Using the fundamentals of the theory of measurement errors in performing geodesic measurement and calculation works

Bakhriddin Akhmedov[1*]

[1]Fergana Polytechnic Institute, Fergana str., 86, 150107 Fergana, Uzbekistan

Abstract. In this article, the tasks of the theory of measurement errors during geodetic measurement and calculation practices, equal and unequal measurements and measurement errors, i.e. deviations from the actual values of the measured quantity as a result of measurement issues aimed at elucidating the work and their classifications are covered. Also, the criteria of measurement accuracy, expressions for their determination, as well as the procedure for processing one quantity measured with equal accuracy and a sample example of its implementation are given.

1 Introduction

Measurements play a very important role in all fields of technology, they provide basic information for exact sciences. Measurements are the main content of all works performed for the purpose of studying the surface of the cartographic and geodetic zone.

Geodesy is an ancient science, which was created according to the requirements of people's living conditions and continues to develop in accordance with it. So, measurement practices in this science also go back to ancient human societies. The peoples of ancient Egypt, India, Central Asia, and other countries used measurement to solve issues necessary for irrigation networks, construction of various buildings, distribution of cropland, and similar needs.

Thinkers who lived in the 9th-10th centuries - Al-Khorazmi, the author of the book "The Image of the Earth" and the founder of the science of algebra, Al-Farghani, the astronomer and geodesist, the author of the books "The Art of Astrolabia" and "The Basics of Astronomy" and "Geodesiya The author of the work, the encyclopedist Abu Rayhan Beruni, made a great contribution to solving the above problems in geodesy.

The measurement of any value is considered from two points of view: quantitative - representing the numerical value of the measured value and qualitative - representing the accuracy of the measurement.

With the development of science and technology, especially geodesy, the accuracy of measurements increases and the methods of their mathematical processing are improved.

* Corresponding author: zohidjon20.06@gmail.com

There should be no gross errors in the measurements - no errors or miscalculations. In geodetic practice, at least two measurements of each value are always performed to determine the result, and in addition, mathematical relationships between the measured values are used.

It is known from experience that multiple (repeated) measurements of any constant quantity, even with the most care and accuracy, always give different results and mathematically related quantities, i.e., measurement result differences (will generate a no-connection error). The difference between the results of repeated measurements and the order of magnitude of the differences should correspond to the accuracy of the measurements, which indicates the absence of gross errors in the measurements.

It is necessary to pay special attention to the issue of measurement accuracy and their mathematical processing. Excess measurements lead to an increase in the volume and time of work and an increase in their cost. Therefore, the problem of determining the appropriate, i.e. necessary and sufficient, accuracy of measurements and processing their results arises.

The theory of mathematical processing of geodetic measurements, the quality of geodetic measurements, the occurrence of inevitable small errors and the laws of motion, the development of calculation rules for the assessment and calculation of the necessary accuracy, as well as calculation methods that allow obtaining the best final results with the economic expenditure of computational labor and related ways.

2 Discussion

2.1 Tasks of the theory of measurement errors

Any measurements, no matter how carefully they are performed, come with some deviation from the true value being measured, that is, errors. By improving the measuring methodology, constructions of measuring instruments and equipment, and improving the qualifications of observers, experience and practice will be a positive factor in reducing the deviation of the results of geodetic measurements from the real value. In practice, it is impossible to obtain completely error-free results. Therefore, in practice, measurements are made in such a way that the results are within a certain "specified accuracy". The concept of "specified accuracy" should be based on certain defined numerical criteria . [1]

The main tasks of the theory of measurement errors are as follows: establishing the specified criteria, developing methods for obtaining and evaluating the criteria.

2.2 General information about measurements

All quantities used in geodetic work can be divided into measured and calculated measurements, that is, their approximate values are obtained as a result of measurements or calculated using the function of measured values. [1]

Quantification is the comparison of two quantities with the same units of measurement and the determination of how much of that unit there is.

