Hydrodynamic pressure of water on the head face of an earth dam during an earthquake

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Abstract. The loads acting on the pressure face of hydraulic structures from the water mass of the reservoir during earthquakes are usually set in the form of a diagram of the coefficients of the added mass of water, while the hydrodynamic pressure of water along the height of the pressure face is distributed in proportion to the diagram of the obtained coefficients. The paper presents a solution to the problem of hydrodynamic water pressure on the pressure face of an earth dam at slope angles from 0 to 75 degrees (with respect to the vertical). Construction of isolines of the streamline function \( \psi(x,z) \) in the computational domain was carried out in PC MATLAB. Solution \( \Delta \psi = 0 \) allows to build isolines of the distribution of the hydrodynamic pressure function \( \phi(x,z) \) in the computational area. The paper presents hydrodynamic grids and graphs that make it possible to construct a diagram of the added mass of water for the entire range of emplacement of the upper slopes of dams. The developed method for solving the problem of hydrodynamic water pressure on the pressure face of the dam can be applied to any shape of the slope profile of the dam.

Keywords: hydrodynamic pressure, added mass of water, Laplace equation, seismic stability, hydrodynamic grid

1 Introduction

In recent years, in the world practice of hydraulic engineering construction, there has been a trend in the design of combined dam structures. At the same time, new designs of concrete dams appear, including soil elements - dams made of soil concrete (CSG) [1-3], at the same time, elements made of cement, concrete and reinforced concrete materials appear in the designs of soil dams. The emergence of new dam designs is caused by the expansion of construction regions, covering the territories of the Northern climatic zone, foothills and mountainous regions, where there is often no developed transport infrastructure, there are no

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quarries of special building materials, where the conditions for conducting construction work and skipping construction costs are complicated,

The studies of the stress-strain state of such dams are the subject of V.V. [4, 5], Lyapicheva Yu.P. [6], in the field of earth dams with non-soil elements - Rasskazova L.N. [7], Sainova M.P. [8, 9], Orishchuk R.N. [10], Anakhaeva K.N. [11], Bestuzheva A.S. [12] and others. At the same time, not many of the proposed designs of combined dams can be adequately substantiated from the point of view of the hydrodynamic loads acting on them, caused by the interaction of structures with a reservoir during an earthquake. Until now, there is uncertainty in setting the hydrodynamic load acting on the pressure face of the dam during an earthquake, if it is not vertical, since, according to Westergard's decision, it is taken into account with a clear excess of the actual value, and the failure to take into account the hydrodynamic pressure for high dams does not allow with certainty give a justification for the seismic resistance of a dam with a sloping profile. Thus,

2 Methods

The theoretical foundations for taking into account the interaction of structures with the aquatic environment during vibrations were developed in the 30s of the last century in the works of Westergard, Karman, Zangar, Lamb [13-16] and others. structure is a model of an ideal incompressible fluid. Westergard's solution [13], obtained for the seismic water pressure on the rigid vertical pressure face of the dam for a structure located in a rectangular canyon, is a reference in all calculations of hydraulic structures and underlies the methods used. In the works of domestic scientists Napetvaridze Sh.G., Grishin M.M., Kulmach P.P., Sheinin I.S., Shulman S.G., Eisler L.A., Konstantinov I.A., Urazbaeva N.T. [17-21] and others.

Lombardo V.N., Mirsaidov M.M., Eshmatov Kh., Abdikarimova R.A., Muzaeva I.D., Belostotsky A.M. are devoted to the issues of numerical modeling of the interaction of hydraulic structures with the aquatic environment. [22-25], including with the use of universal software systems, these issues were considered by Kozinets G.L., Zaborova D.D. and others [26, 27].

In modern Russian regulatory literature, the definition of hydrodynamic pressure on the contact surface of a structure with water is regulated as additional seismic loads for the calculation of hydraulic structures and their foundations. It is recommended to take into account the hydrodynamic pressure of water in the form of determining an additional "added mass of water", which creates additional inertial loads on the dam during an earthquake. Thus, despite the large amount of research and the depth of elaboration of issues on the interaction of structures with liquid during vibrations, the method of "added masses" is fundamental in the calculations of hydraulic structures due to its simplicity and good result of approximating the analytical solution.

\[ \mu_\alpha = \mu \cos^3 \alpha \] (1)

Where \( \alpha \) - the angle of inclination of the pressure face with respect to the vertical, degrees;

\( \mu \) is the coefficient of the added mass of water for the vertical face according to the Westergaard solution [13].

