceptible subpopulations, exposed subpopulations, and infected subpopulations as model parameters. The SEAIQHR model proposed by Dipo Aldila et al. [14] considers non-pharmaceutical interventions (use of medical masks, rapid tests) and pharmaceutical interventions (improvement of medical services in hospitals) as control variables. The SEIQ1Q2HR model was proposed by Widowati et al. [15] by adding quarantine and hospitalization as variables. Nurul Aini Istiqomah et al.'s STQIR model [16] considers physical distancing and self-precaution interventions as model parameters. Finally, U. A. Fitriani [17] proposed the STQIR model with a case study in Indonesia’s Central Java Province.

In this study, the authors discuss the development of the SEAIQHR model proposed by Dipo Aldila et al. [14] by adding the deceased variable to COVID-19 illness dispersion. Additionally, we performed a stability study on the model to produce two equilibrium points: endemic equilibrium (EE) and disease-free equilibrium (DFE) point. We can use basic reproduction number ($\mathcal{R}_0$) to calculate the stability analysis and the Routh-Hurwitz criterion method to define the local stability system. The results of $\mathcal{R}_0$ can be employed to determine the model stability. In this article, numerical simulations demonstrate the dynamics of the COVID-19 illness's propagation, based on data in DKI Jakarta Province obtained from the official website [18] from 1 March to 31 August 2022.

2 Model Formulation

Based on the pandemic conditions in DKI Jakarta province, we developed the SEAIQHR (susceptible-exposed-asymptomatic-infected-quarantined-hospitalized-recovered) model by adding the deceased variable. We divided the total human population into eight subpopulations: susceptible/vulnerable subpopulation ($S$), exposed/latent subpopulation ($E$), an asymptomatic subpopulation ($A$), a symptomatic/infected subpopulation ($I$), quarantine subpopulation ($Q$), hospitalized subpopulation ($H$), a recovered subpopulation ($R$), a deceased subpopulation ($D$).

Subpopulations in the model are classified based on their infection status, where Susceptible are individuals who are susceptible to infection, Exposed are individuals who are suspected of being infected, Asymptomatic people are those who are afflicted with the COVID-19 virus but have no symptoms and can still spread the illness to other people, Symptomatic people are those who have the COVID-19 viral infection and show symptoms as well as the ability to spread the illness to others, Quarantined are individuals who are under surveillance in quarantine, Hospitalized are infected individuals who require treatment in a hospital, Recovered people are infected people who managed to survive the infection without suffering any long-term health consequences, Deceased people are those who contracted the virus and died.

Table 1. Descriptions of parameters.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Interpretations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>enrollment rate of vulnerable subpopulations</td>
</tr>
<tr>
<td>$\mu$</td>
<td>typical passing rate</td>
</tr>
<tr>
<td>$\beta$</td>
<td>an infection rate of vulnerable subpopulations becomes latent subpopulations</td>
</tr>
<tr>
<td>$\zeta_1$</td>
<td>rate of latent subpopulations becomes asymptomatic subpopulations</td>
</tr>
<tr>
<td>$\zeta_2$</td>
<td>rate of latent subpopulations becomes symptomatic subpopulations</td>
</tr>
</tbody>
</table>
The following is given the SEAIQHRD model that has been developed:

\[
\begin{align*}
\frac{dS}{dt} &= \lambda - \beta S (A + I) - \mu S \\
\frac{dE}{dt} &= \beta S (A + I) - (\zeta_1 + \zeta_2 + \zeta_3) E - \mu E \\
\frac{dA}{dt} &= \zeta_1 E - \eta A - (1 - \eta) A - \mu A \\
\frac{dI}{dt} &= \zeta_2 E + \eta A - (\kappa_1 + \kappa_2 + \kappa_3) I - \mu I \\
\frac{dQ}{dt} &= \zeta_3 E + \kappa_1 I - \delta Q - (1 - \delta) Q - \mu Q \\
\frac{dH}{dt} &= \kappa_2 I + \delta Q - \tau H - (1 - \tau) H - \mu H \\
\frac{dR}{dt} &= (1 - \eta) A + \kappa_3 I + (1 - \delta) Q + (1 - \tau) H - \mu R \\
\frac{dD}{dt} &= \tau H.
\end{align*}
\]

With initial state \(S(0) \geq 0, E(0) \geq 0, A(0) \geq 0, I(0) \geq 0, Q(0) \geq 0, H(0) \geq 0, R(0) \geq 0, D(0) \geq 0\),

