

Development of models of ice interaction with waves and structures

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Abstract. Mathematical models used to study the interaction of sea ice with ocean waves and hydraulic structures are analyzed. Works focusing on general and specific scenarios of this interaction are reviewed. General scenarios include wave-ice interactions with a focus on various prevailing physical and mechanical processes, while specific scenarios include ice-structure interactions. The specific scenarios within the framework of this work include changes in ice pressure as the ice field thickens between the offshore platform supports, the effect of seasonal variations on the resulting ice load, and ice floe bending failure using the Hoek-Brown model. Based on the results obtained, a classification of existing models was developed according to modeling methods, details of accounting for various natural phenomena, complexity of use, and predictive accuracy. For each model in the classification, advantages and disadvantages were identified, and recommendations for their improvement for further application in scientific research and practical calculations were proposed.

Keywords: mathematical models, ice, hydraulic structures, waves, numerical modeling, ice loads

1 Introduction

Studies in the field of interaction of sea ice with waves and hydraulic structures on the Arctic part of the Russian shelf have become especially relevant against the background of increasing development activity in this territory. In recent years, significant progress has been made in this area, especially in the areas related to numerical modeling of these processes.

Despite the large amount of accumulated data and achievements, there are still areas that require further study. Predicting the dynamic interaction between ice and offshore structures, especially under the influence of waves, is still a challenging task requiring the application of modern numerical modeling methods and improvement of the regulatory framework.

The purpose of this study is to provide an overview of the currently existing research in this area and critically analyze it, identify the advantages and disadvantages of the models proposed in the literature, as well as identify "gaps" in the current state of scientific knowledge and propose functional alternatives and recommendations.

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2 Materials and Methods

The methodological apparatus of the research includes a qualitative approach, which is based on a thorough analysis and interpretation of research results.

The main methods of data processing are synthesis and generalization. Using the synthesis method, different models similar in some qualitative characteristics are considered. The generalization method is used to classify the selected models.

The methodological limitations of the study are primarily related to the complexity of the mechanics of the interaction between waves, ice, and structures, which complicates the development of accurate classifications reflecting all relevant factors and makes a number of assumptions necessary.

The Results section presents a review of the literature related to modeling the interaction of sea waves and ice, as well as ice and offshore structures.

The Discussion section provides a classification of the models considered in this review.

The Conclusion section draws conclusions regarding the current progress of research related to modeling the interaction of waves, ice, and structures, identifies general advantages and limitations, and formulates recommendations for further research.

3 Results

Current research is focused on developing a variety of models to study the propagation of sea waves in the ocean, in the presence of ice. Most models focus on investigating the dynamics of the interaction between ice and waves in the boundary zones where permanent ice is converted into drifting ice. Researchers [1] distinguish three main classes of models: ice floe models, effective medium models, and transport equation models.

In ice floe models, individual ice floes are treated as fundamental degrees of freedom. These models study wave diffusion in two-dimensional and three-dimensional zones from a floating ice floe of indeterminate size. The size used in them is a random number that is determined by a predetermined probability distribution. Thin elastic plates floating under Archimedean force are used in most wave-ice interaction models. In these models, the mean and variance of the reflection and wave-passing coefficients through ice are calculated. Ice models propose that wave motion is a time-harmonic process, so the solutions offered by such models are only suitable for stationary processes.

Sea ice in ice floe models is a heterogeneous material with constituents of crystalline, granular, and fibrous ice. Its properties are influenced by temperature, age, dynamic processes, and salt content. However, to predict wave scattering, many of these parameters can be ignored and the ice sheet can be treated as a thin elastic plate. This assumption is based on three key points: the short observation time, the relative linearity of the deformation, and the lack of information on the internal heterogeneity of the ice. In this case, the small thickness of the ice floe compared to the wavelength is considered. Despite this, it should be taken into account that these are rather crude assumptions applicable only for approximate calculations.

The first model of an ice floe, representing a thin elastic plate floating on a non-viscous fluid, was proposed by Greenhill [2] in 1886. It was this model that served as the basis for further ice floe modeling concepts. In 1986, Wadhams [3] developed a technique to determine the decrease in wave energy density when traveling through an ice field of arbitrary geometry. In the model he developed, each incoming wave is partially reflected from each ice floe.

In 2007, Bennetts [4] provided the ability to model an ice floe of variable thickness. In 2009, Williams and Porter [5] improved this model by introducing the ability to calculate

the water velocity under an ice floe of variable thickness. The process of model complexity continued in the 2012 work of Bennetts and Squire [6]. They modeled the individual characteristics of each ice floe, such as cracks, protrusions, ridges, and others. Finally, by 2017, Meylan [7] included the porosity parameter in the ice floe model.

At the same time, there have been studies aimed at simplifying the models without significantly reducing the accuracy of the results. In 2008, Kohout [8] proposed a method for determining the predicted values of the wave energy attenuation coefficient as a function depending only on the ice floe thickness. And in 2010, Bennetts [9] revealed that replacing complex ice floes with square ice floes has a barely noticeable effect on the wave attenuation rate.

Even so, because of the continued increase in complexity of the models, their computational capabilities have become computationally demanding, which limits their use in a multitude of production areas.

Effective medium models describe the surface layer of the ocean (including ice floes, ice, etc.) as a single viscoelastic substance, causing waves to attenuate as they travel through the medium.

In 1998, Keller [10] transformed the understanding of sea ice by visualizing it as a continuous upper layer of a viscous fluid on top of a main layer of non-viscous fluid representing most of the ocean. The upper, more viscous layer describes the ice floes gathered as a suspension of particles. Based on the Keller model, wave energy attenuation can be expressed in terms of the effective viscosity of the upper layer. This statement is supported by most observations and studies, with respect to ice fields consisting of icesheet or pancake ice.