It is important to note that recommendation (1) exists only for the inclination of the pressure face of no more than 15 degrees, which is not fulfilled for many structures of concrete gravity, buttress, and rolled concrete dams. For earth dams, all emplacements of upstream pressure faces have an angle significantly greater than 15 degrees.
The paper presents a technique for constructing a diagram of hydrodynamic pressure on the pressure face of a dam for any slope profiles of a dam, based on the analogy of solving similar problems on electrohydrodynamic simulation units [15]. In solving the problem, numerical methods implemented in the MATLAB software package, as well as graphic-analytical methods for constructing hydrodynamic grids in a homogeneous isotropic medium, are used.

The solution of the contact problem of hydroelasticity for a hydraulic structure is based on the principles of continuum mechanics and the theory of small deformations, while certain prerequisites are used to simplify the task. To determine the hydrodynamic pressure exerted by the aquatic environment during earthquakes on the upstream face of a structure, one proceeds from the idea that, during vibrations, part of the water mass of the reservoir makes joint vibrations with the structure, which determines the conditions for the compatibility of displacements and velocities at the boundary of two bodies. Due to the smallness of the amplitudes of seismic vibrations, and during the strongest earthquakes they are measured in centimeters, and taking into account the fact that the dimensions of the dam are much larger and are measured in tens of meters, it is possible to neglect the shape of the deformed axis of the pressure face of the dam during vibrations and take it in the form of a displacement diagram uniform in height, which corresponds to the model of "translational motion of a rigid wall". When setting the problem, it is assumed that it is equivalent to consider a problem in which the hydrodynamic pressure that a moving body experiences from a stationary fluid can be described by equations for a moving flow acting on a stationary body, while the body itself is assumed to be rigid, and the aquatic environment is incompressible. The assumption of fluid incompressibility is often used in solving hydrodynamic problems due to the high value of the bulk modulus of elasticity for water [13], however, a possible increase in pressure in the region of acoustic resonances at high-frequency components of seismic vibrations has not been fully studied, although it is noted that the presence of bottom sediments contributes to the damping of oscillations and can be considered as “added resistance” [20]. The fluid in the volume of interaction with the structure is considered to be homogeneous, inviscid, its motion is assumed to be irrotational. In fact, the manifestation of viscous forces will only affect the boundary layer with the dam, the thickness of which is extremely small (1-3 mm) compared to the dimensions of the dam [16], while setting the boundary conditions in solving the problem of small vibrations for a viscous fluid is also based on the conditions applicable to an ideal fluid [19].

To test Westergaard's theoretical solutions at Stanford University (USA), experiments were carried out on a mobile seismic platform, which confirmed the results obtained and the main conclusions that when modeling similar phenomena with water, it can be considered an inviscid and incompressible liquid, and wave phenomena on the surface can be neglected [20].

The solution of the problem of pressure distribution in a fluid under given boundary conditions and a known displacement on the contour (pressure face of the dam) is based on the dynamic equilibrium equations in the Euler form for an irrotational flow in the absence of external forces, which is written (for a plane problem) as:

\[
\begin{align*}
\frac{\partial P_0}{\partial x} &= -\rho \frac{\partial^2 u_x}{\partial t^2} \\
\frac{\partial P_0}{\partial z} &= -\rho \frac{\partial^2 u_z}{\partial t^2}
\end{align*}
\]

Where \(P_0\) - excess hydrodynamic pressure caused by dam vibrations;

\(u_x, u_z\) - horizontal and vertical displacement components
Since the function of the speed potential \( \phi(x, z, t) \) is a potential function for the problem posed, the values of the speeds in the calculated nodes are related to the speed potential by known relations:

\[
\begin{align*}
    v_x &= -\frac{\partial u_x}{\partial t} = \frac{\partial \phi}{\partial x} \\
    v_z &= -\frac{\partial u_z}{\partial t} = \frac{\partial \phi}{\partial z}
\end{align*}
\]

(3)

After substituting (3) into (2) and replacing \( \frac{\partial u_x}{\partial t} \) on \( \frac{\partial \phi}{\partial x} \) it can be obtained that the hydrodynamic pressure is related to the velocity potential by the relation:

\[ P_0 = \rho \frac{\partial \phi}{\partial t} \]