\(\mathbb{R}_0\) is crucial. The average number of new incidents brought on by those who can spread the illness is known as \(\mathbb{R}_0\). If \(\mathbb{R}_0 > 1\), it signifies that one diseased person can spread the disease to more than one vulnerable person, which makes the disease-free equilibrium unstable and leads to an epidemic. However, if \(\mathbb{R}_0 < 1\), the DFE points will be locally asymptotically stable.
\[ \frac{dx}{dt} = F(x) - V(x) \]

\[ F(x) = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \end{bmatrix} = \begin{bmatrix} \beta S(A + I) \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad V(x) = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} (\zeta_1 + \zeta_2 + \zeta_3)E + \mu E \\ -\zeta_1E + (1 + \mu)A \\ -\zeta_3E - (\kappa_1 + \kappa_2 + \kappa_3)I + \mu I \\ -\kappa_2I - \delta Q + (1 + \mu)H \end{bmatrix} \]

\[ \mathcal{R}_0 = \frac{\beta \lambda (\eta + \mu + \kappa_1 + \kappa_2 + \kappa_3)\zeta_1 + (1 + \mu)\zeta_2}{\mu (1 + \mu)(\kappa_1 + \kappa_2 + \kappa_3 + \mu)(\zeta_1 + \zeta_2 + \zeta_3 + \mu)} \]

### 4 Equilibrium Point

The equilibrium point is a state where the total subpopulations do not change over time. COVID-19 spread model is said to be equilibrium if it satisfies:

\[ \frac{dS}{dt} = 0, \quad \frac{dA}{dt} = 0, \quad \frac{dI}{dt} = 0, \quad \frac{dQ}{dt} = 0, \quad \frac{dH}{dt} = 0 \]

**E\textsuperscript{*} (S\textsuperscript{*}, E\textsuperscript{*}, A\textsuperscript{*}, I\textsuperscript{*}, Q\textsuperscript{*}, H\textsuperscript{*})**

**E\textsuperscript{0} (S\textsuperscript{0}, E\textsuperscript{0}, A\textsuperscript{0}, I\textsuperscript{0}, Q\textsuperscript{0}, H\textsuperscript{0})**

\[ S^* = \frac{(\mu + \kappa_1 + \kappa_2 + \kappa_3)(\mu + 1)(\mu + \zeta_1 + \zeta_2 + \zeta_3)}{((\eta + \mu + \kappa_1 + \kappa_2 + \kappa_3)\zeta_1 + (1 + \mu)\zeta_2)\lambda} \]

\[ E^* = \frac{\mu + \zeta_1 + \zeta_2 + \zeta_3}{\lambda} \]

\[ A^* = \frac{\mu + \zeta_1 + \zeta_2 + \zeta_3}{\zeta_1 \lambda} - \frac{(\eta + \mu + \kappa_1 + \kappa_2 + \kappa_3)\zeta_1 + (1 + \mu)\zeta_2}{\zeta_1 (\mu + \kappa_1 + \kappa_2 + \kappa_3)\mu} \]

\[ I^* = \frac{(\mu + \kappa_1 + \kappa_2 + \kappa_3)(\mu + 1)\zeta_1 + \zeta_2 + \zeta_3}{(\eta + \mu + \kappa_1 + \kappa_2 + \kappa_3)\zeta_1 + (1 + \mu)\zeta_2} \]

\[ Q^* = \frac{(\eta + \mu + \kappa_1 + \kappa_2 + \kappa_3)\zeta_1 + (1 + \mu)\zeta_2}{(\mu + 1)(\eta + \mu + \kappa_1 + \kappa_2 + \kappa_3)\zeta_1 + (1 + \mu)\zeta_2} \]

\[ H^* = \left( \frac{(\eta + \mu + \kappa_1 + \kappa_2 + \kappa_3)\zeta_1 + (1 + \mu)\zeta_2}{(\mu + 1)^2(\mu + \kappa_1 + \kappa_2 + \kappa_3)(\mu + \zeta_1 + \zeta_2 + \zeta_3)} \right) \lambda \]
\[
- \left( \frac{\eta \zeta, \kappa_3 + (\mu + \kappa_1 + \kappa_2 + \kappa_3)(1 + \mu), \zeta_3) \mu \delta}{(\mu + 1)^2(\eta + \mu + \kappa_1 + \kappa_2 + \kappa_3) \zeta_1 + (1 + \mu) \zeta_2)}, \beta \right)
- \left( \frac{1 + \mu)((\eta + \mu + \kappa_1 + \kappa_2 + \kappa_3) \zeta_1 + (\mu + 1) \zeta_2), \beta \right)
\]

