Numerical solution of the heat transfer equation using different schemes

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1 Introduction

Heat dissipation is a type of heat transfer. Heat dissipation occurs when an object that is hotter than other objects is placed in an environment where the heat of the hotter object is transferred to the colder objects and the surrounding environment. Heat dissipation occurs in a variety of ways. Heat dissipation is unavoidable and sometimes is considered a negative process in a range of industries and applications. In the corrosion mitigation and prevention industry, heat dissipation can lead to corrosion under insulation. Heat transfer can also be detrimental to alloy performance, which is a consideration in high temperature environments. Temperature difference is the major factor that determines the mode and rate of heat transfer in a given application. There are other factors relevant to the design, operation and environment that need to be taken into consideration to manage heat dissipation, and aid in the selection of suitable thermal insulation means wherever applicable.

Heat transfer is the process of transferring heat from one coolant to another through a partition wall. Heat exchange is related to very complex processes, and in its study it is necessary to know the laws of the theory of heat exchange and analytical methods used in physics, thermodynamics, hydrodynamics and chemistry. The complex process of heat transfer is divided into several simple processes. This approach makes learning easier. In addition, each simple process of heat transfer is subject to its own laws. There are three basic methods of heat transfer: conduction, convection, and radiation. The phenomenon of heat conduction is the transfer of heat through microparticles (molecules, atoms, electrons, etc.). Such heat exchange can occur in any body with a non-uniform temperature distribution. Convective heat transfer (convection) is observed only in liquids and gases. Convection is...
the transfer of heat with macroscopic volumes. It should be remembered that heat conduction always exists simultaneously with convection. However, convection is usually the deciding factor because it is stronger than heat conduction. Convection can transfer heat over very long distances (for example, when gas moves through pipes). A moving medium (liquid or gas) used to transfer heat is called a heat carrier. The third method of heat transfer is radiation. Due to radiation, heat is transferred in all radiative media, including vacuum. In radiation heat transfer, the energy carriers are photons emitted and absorbed by the bodies participating in the heat transfer. In most cases, heat transfer is carried out in several ways at the same time. For example, convective heat transfer from a gas to a wall is almost always accompanied by parallel heat transfer with radiation.

Fourier's law is the basic law of heat conduction. Its essence lies in the proportionality of the heat flux density to the temperature gradient. As a prerequisite for this process, the temperature difference of the wall surfaces is taken into account. In such a situation, a heat flow directed from a high-temperature wall to a low-temperature wall surface is created. According to Fourier's law, the heat flow is proportional to the area of the wall, as well as the temperature difference, and it is inversely proportional to the wall thickness. Decrease depends on the thermal conductivity of the material from which the wall is made. If they contain several different layers, they are multi-layered surfaces. As an example of such materials, we can name the walls of houses where plaster is applied to the brick layer, as well as the exterior coating. It is because of this that the heat transfer is reduced, there is a risk of overheating of the working engine. With an increase in the number of wall layers, its maximum thermal resistance increases and the value of heat flow decreases. The temperature distribution for multi-layered walls is a broken line. In most heat exchangers, the heat flow passes through the walls of circular tubes. If the heating body moves inside such pipes, then in this case the heat flow is directed from the inner parts to the outer walls. With the external version, the opposite process is observed.

The main goal of this work is to compare the results using explicit and implicit schemes for numerically solving the heat diffusion equation.

2 Setting a mathematical problem

To calculate this equation of heat transfer, a steel sheet with dimensions of $0.2$ meters in $x$-axis length and $0.1$ meter in $y$-axis width was taken. Its temperature was given.

Three different materials with different thermal conductivity were placed inside this steel sheet.

1. Aluminum - its thermal conductivity coefficient is $5.848 \times 10^3$ K$^{-1}$

2. Copper - its thermal conductivity coefficient is $1.6563 \times 10^3$ K$^{-1}$

3. Silver - its thermal conductivity coefficient is $1.11 \times 10^3$ K$^{-1}$

To heat the steel sheet from all sides, temperature was given.
3 Heat transfer equation for parabolic equations

\[
\frac{\partial T}{\partial t} = \frac{\lambda}{\rho c} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)
\]

\[
\frac{T_{i,j}^{n+1} - T_{i,j}^n}{\Delta t} = \frac{\lambda}{\rho c} \left( \frac{T_{i+1,j}^n - T_{i,j}^n + T_{i-1,j}^n}{\Delta x} + \frac{T_{i,j+1}^n - T_{i,j}^n + T_{i,j-1}^n}{\Delta y} \right)
\]

\[
T_{i,j}^{n+1} = T_{i,j}^n + \Delta t \left( \frac{T_{i+1,j}^n - T_{i,j}^n + T_{i-1,j}^n}{\Delta x} + \frac{T_{i,j+1}^n - T_{i,j}^n + T_{i,j-1}^n}{\Delta y} \right)
\]