(4)

Thus, at each fixed point in time, the hydrodynamic pressure function is associated with the distribution of the velocity potential, therefore its distribution in space also obeys the solution of the Laplace equation, on the basis of which we can write:

\[ \frac{\partial^2 P_0}{\partial x^2} + \frac{\partial^2 P_0}{\partial z^2} = 0 \quad \text{or} \quad \Box P_0 = 0 \]

(5)

The solution of the Dirichlet problem for the Laplace equation is completely determined by the boundary conditions (boundary value problem), which are given in the form of the values of the potential function itself on the contour and in the form of conditions for immobility of the sections of the boundary contour \( (v_n = 0 \quad \text{or} \quad \frac{\partial \phi}{\partial n} = 0) \). In the above calculations, it is assumed that the boundaries of the reservoir bed are immobile, and there is no wave pressure on the free surface of the water, i.e. hydrodynamic pressure is zero \((P_0 = 0)\).

The Laplace equation (5) is an elliptic equation that can be solved by analytical methods using the apparatus of conformal transformations [17, 28], numerical methods [29], as well as experimental methods based on the electrohydrodynamic similarity principle, for which the identity of differential equations allows one to obtain solutions hydrodynamic problem by solving an electrodynamic problem. The method of electrohydrodynamic analogy (EGDA) has been widely used in various engineering problems: to obtain a picture of the flow around a wing in aircraft construction [30], to solve problems of fluid flow in a porous medium [31], as well as to solve a number of hydrodynamic problems for dams in a model basin filled with electrolyte [15].

In this work, to solve the Laplace equation in a flat formulation, the apparatus of numerical methods was used in the MATLAB software package [32,33]. Modeling of the computational domain with the definition of boundary conditions is shown in Figure 1. In the problem posed, based on the physical concepts of the stationary distribution of the velocity potential function in an ideal, incompressible fluid, we can assume that the boundary of the computational domain along the bottom of the reservoir is the boundary streamline \((\psi = 1)\), and the surface of the reservoir at the WB level corresponds to the zero value of the velocity potential \((\phi = 0)\), and hence the hydrodynamic pressure. Since the water surface is the equipotential of the function \(\phi = 0\), then the following condition must be satisfied for the
stream functions on this boundary: \( \frac{\partial \psi}{\partial n} = 0 \). On the lateral boundaries, the calculation scheme is limited: on the one hand, by the water pressure face of the dam, on the other, by the reservoir coastline, which is assumed to be fixed ( \( \frac{\partial \phi}{\partial n} = 0 \)).

![Diagram](image)

**Fig 1. Problem solution area**

The accepted formulation of the boundary conditions does not allow us to completely determine the boundary value problem on the full contour of the computational domain, since the desired values of the potential function on the pressure face of the dam are unknown. Thus, it is necessary to turn to the essence of the orthogonal functions of the velocity potential \( \phi = \phi(x, z) \) and current functions \( \psi = \psi(x, z) \), which are complex-conjugate harmonic functions of the function of the complex potential of the flow \( W = \phi + i\psi \). Complex conjugate functions are related by the Cauchy-Riemann relations:

\[
\begin{align*}
\frac{\partial \phi}{\partial x} &= \frac{\partial \psi}{\partial z} \\
\frac{\partial \psi}{\partial x} &= -\frac{\partial \phi}{\partial z}
\end{align*}
\]

In this case, the system of equipotentials \( \phi = \text{const} \) and current lines \( \psi = \text{const} \) form an orthogonal hydrodynamic grid in which the position of one of the functions uniquely determines the position of the other. Based on this position, it is possible, with known boundary conditions on the contour of the computational domain, to construct equipotentials for the stream function, on the basis of which the equipotentials of the velocity potential function can be obtained using the graphical-analytical method, which determines the distribution law of hydrodynamic pressure in a stationary field.