5 Stability Analysis

\[ E^*(S^*, E^*, A^*, I^*, Q^*, H^*) \]

\[ \mathcal{E}_0(S_0, E_0, A_0, I_0, Q_0, H_0) \]

Theorem 5.1

\[ \mathcal{R}_0 = \frac{\beta \lambda((\eta + \mu + \kappa_1 + \kappa_2 + \kappa_3) \zeta_1 + (1 + \mu) \zeta_2)}{\mu(1 + \mu)((\kappa_1 + \kappa_2 + \kappa_3 + \mu) \zeta_1 + \zeta_2 + \zeta_3 + \mu)} \]

\[ \mathcal{R}_0 < 1, \mathcal{E}_0(S_0, E_0, A_0, I_0, Q_0, H_0) = \left( \frac{2}{\mu}, 0, 0, 0, 0 \right) \]

\[ \mathcal{R}_0 > 1, \mathcal{E}_0(S_0, E_0, A_0, I_0, Q_0, H_0) = \left( \frac{2}{\mu}, 0, 0, 0, 0, 0 \right) \]

Proof:

\[ \mathcal{E}_0(S_0, E_0, A_0, I_0, Q_0, H_0) = \left( \frac{2}{\mu}, 0, 0, 0, 0 \right) \]

\[ |J(\mathcal{E}_0)| - \lambda \]

\[ (\mu + x)(\mu + x + 1)^2(a_0 x^2 + a_1 x^2 + a_2 x + a_3) = 0 \]

\[ a_0 = 1 \]
\[ a_1 = 3\mu + \zeta_1 + \zeta_2 + \zeta_3 + \kappa_1 + \kappa_2 + \kappa_3 + 1 \]
\[ a_2 = \frac{\beta \lambda(\zeta_1 + \zeta_2)}{\mu} + 3\mu^2 + 2\mu(\zeta_1 + \zeta_2 + \zeta_3 + \kappa_1 + \kappa_2 + \kappa_3 + 1) \]
\[ + ((\kappa_1 + \kappa_2 + \kappa_3 + 1)(\zeta_1 + \zeta_2 + \zeta_3) + (\kappa_1 + \kappa_2 + \kappa_3)) \]
\[ a_3 = \frac{-\beta \lambda((\eta + \mu + \kappa_1 + \kappa_2 + \kappa_3) \zeta_1 + (\mu + 1) \zeta_2)}{\mu} + (\mu + \kappa_1 + \kappa_2 + \kappa_3) \]
\[ + 1)(\mu + \zeta_1 + \zeta_2 + \zeta_3) \]

\[ a_1, a_2, a_3 > 0 \]

\[ a_1, a_2 - a_3 > 0 \]

\[ a_1 > 0, a_2 > 0, a_3 > 0 \]

\[ \mathcal{E}_0(S_0, E_0, A_0, I_0, Q_0, H_0) \]

\[ \mathcal{R}_0 < 1 \]

\[ a_3 > 0 \]

\[ a_1 > 0, a_2 > 0, a_3 > 0, \mathcal{E}_0(S_0, E_0, A_0, I_0, Q_0, H_0) \]

\[ \mathcal{R}_0 < 1 \]

\[ a_3 = \frac{-\beta \lambda((\eta + \mu + \kappa_1 + \kappa_2 + \kappa_3) \zeta_1 + (\mu + 1) \zeta_2)}{\mu} \]
\[ + (\mu + \kappa_1 + \kappa_2 + \kappa_3)(\mu + 1)(\mu + \zeta_1 + \zeta_2 + \zeta_3) \]

\[ \mathcal{E}_0(S_0, E_0, A_0, I_0, Q_0, H_0) \]

\[ a_3 > 0 \]
\(-\beta \lambda (\eta + \mu + \kappa_1 + \kappa_2 + \kappa_3)\zeta_1 + (\mu + 1)\zeta_2)\\+ \mu(\mu + 1)(\mu + \kappa_1 + \kappa_2 + \kappa_3)(\mu + \zeta_1 + \zeta_2 + \zeta_3) > 0
\)
\(\beta \lambda (\eta + \mu + \kappa_1 + \kappa_2 + \kappa_3)\zeta_1 + (\mu + 1)\zeta_2 < \mu(\mu + 1)(\mu + \kappa_1 + \kappa_2 + \kappa_3)(\mu + \zeta_1 + \zeta_2 + \zeta_3)
\)
\(\beta \lambda ((\eta + \mu + \kappa_1 + \kappa_2 + \kappa_3)\zeta_1 + (1 + \mu)\zeta_2) = \Re_0 < 1
\)