\[
\frac{T_{i,j}^{n+1} - T_{i,j}^n}{\Delta t} = \frac{\lambda}{\rho c} \left( \frac{T_{i+1,j}^{n+1} - T_{i,j}^{n+1} + T_{i-1,j}^{n+1}}{\Delta x} + \frac{T_{i,j+1}^{n+1} - T_{i,j}^{n+1} + T_{i,j-1}^{n+1}}{\Delta y} \right)
\]

\[
k = \frac{\lambda \Delta t}{\rho c} \quad A = \frac{T_{i+1,j}^{n+1} - T_{i,j}^{n+1} + T_{i-1,j}^{n+1}}{\Delta y}
\]

\[
T_{i,j}^{n+1} = T_{i,j}^n + k \left( \frac{T_{i+1,j}^{n+1} - T_{i,j}^{n+1} + T_{i-1,j}^{n+1}}{\Delta x} + A \right)
\]

\[
T_{i,j}^{n+1} = T_{i,j}^n + k \left( \frac{T_{i+1,j}^{n+1} - T_{i,j}^{n+1} + \alpha_{i-1,j} T_{i,j}^n + \beta_{i-1,j} T_{i-1,j}^n}{\Delta x} \right) + kA
\]
\[
T_{i,j}^{n+1} = T_{i,j}^n + \frac{k}{\Delta x^2} T_{i+1,j}^{n+1} - \frac{k}{\Delta x^2} T_{i-1,j}^{n+1} + \frac{k}{\Delta y^2} \alpha_{i,j} T_{i,j}^n + \frac{k}{\Delta y^2} \beta_{i,j} + kA
\]

\[
T_{i,j}^n = \frac{k}{\Delta x^2} T_{i+1,j}^n - \frac{k}{\Delta x^2} T_{i-1,j}^n + \frac{k}{\Delta y^2} \alpha_{i,j} T_{i,j}^n + \frac{k}{\Delta y^2} \beta_{i,j} + kA
\]

\[
\frac{\Delta T_{i,j}}{\Delta t} = \frac{\lambda}{\rho c} \left( \frac{T_{i,j}^{n+1} - T_{i,j}^n}{\Delta x^2} + \frac{T_{i,j}^{n+1} - T_{i,j}^n}{\Delta y^2} \right)
\]

\[
T_{i,j}^{n+1} = T_{i,j}^n + \left( \frac{T_{i,j}^{n+1} - T_{i,j}^n}{\Delta x^2} + \frac{T_{i,j}^{n+1} - T_{i,j}^n}{\Delta y^2} \right) + kB
\]

\[
T_{i,j}^{n+1} = T_{i,j}^n + \frac{k}{\Delta x^2} T_{i+1,j}^{n+1} - \frac{k}{\Delta y^2} T_{i,j}^{n+1} + \frac{k}{\Delta y^2} \alpha_{i,j} T_{i,j}^n + \frac{k}{\Delta y^2} \beta_{i,j} + kB
\]

\[
T_{i,j}^n = \frac{k}{\Delta x^2} T_{i+1,j}^n - \frac{k}{\Delta y^2} T_{i,j}^n + \frac{k}{\Delta y^2} \alpha_{i,j} T_{i,j}^n + \frac{k}{\Delta y^2} \beta_{i,j} + kB
\]

\[
T_{i,j}^n = \frac{k}{\Delta y^2} T_{i,j+1}^n - \frac{k}{\Delta y^2} T_{i,j-1}^n + \frac{k}{\Delta y^2} \alpha_{i,j} T_{i,j}^n + \frac{k}{\Delta y^2} \beta_{i,j} + kB
\]

\[
T_{i,j}^n = \frac{k}{\Delta x^2} T_{i+1,j}^n + \frac{k}{\Delta y^2} T_{i,j+1}^n - \frac{k}{\Delta x^2} T_{i-1,j}^n - \frac{k}{\Delta y^2} T_{i,j-1}^n + \frac{k}{\Delta x^2} \alpha_{i,j} T_{i,j}^n + \frac{k}{\Delta y^2} \beta_{i,j} + kB
\]

\[
T_{i,j}^n = \frac{\Delta T_{i,j}}{\Delta t} = \frac{\lambda}{\rho c} \left( \frac{T_{i,j}^{n+1} - T_{i,j}^n}{\Delta x^2} + \frac{T_{i,j}^{n+1} - T_{i,j}^n}{\Delta y^2} \right)
\]

\[
T_{i,j}^n = \frac{D}{C} T_{i,j+1}^n + \frac{E}{C} \alpha_{i,j} = \frac{D}{C} \beta_{i,j} = \frac{E}{C}
\]

4 Calculation results and their discussion
in dt=0.1 second and showed the solution clearly [36-44]. In this case, it gave an opportunity to find out their advantages and disadvantages by applying various schemes to the problem numerically [18-19].
5 Conclusion

In conclusion, it can be said that this problem of heat dissipation was calculated numerically in explicit and implicit schemes. The results of the comparison show that the application of...
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