According to the accuracy of the measurement results or depending on whether the measurement conditions change or not, measurements are divided into measurements with *equal accuracy* and measurements without *equal accuracy* .

In the mathematical processing of measurements, the concepts of *necessary measurements* and *excess measurements* play an important role. [5,6,7]

To solve geodetic problems, it is usually necessary to measure several quantities.

The minimum number of magnitudes required to solve a given geodesic problem is called the number of required magnitudes, often this term is called *the number of required measurements* .

When solving geodetic problems, obtaining additional measurement values increases the accuracy of the results of the sought quantity and allows to identify errors in measurement and calculation work.

2.3 Measurement errors

As mentioned above, any measurements are accompanied by errors, that is, the deviation of the values of the measurement results from the true values of the measured quantity. [1]

These errors occur in the measurement process, in addition to the object being measured, the observer-measurer, measuring instruments and equipment, and the external environment. These complex conditions are constantly changing, and it is impossible to account for these changes without errors. Therefore, the deviation of the measurement results from the actual value happens differently every time.

The reflection of the fluctuations in the measurement results, which are among the total factors that appear in the measurement results, are permanent changes in the measurement conditions. [8-10]

The actual value of the measured quantity by X, and the measurement result x_i by . The difference between the measured value and the actual value of the measurand is *the measurement error* , often called *the true error* .

$$\theta\ (D) = x_i - X.$$

Having a large number of real errors in measurement, we can study the laws of their occurrence.

The results of the measurement of one quantity can be considered as discrete values of some continuous random variable obeying the law of normal distribution. At the same time , to describe the characteristics of the measurement results, it is necessary to know the main distribution parameters of the indicated random variable: *mathematical expectation (arithmetic mean)* and *standard (mean squared deviation)* . [4]

(classification) of measurement errors . Measurement errors are divided into "instrumental", external or environmental and personal types according to their source of origin.[1]

A gross error. The theory of mathematical processing of measurements does not take into account gross errors caused by mistakes or miscalculations of the observer, malfunctions of equipment and devices, sudden changes in external environmental conditions, etc.

Random and systematic errors. From the set of elementary errors in the component of the total error of measurements, errors are divided into two main categories: random and systematic.

Elementary errors that represent the values of random variables with a mathematical expectation that differs very little from zero are called random errors. [11-13]

Elementary errors, which are values of random variables with mathematical expectations significantly different from zero, are called systematic errors.

2.4 Measurement accuracy criteria

To assess the accuracy of measurements of certain quantities, it is necessary to determine the possible deviations of the measurement results from the actual value of this quantity. The deviation of the measured value of a certain quantity from its actual value is expressed in two different quantities: *the deviation of the measured value from the mathematical expectation*

- through the result of random errors, *the deviation of the mathematical expectation from the actual value* - through systematic (systematic) errors.

m - mean square error is adopted as the main feature of measurement accuracy , its exact value corresponds to the following expression

$$m = \sqrt{m_{\Delta}^{2} + m_{\delta}^{2}} \tag{1}$$

The root mean square error ***m of one measurement*** is characterized by the combined effects of -random error and -systematic error m_{δ} of measurement . m_{Δ}

m_{Δ} - the approximate value of the mean square error of random error in one measurement is found by the following formula

$$m_{\Delta} = \sqrt{\frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)2}{n-1}}, \tag{2}$$

m_{δ} - the mean square error value of the systematic (systematic) error in one measurement is found by the following formula

$$m_{\delta} = \sqrt{m^{2} - m_{\Delta}^{2}}. \tag{3}$$

m , m_{Δ} and m_{δ} determining the amount of errors is one of the most important issues in the theory of measurement errors.

2.5 Survey of measuring lines

The specific objectives of *measurement series research are, firstly, to eliminate gross errors, secondly, to determine the accuracy of a single measurement and, if possible, to identify the characteristics of systematic effects, and finally, to obtain a normal distribution.* is to determine the criteria of agreement with the law.