\(a_2 > 0 \quad a_1, a_2 - a_3 > 0 \quad a, a_2 - a_3 > 0
\)

\[a_2 = -\frac{\beta \lambda (\zeta_1 + \zeta_2) + 3\mu^3 + 2\mu^2(\zeta_1 + \zeta_2 + \zeta_3 + \kappa_1 + \kappa_2 + \kappa_3 + 1)}{\mu(\kappa_1 + \kappa_2 + \kappa_3 + 1)(\zeta_1 + \zeta_2 + \zeta_3) + (\kappa_1 + \kappa_2 + \kappa_3)} \]

\[A = (\zeta_1 + \zeta_2), B = (3\mu^3 + 2\mu^2(\zeta_1 + \zeta_2 + \zeta_3 + \kappa_1 + \kappa_2 + \kappa_3 + 1) + \mu((\kappa_1 + \kappa_2 + \kappa_3 + 1))
\]

\(a_1, a_2 - a_3 = (3\mu + \zeta_1 + \zeta_2 + \zeta_3 + \kappa_1 + \kappa_2 + \kappa_3 + 1)
\]

\[a_1, a_2 - a_3 = (3\mu + \zeta_1 + \zeta_2 + \zeta_3 + \kappa_1 + \kappa_2 + \kappa_3 + 1)
\]

\[J = (3\mu + \zeta_1 + \zeta_2 + \zeta_3 + \kappa_1 + \kappa_2 + \kappa_3 + 1)
\]

\[K = \beta \lambda (\zeta_1 + \zeta_2) + 3\mu^3 + 2\mu^2(\zeta_1 + \zeta_2 + \zeta_3 + \kappa_1 + \kappa_2 + \kappa_3 + 1)
\]

\[M = \beta \lambda ((\eta + \mu + \kappa_1 + \kappa_2 + \kappa_3)\zeta_1 + (\mu + 1)\zeta_2) \]

\[a_1, a_2 - a_3 = -\frac{J}{\mu} + \frac{M}{\mu(\mu + 1)(\mu + \kappa_1 + \kappa_2 + \kappa_3)(\mu + \zeta_1 + \zeta_2 + \zeta_3)}
\]

\[-\frac{\mu(1 + \mu)(\mu + \kappa_1 + \kappa_2 + \kappa_3)(\mu + \zeta_1 + \zeta_2 + \zeta_3)}{\mu} < 0
\]

\[J + \mu(1 + \mu)(\mu + \kappa_1 + \kappa_2 + \kappa_3)(\mu + \zeta_1 + \zeta_2 + \zeta_3)(\Re_0 - 1) < 0
\]
\[a_1, a_2 - a_3 > 0 \quad \text{and} \quad JK + \mu(1 + \mu)(\mu + \kappa_1 + 
\kappa_2 + \kappa_3)(\mu + \xi + \xi_2 + \xi_3)(\mathbb{R}_0 - 1) < 0. \]

\[\mathbb{E}_0(s_0, E_0, A_0, I_0, Q_0, H_0) = \left(\frac{1}{\mu}, 0, 0, 0, 0, 0\right) \quad \text{and} \quad \mathbb{R}_0 < 1. \]

\[\mathbb{E}^*(S', E', A', I', \mathbb{R}' , H') \quad \text{is locally asymptotically stable.} \]

**Theorem 5.2**

\[\mathbb{R}_0 = \frac{\beta(\eta + \mu + \kappa_1 + \kappa_2 + \kappa_3)}{\mu(1 + \mu)(\kappa_1 + \kappa_2 + \kappa_3)} \]

\[\mathbb{R}_0 > 1, \quad \mathbb{E}^*(S', E', A', I', \mathbb{R}' , H') \quad \text{is locally asymptotically stable.} \]

\[\mathbb{R}_0 < 1, \quad \mathbb{E}^*(S', E', A', I', \mathbb{R}' , H') \quad \text{is locally asymptotically stable.} \]