For the potential function of the streamlines, the boundary conditions are established from the following considerations: the bottom of the reservoir is a boundary streamline, so the value of the potential equal to unity can be set along it; at infinity, the stream functions are perpendicular to the boundary; therefore, one can set the condition \( \frac{\partial \psi}{\partial n} = 0 \); the pressure face of the dam is a surface with a uniform gradient of the potential, changing from 1 at the base to 0 at the water level in the reservoir, while the accuracy of the solution of the problem in the representation of the hydrodynamic grid will be associated with a given step \( \Delta \psi \) on
isolines of equal values of stream functions. The constant gradient for the stream functions along the height of the pressure face is set according to the assumption of a uniform “penetrating-reflecting” ability of the boundary, which determines the uniform distribution of the flow rates of the “current tubes” along the height of the dam. In the given solutions $\Delta \psi$ accepted 5% $h$ and 10% $h$, where $h$ is the height of the pressure face of the dam.

Boundary conditions in solving the problem of the distribution of the potential stream function in the computational domain "dam-reservoir" are set by the interface of the MATLAB software package. To do this, boundary conditions must be satisfied at the boundaries of the region (lines AB, BC, CD, DA), which in mathematical physics are called the Dirichlet and Neumann conditions [34].

In MATLAB, boundary conditions are specified functionally in the following order:
- Dirichlet boundary conditions (or conditions of the 1st kind) are determined for a given contour of the computational domain in the form:

$$r(x) = V(x)h$$

where at $r(x) = h = 0$ potential function $V(x)$ can take any value.
- the Neumann condition (or condition of the 2nd kind) is the method of specifying the boundary condition, in which the distribution of the desired value is given by its normal derivative:

$$\frac{\partial V(x)}{\partial n} = 0$$

For the considered calculation scheme "dam-reservoir" (Figure 1), the boundary conditions are set in the form of values for the potential function $V(x)$ in areas:

AB - reservoir bottom $h = 1$ and $r(x) = 1$ (Dirichlet condition);

DA - condition at infinity $\frac{\partial V(x)}{\partial n} = 0$ (Neumann condition);

CD is the free water surface of the reservoir, where for the stream functions $V(x)$ Neumann condition $\frac{\partial V(x)}{\partial n} = 0$;

BC is the pressure face of the dam, where for the function $V(x)$ the linear distribution of the potential is set depending on the angle of inclination of the pressure face:

$$V(x) = x \cdot \text{ctg}(\alpha)$$

$h$ is the relative height of the dam, taken as a unit.

The hydrodynamic pressure on the pressure head of the dam can be represented, based on Newton's II law, as an acceleration caused by wall vibrations $\ddot{U}_0$ multiplied by the mass of water distributed along the pressure face $m_w$ as:

$$p_{wd} = m_w \ddot{U}_0$$

Where $m_w$ - the mass of the "added mass" of water distributed along the pressure face of the dam, which participates in the joint oscillations of the dam and exerts hydrodynamic pressure on the structure, which in the calculations can be interpreted as inertial.
The force of hydrodynamic water pressure applied to the k-element of the pressure head of the dam can be written as:

\[ P_k = \bar{U}_0 \mu_k \rho_0 h \omega_k \]  

(11)

Where \( \omega_k \) - pressure face area for k-element, \( \mu_k \) - coefficient that determines the nature of the distribution of the hydrodynamic pressure diagram along the height of the structure, \( \mu_k = R(\frac{z}{h}) \)

3 Results

Since some limited area of the reservoir takes part in the joint oscillations of the dam, then in order to exclude the influence of the boundaries of the length of the reservoir on the solution of the problem, options with the length of the reservoir were considered \( L = H \), \( L = 2H \), \( L = 3H \) and more, where \( H \) is the height of the dam. Similar studies for free vibrations of elastic walls were carried out by Shulman S.G. [35]. The results on the distribution of hydrodynamic pressure in the computational domain are presented in Figure 2 (ac), from which it can be seen that when the length of the reservoir is only more than three dam heights, the nature of the distribution of the pressure field near the dam does not change, which is consistent with the recommendations of Russian building codes and rules for determining the coefficient influence of reservoir length: \( \Psi = f(L, H) \), which at \( L \geq 3H \) is taken equal to 1.

![Fig 2](image_url)

Fig 2. The influence of the relative length of the reservoir (L/H) on the distribution of the potential function \( \Psi(L, H) \): a) \( L / H = 1 \), b) \( L / H = 2 \), c) \( L / H = 3 \)

According to the described method for dams with inclination angles of the pressure head in the range from to in the MATLAB software package, boundary problems for the Laplace equation in the computational domain (Figure 1) were solved and hydrodynamic grids were constructed for the stream function and velocity potential. The values of the velocity potential at the contact with the pressure face of the dam determine the diagram of the distribution of hydrodynamic pressure along the height of the pressure face in relative units (%), which determines the diagram of the coefficients of the added mass of water.
Fig 3. Hydrodynamic grid and coefficient $\mu_k$ distribution diagram for $\alpha = 0^\circ$ (vertical face)

For the vertical pressure face, the obtained solutions completely coincided with the Westergaard solution, which is shown in Figure 3b.