2.6 Procedure for processing a single quantity measured with equal precision

Strict verification of calculation results in work in geodetic practice has an important value.

When calculating the arithmetic mean x_i , individual results of $\bar{x}$ are checked by its deviation from the arithmetic mean. In geodesy, $x_i - \bar{x}$ the difference is called ***the deviation from the arithmetic mean*** , $\bar{x} - x$ and the difference is called ***the correction*** and v is defined by . With these above definitions, we write as follows

$$v_i = \bar{x} - x_i (i = 1,2,...,n) \tag{4}$$

We get the above sum of equalities as follows

$$[v_i] = \bar{x} * n - [x]$$

But

$$\bar{x} * n = \frac{[x]}{n} * n = [x]$$

Therefore, according to the last equation obtained

$$[v] = [x] - [x]$$

long last

$$[v] = 0 \tag{5}$$

In practice, the calculation of the arithmetic mean is usually obtained by rounding it to a given decimal unit. Therefore, the equality (IV.7) does not hold exactly.

the rounded value of the arithmetic mean $\bar{x}_0$ by and the rounding error Δ_0 by and write as follows

$$\bar{x}_0 = \bar{x} - \Delta_0$$

The correction v_i magnitude $\bar{x}_0$ is found by rounded values, i.e

$$v_i^0 = \bar{x}_0 - x_i = \bar{x} + \Delta_0 - x_i = v_i + \Delta_0$$

v_i^0 Summing all the values of , we get the following

$$[v^0] = [v] + \Delta_0 n$$

known $[v] = 0$, then

$$[v^0] = \Delta_0 n \tag{6}$$

of the obtained equality $\bar{x}_0$ and v_i^0 calculations.

We can simplify the arithmetic calculation in the following way .

of each measurement x_i are summarized as follows

$$x_i = x_0 + \varepsilon_i \tag{7}$$

where, x_0 - is some (exactly chosen) rounded value, which is chosen in such a way that it $\varepsilon_i = x_i - x_0$ is in all small positive numbers.

In that case

$$\bar{x} = \frac{[x]}{n} = x_0 + \frac{[\varepsilon]}{n} \tag{8}$$

In addition $v_i = \bar{x} - x_i (i = 1,2,...,n)$, $[v^2]$ we get the following formulas to control the calculation of deviations

We have the following

$$v_i = \bar{x} - x_i = \frac{[\varepsilon]}{n} - \varepsilon_i$$

from which we get the following

$$[v^2]=[\varepsilon^2]+n\frac{[\varepsilon^2]}{n^2}-2\frac{[\varepsilon]^2}{n}$$

and finally we get the following

$$[v^2]=[\varepsilon^2]-\frac{[\varepsilon]^2}{n} \tag{9}$$

The calculation procedure for processing equal precision measurements is as follows:
1) x_0 -some (exactly selected) rounded value is selected and $\varepsilon_i = x_i - x_0 \, (i=1,2,...,n)$ calculated;
2) the rounded value of the arithmetic mean $\bar{x}_0$ is calculated

$$\bar{x}_0 = x_0 + \frac{[\varepsilon]}{n}$$

and the rounding error is also calculated

$$\Delta_0 = \bar{x}_0 - \bar{x}$$

Δ_0 the size is obtained with some rounding errors, which cannot be taken into account;
3) $v_i^0 = \bar{x}_0 - x_i \, (i=1,2,...,n)$ s i.e. corrections are considered;
4) correction calculations are controlled as follows

$$[v^0]=\Delta_0 n \, * \tag{10}$$

*recommended calculation procedure 1, 2, 3, and 4 are performed again in cases of detection and discarding of measurements with gross error without satisfying the verification condition. [15,16,17]
5) the following values are calculated, i.e

$$[v^{0^2}],\ [\varepsilon^2],\ \frac{[\varepsilon]^2}{n};$$

6) the calculation is controlled as follows

$$[v^2]=[\varepsilon^2]-\frac{[\varepsilon]^2}{n}; \tag{11}$$

7) is the root mean square error of one measurement

$$m_\Delta = \sqrt{\frac{[v^2]}{n-1}}; \tag{12}$$

8) is the root mean square error of the mean arithmetic value

$$M_\Delta = \frac{m_\Delta}{\sqrt{n}} \tag{13}$$

and if it has systematic errors, then it is calculated by the following expression.