**Proof:**

\[|J(\mathbb{E}^*) - x| = 0 \]

\[(\mu + x + 1)(a_0 x^4 + a_1 x^3 + a_2 x^2 + a_3 x + a_4) = 0 \]

\[a_0 = 1 \quad a_1 = \beta(A' + I') + 4\mu + \xi_1 + \xi_2 + \xi_3 + \kappa_1 + \kappa_2 + \kappa_3 + 1 \]

\[a_2 = \beta(3\mu(A' + I') + A' + I' - S')(\xi_1 + \xi_2 + \xi_3 + \kappa_1 + \kappa_2 + \kappa_3 + 1) \]

\[a_3 = \beta(3\mu^2(A' + I') + (2A' + I' - S')(\xi_1 + \xi_2 + \xi_3 + \kappa_1 + \kappa_2 + \kappa_3 + 1)(A' + I')) \]

\[a_4 = \beta((A' + I')^3 + (A' + I' - S')(\xi_1 + \xi_2 + \xi_3 + \kappa_1 + \kappa_2 + \kappa_3 + 1)(A' + I')) \]

\[x_1 + x_2 + x_3 + x_4 = -\frac{b}{a} < 0 \]

\[-(\beta(A' + I') + 4\mu + \xi_1 + \xi_2 + \xi_3 + \kappa_1 + \kappa_2 + \kappa_3 + 1) < 0 \]

\[\Rightarrow -(\beta(\eta + \mu + \kappa_1 + \kappa_2 + \kappa_3)\xi_1 + (\mu + 1)\xi_2 - (4\mu + \xi_1 + \xi_2 + \xi_3 + \kappa_1 + \kappa_2 + \kappa_3 + 1) < 0 \]
\[ f = ((\eta + \mu + \kappa_1 + \kappa_2 + \kappa_3)\zeta_1 + (\mu + 1)\zeta_2) \]
\[ g = (4\mu + \zeta_1 + \zeta_2 + \zeta_3 + \kappa_1 + \kappa_2 + \kappa_3 + 1) \]
\[ \Leftrightarrow - (\Re_0 - 1) \zeta_1 \mu (\mu + \kappa_1 + \kappa_2 + \kappa_3) - (\Re_0 - 1) \mu - gf < 0 \]
\[ \Leftrightarrow - (\Re_0 - 1) \mu (\zeta_1 (\mu + \kappa_1 + \kappa_2 + \kappa_3) + 1) - gf < 0 \]

\[ \Re_0 - 1 > 0 \]
\[ \Re_0 > 1 \]

\[ x_1 x_2 + x_1 x_3 + x_2 x_3 + x_1 x_4 + x_2 x_4 + x_3 x_4 = \frac{c}{a} > 0 \]
\[ c = a \]
\[ x_1 x_2 x_3 + x_1 x_2 x_4 + x_1 x_3 x_4 + x_2 x_3 x_4 = \frac{d}{a} < 0 \]
\[ d = a \]
\[ x_1 x_2 x_3 x_4 = \frac{e}{a} > 0 \]
\[ e = a \]

\[ E^*(S^*, E^*, A^*, I^*, Q^*, H^*) \]

6 Numerical Simulation

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
\textbf{Parameter} & \textbf{Value} & \textbf{Parameter} & \textbf{Value} & \textbf{Parameter} & \textbf{Value} \\
\hline
\textbf{Estimated} & \textbf{Estimated} & \textbf{Estimated} & \textbf{Estimated} & \textbf{Estimated} & \textbf{Estimated} \\
\hline
\hline
\textbf{\( \lambda \)} & \textbf{10644776} & \textbf{\( \Re_0 \)} & \textbf{1.00637620} & \textbf{\( \kappa_1 \)} & \textbf{1.99628677} \\
& \textbf{70 \times 365} & & & & \textbf{0.00371323} \\
\hline
\textbf{\( \mu \)} & \textbf{416,62528} & \textbf{\( \Re_0 \)} & \textbf{0.00177894} & \textbf{\( \kappa_2 \)} & \textbf{0.01218} \\
& \textbf{391398 \times 10^{-5}} & & & & \textbf{0.00471323} \\
\hline
\textbf{\( \beta \)} & \textbf{2.00142 \times 10^{-7}} & \textbf{\( \kappa_3 \)} & \textbf{1.99628677} & \textbf{\( \eta \)} & \textbf{0.32160} \\
& \textbf{0.00920715} & & & & \textbf{0.0471323} \\
\hline
\end{tabular}
\caption{Numerical parameter estimation}
\end{table}

\[ S(0) = 7.191233 ; \quad E(0) = 1.928799 ; \quad A(0) = 1.091179 ; \quad I(0) = 87.294 ; \quad Q(0) = 32.698 ; \quad H(0) = 5.077 ; \quad R(0) = 1.126008 ; \quad D(0) = 14.690 \]
The value $R_0 = 2,1966 > 1$ is obtained using the parameters estimated in Table 2, with $R_0 = 2,1966 > 1$. The following figure shows the dynamic conduct of the SEAIQHRD model (susceptible, exposed, asymptomatic, symptomatic, quarantined, hospitalized, recovered, and deceased) concerning the level of COVID-19 disease spread.