For inclined pressure face $\alpha = 40^\circ$ and $\alpha = 60^\circ$, the hydrodynamic grid and the diagram of the coefficient of the added mass of water are presented in Figures 4 and 5.

Fig 4. Hydrodynamic grid and coefficient $\mu_k$ distribution diagram for $\alpha = 40^\circ$
Fig 5. Hydrodynamic grid and coefficient $\mu_k$ distribution diagram for $\alpha = 60^\circ$

4 Discussion

The obtained solutions (Fig. 3 - Fig. 5) clearly demonstrate that a change in the angle of inclination leads to a decrease in the amplitudes on the hydrodynamic pressure diagram, and the diagram takes the form of a parabola with an extremum not at the bottom mark, but somewhat higher, while the extremum point with increasing laying edge moves up. The maximum ordinates in the diagrams decrease almost linearly from 0.74 for the vertical face to 0.2 when the pressure face is about 2.5.

All obtained solutions for the angles of inclination of the pressure face $0^\circ$...$75^\circ$ are presented in the form of a nomogram, according to which it is possible to reproduce the diagram of the added masses of water for any angles of inclination of the pressure face of the dam (Figure 6).

From the plot of added masses, one can estimate the total value of the hydrodynamic pressure exerted on the dam during seismic vibrations from the reservoir water. In this case, the hydrodynamic pressure force can be obtained:

$$P_{hd} = \bar{U}_0 \rho_0 h \int_0^h \mu(z) dz$$

Calculation of hydrodynamic forces from the obtained solutions for the function $u(z)$ at different angles of inclination of the pressure face of the dam made it possible to plot the change $P_{hd} = f(\alpha)$. In this case, traditionally, the function is represented in fractions of the hydrostatic pressure $\gamma h^2$. For this reason, the resulting plots $u(z)$ were numerically integrated over height, which made it possible to write the hydrodynamic force in the traditional form:

$$P_{hd} = k_c \gamma_0 h \int_0^h \mu(z) dz = Mk_c \gamma_0 h^2$$

Where $M$ – coefficient characterizing the change in the hydrodynamic pressure force depending on the inclination of the pressure face. Schedule $M = f(\alpha)$ is presented in Figure 7. If we write the expression for the horizontal hydrostatic pressure of water on the pressure face of the dam as:

$$W_{hd} = \frac{1}{2} \gamma_0 h^2$$

then the hydrodynamic pressure force in fractions of the hydrostatic pressure will be: $P_{hd}(\alpha) = 2M(\alpha)k_cW_{hd}$, which during a strong earthquake and a vertical pressure face will be up to half of the hydrostatic force, and with $\alpha = 45^\circ$ about a quarter.

5 Conclusions
1. A technique for constructing diagrams of the added mass of water is presented, which makes it possible to calculate the hydrodynamic pressure during an earthquake for any outline of the dam's pressure face.

2. Based on the solutions of the Dirichlet boundary value problem for the Laplace equation using the MATLAB software package, hydrodynamic grids and plots of the coefficient of the added mass of water for the pressure faces of dams with vertical angles of inclination of the faces in the range from $0^\circ$ before $75^\circ$.

3. Based on the solutions obtained, a nomogram was constructed for practical use in determining the hydrodynamic pressure during an earthquake for the pressure faces of dams with vertical angles of inclination of the faces in the range $0^\circ$ before $75^\circ$.

4. Verification of the method for the vertical pressure face of the dam gave the coincidence of the obtained solution with the well-known Westergaard solution, which proved the correctness and adequacy of the chosen method for constructing diagrams of the coefficients of the added masses of water $H_k$.

5. For dams with inclination angles of pressure faces in the range from $0^\circ$ before $75^\circ$ a graph was obtained to determine the force of hydrodynamic pressure exerted by the reservoir on structures during an earthquake.

![Figure 6. Nomogram for plotting the coefficient $H_k$ distribution diagram](image)
Fig 7. Total force of the hydrodynamic pressure

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