$$M = \sqrt{M_{\Delta}^{2} + M_{\delta}^{2}} = \sqrt{\frac{m_{\Delta}^{2}}{n} + \frac{m_{\delta}^{2}}{K}}. \tag{14}$$

2.7 An example of processing a single measured quantity

The results of measuring one angle with equal accuracy in two complex conditions (morning and evening) are given (Table 1): the number of measurements is $n = 12$ equal to. [18]

3 Results analysis

The following is to determine the rounded value of the arithmetic mean $\overline{x}_0$, the root mean square error of a single measurement m_{Δ}, the root mean square deviation of the root mean, M_{Δ} and the root mean square error of the root mean under K complex conditions M (m_{δ} given is $0{,}85^{//}$ equal to , and $K = 2$ is equal to) was asked.

Table 1. Results of measuring one angle with equal accuracy in two complex conditions

Measurement results x_i	ε_i	ε_i^2	v_i	v_i^2	**Solving**
$27^0 23' 44''$	+4	16	+0.7	0.49	Check:
$27^0 23' 40''$	0	0	+4.7	22.1	1) $[v^0] = \Delta_0 n = 0.03 * 12 = +$
$27^0 23' 43''$	+3	9	+1.7	2.89	;
$27^0 23' 45''$	+5	25	-0.3	0.09	2 $[v^2] = [\varepsilon^2] - \frac{[\varepsilon]^2}{n}$),
$27^0 23' 46''$	+6	36	-1.3	1.69	$[v^2] = 334 - \frac{56^2}{12} = 72$;
$27^0 23' 43''$	+3	9	+1.7	2.89	Accuracy assessment:
$27^0 23' 48''$	+8	64	-3.3	10.9	1) $m_{\Delta} = \sqrt{\frac{72{,}9}{11}} = 2{,}6^{//}$;
$27^0 23' 45''$	+5	25	-0.3	0.09	$m_{m_{\Delta}} = \frac{m_{\Delta}}{\sqrt{2(n-1)}} =$
$27^0 23' 48''$	+8	64	-3.3	10.9	$= \frac{2{,}6^{//}}{\sqrt{2*11}} = 0{,}55^{//}$
$27^0 23' 46''$	+6	36	-1.3	1.69	2 $M_{\Delta} = \frac{2{,}6^{//}}{\sqrt{12}} = 0{,}75^{//}$)
$27^0 23' 47''$	+7	49	-2.3	5.29	$m_{M_{\Delta}} = \frac{M_{\Delta}}{\sqrt{2n}} =$
$27^0 23' 41''$	+1	1	+3.7	13.7	; $= \frac{0{,}75}{4{,}9} = 0{,}15^{//}$

					3) $m_\delta = 0{,}85^{//}$, $M_\delta^2 = \frac{0{,}85^2}{2} = 0{,}36^{//}$; 4 $M = \sqrt{0{,}56+0{,}36} = 0.96^{//}$) $N = 12+12*(0.36:0.56)$
$x_0 = 27^0 23' 40^{//}$ $\bar{x} = 27^0 23' 44{,}666^{//}$ $\bar{x}_0 = 27^0 23' 44{,}70^{//}$ $\Delta_0 \approx +0.03^{//}$	+56	334	+12.5	72.9	
			-12.1	answer: $\bar{x} = 27^0 23' 44{,}7^{//} \pm 0{,}96^{//}$	

4 Conclusion

With the development of science and technology, especially geodesy, the accuracy of measurements increases and the methods of their mathematical processing are improved.