Fig. 1. COVID-19 Disease Simulation Model in DKI Jakarta Province.

Accordingly, $E^*(S^*, E^*, A^*, I^*, Q^*, H^*)$ is locally asymptotically stable because $R_0 > 1$, as a result, infected subpopulations, both asymptomatic and symptomatic, can transmit the virus to susceptible subpopulations so that COVID-19 disease remains in a population over time.

7 Sensitivity Analysis

Sensitivity analysis was used to find the parameters that significantly predispose the value of $R_0$ in the dispersion of the COVID-19 illness. The first step to do is to find an index of sensitivity of each parameter to $R_0$.

Definition 7.1 [30, 31]

The sensitivity index is obtained by finding the partial derivative of $R_0$ to the parameter $P$, i.e.

$$C_P^{R_0} = \frac{\partial R_0}{\partial P} \times \frac{P}{R_0}$$
Next, substitute the parameter values \( \lambda, \mu, \beta, \zeta_1, \zeta_2, \zeta_3, \eta, \kappa_1, \kappa_2, \kappa_3 \) to their respective sensitivity index analysis equation, an index of sensitivity value to \( R_0 \) is presented in the following table:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Index of Sensitivity</th>
<th>Symbol</th>
<th>Index of Sensitivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda )</td>
<td>1</td>
<td>( \kappa_2 )</td>
<td>(-0.001483658433)</td>
</tr>
<tr>
<td>( \beta )</td>
<td></td>
<td>( \zeta_3 )</td>
<td>(-0.007141996730)</td>
</tr>
<tr>
<td>( \eta )</td>
<td></td>
<td>( \zeta_2 )</td>
<td>(-0.03679017198)</td>
</tr>
<tr>
<td>( \zeta_3 )</td>
<td></td>
<td>( \kappa_3 )</td>
<td>(-0.8441709064)</td>
</tr>
<tr>
<td>( \kappa_1 )</td>
<td>(-0.0005317831347)</td>
<td>( \mu )</td>
<td>(-1.000070928)</td>
</tr>
</tbody>
</table>

The value of sensitivity index \( \eta = 0.04874259046 \) shows that if we increase/decrease the parameter value \( \eta \) by 10\%, it will increase/decrease score of \( R_0 \) by 0.04874259046\%.

According to the index of parameter sensitivity, \( \lambda, \beta, \zeta_1, \zeta_2, \zeta_3, \eta, \kappa_1, \kappa_2, \kappa_3 \) are positive, so the parameters have an effect on \( R_0 \) due to the disease's propagation. Then, the value of the sensitivity index \( \kappa_3 = -0.8441709064 \) is obtained, which shows that if we increase/decrease the parameter value \( \kappa_3 \) by 10\%, it will decrease/increase the value of \( R_0 \) by 8.441709064\%.

Based on Table 3, the parameters sensitivity index \( \kappa_1, \kappa_2, \zeta_2, \zeta_3, \kappa_3, \mu \) is negative, so the parameters play a role in disease control.

8 Conclusion

The SEAIQHR model has been modified to create the SEAIQHRD model for transmitting COVID-19 disease by including the deceased variable (D). The indicated model includes eight subpopulations: susceptible, latent, asymptomatic, symptomatic, quarantined, hospitalized, recovered, and deceased. In this study, the dispersion of the COVID-19 virus is then shown analytically to be dependent on \( R_0 \) at the equilibrium point, and its value helps determine the model’s stability. Routh-Hurwitz method is the approach used to analyze local stability. According to the analysis, \( R_0 > 1 \), and the model is locally asymptotically stable at the endemic equilibrium point. The sensitivity analysis reveals that the COVID-19 disease can be contained by increasing the pace at which symptomatic subpopulations recover. This implies that the spread of COVID-19 will slow as more people recover from symptomatic persons, allowing COVID-19 to be controlled.

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10 References