There should be no errors or miscalculations in measurements. In geodetic practice, at least two measurements of each value are always performed to determine the result, and in addition, mathematical relationships between the measured values are used.

The theory of mathematical processing of geodetic measurements, the quality of geodetic measurements, the occurrence of inevitable small errors and the laws of motion, the development of calculation rules for the assessment and calculation of the necessary accuracy, as well as the calculation methods that allow obtaining the best final results with the economic expenditure of computational labor and related ways.

References

1. V.D. Bolshakov, P.A. Gaydaev, Theory mathematics obrabotki geodezicheskih izmereniy (Moscow, Nedra, 1977)
2. Q.N. Norkho'jaev, Engineering geodesy (Tashkent, Teacher, 1984)
3. V.V. Golubev, Geodesy - Theory of mathematical processes of geodesy (Moscow Izdatelstvo, MIIGAiK, 2016)
4. D.O. Jorayev, H.D. Jorayeva, "Theory of mathematical processing of geodesic measurements" Part I.. (Tashkent, 2014)
5. A. Marupov, B. Akhmedov, General characteristics of zones with special conditions for using the territory of the city of Fergana . Zbírnik scientist prats LÓG O S, 7-10 (2020)
6. A.B. Madaminovich, The use of GIS technology to create electronic environmental maps. Academicia: An International Multidisciplinary Research Journal, **10(5)**, 438-440 (2020)

7. A.A. Marupov, B.M. Ahmedov, General Characteristics of Zones with Special Conditions of Use of the Territory. Middle European Scientific Bulletin , **18**, 446-451 (2021)
8. B. M. Axmedov, Knauf Insulation is Effective Isolation. Central Asian Journal of Theoretical and Applied Science, **3(6)**, 298-302 (2022)
9. E. M. Mamanazarovna, M. A. Abbosxonovich, Analysis of Agricultural Soils Designation of Different Linear Protected Zones using GIS Technology. Central Asian Journal of Theoretical and Applied Science, **2(11)**, 188-192 (2021)
10. M. A. Abbosxonovich, Analysis of Geographic Information Programs Freehand and MAPINFO for Sensing the Area using the Example of the City of Kokand. Central Asian Journal of Theoretical and Applied Science, **3(11)**, 84-89 (2022)
11. M. A. Abbosxonovich, Designing and Drawing up Employment Maps the Example of the City of Kokand. Central Asian Journal of Theoretical and Applied Science, **3(11)**, 79-83 (2022)
12. M. A. Abdukadirova, The Role of Builder and Building in the Development of the Country is Invaluable. The American Journal of Interdisciplinary Innovations and Research, **3(05)**, 81-84 (2021)
13. M. A. Abdukadirova, G. M. Mirzakarimova, The importance of installation of base gps stations in permanent activity in Fergana region. Asian Journal of Multidimensional Research, **10(9)**, 483-488 (2021)
14. M. A. Abdukadirova, G. M. Mirzakarimova, Value of geodetic works in construction of hydrotechnical structures. Academicia: An International Multidisciplinary Research Journal, **10(6)**, 1307-1312 (2020)
15. K. Khakimova, B. Abdukhalilov, L. Qosimov, A. Abdusalomov, S. Yokubov, *Application of GIS technologies for improving the content of the tourist map of Fergana province, Uzbekistan*, E3S Web of Conferences, **386** (2023)
16. S. Z. Mavlyankulova, The essence of cartographic maps is that they are used for cartographic description of the terrain. generalizing works in the preparation of maps. research and education, **1(4)**, 27-33 (2022)
17. Z. M. Alakhanov, D. S. qizi Olimova, The state cadastre for the regulation of information resources for the formation and improvement. Educational Research in Universal Sciences, **1(1)**, 47-53 (2022)
18. K. R. Khakimova, S. S. Madaminova, S. S. Yokubov, Y. K. Berdaliyeva, Some technological issues of using gis in mapping of irrigated lands. Galaxy International Interdisciplinary Research Journal, **10(4)**, 226-233 (2022)