However, the relationships between effective viscosity and wave parameters, such as attenuation, period, and amplitude, have yet to be established. A notable difficulty is that such relationships can only be identified through in situ studies, which limit the scope of applicability of the Keller model.

Carrying out calculations for different types of ice (frazil, granular, grease, pancake, etc.) and in a variety of weather conditions presents an even greater challenge in identifying the relationship between effective viscosity and wave parameters.

The Wang-Shen model [11], developed in 2010, adds elasticity to the upper layer by considering two limiting cases: when the ocean is either covered with brine or pancake ice, or completely covered with solid sea ice. Wave propagation in an effective viscoelastic medium is characterized by the law of dispersion, in the form of an equation of dependence of the wave frequency on the wave number. There is no consensus in the scientific community whether this type of parameterization can be used to model wave propagation in the ice zone. However, this does not prevent the Wang-Shen model from remaining popular and being used in many programs [12]. Some experiments were conducted [13], according to which the Wang-Shen model gave the smallest discrepancy between the modeled and measured wave heights for both calm and storm conditions. However, it should be considered that the other models were very simplified, which does not give a definitive reason to fully trust the Wang-Shen model.

The Fox-Squire model [14] presents a new approach in which the ice plate is converted into a beam and the elastic modulus is limited to two dimensions. The effective medium in this model is divided into two layers: the bottom layer is an incompressible non-viscous fluid (water) and the top layer is a viscous mixture of water and ice. The fundamental difference between this model and the Wang-Shen model is that the solutions can be obtained analytically rather than numerically, which distinguishes the Fox-Squire model from others because of the relative simplicity of their application.

Transport equation models, whose task is to describe the process of wave energy propagation, apply a specific transport equation to model the dynamics of ocean waves. For

these models, the basis used is not a dense cluster of ice floes, but rather ice floes that are spaced at a certain distance from each other.

There are many developed models of the transport equation. The Masson-Leblond model [15], created in 1989, is based on the Hasselmann transfer equation [16] (1). The left side of this equation contains terms describing the variation of the wave flux density in time and the group velocity of the wave train, which reflects the transformation of the spatial configuration of the waves in the overall dynamics of the system. In the right part of the equation, there is a term that describes the influence of various factors on the behavior of waves in the medium, causing the processes of absorption and dissipation of wave energy. In the context of the model, the right-hand side of the equation is the sum of the factors exerted on the wave field by each individual ice floe.

$$\partial_t I(t, x, k) + \partial_t \omega(t, x, k) = \sum_i S_i \quad (1)$$

where I is the wave impact density, t is time, x is the coordinate, k is the wave number, ω is the wave frequency, and S_i is a term describing the effect of ice on the wave field, which includes the geometric and physical-mechanical characteristics of the ice floe, coordinates, wave approach angle, and so on.

In 2016, a new theory by Zhao and Shen [17] was proposed, which proposes to simplify the Masson-Leblond transport equation by using the diffusion equation. In this case, the waves do not interact with individual ice floes but with a cluster of ice floes.

Considering the restriction on the low concentration of ice floes, these models may be ineffective when the concentration increases. In this case, it will be necessary to consider the effects of wave reflection, which is not included in the transport equation. Finding ways to modify the transport equation models for higher ice densities is one of the key tasks in the study of wave-ice interaction today.

Current research on modeling ice-structure interaction is difficult to summarize because each new model focuses on a different physical phenomenon and uses different computational programs. For this reason, three papers [7-9] were studied to perform an analytical review and general conclusions were formulated on their basis.

The analysis presented in the study [18] concentrates on measuring the total ice pressure on three- and four-leg structures, which are often used in offshore field development (Fig. 1). The application of a three-dimensional numerical model presented in the ANSYS program allowed a detailed assessment of the influence of different aspects on the ice pressure. This includes the thickness of the ice layer, the distance between the supports, the angle of ice drift relative to the structure, and ice wedging between the supports.

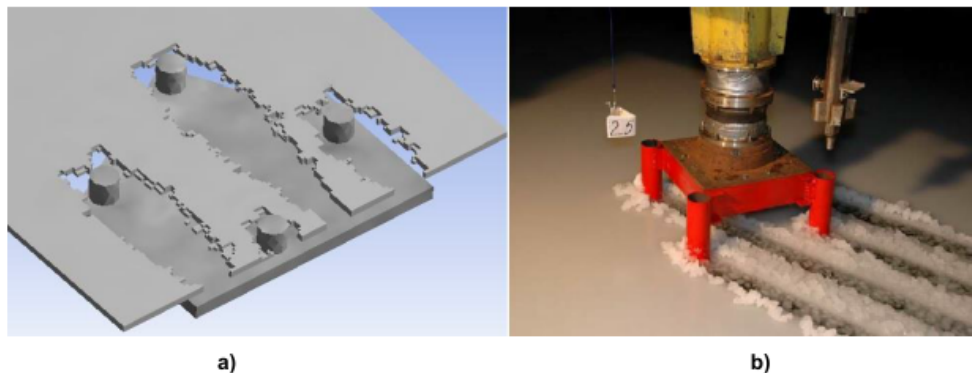


Fig. 1. Picture of the penetration of a 4-legged structure into an ice field: a) numerical simulations; b) model tests in a pool. *Source:* [18].

According to the numerical study, when the ice thickness is up to 1 meter, the ice pressure on a four-post structure is slightly higher than on a three-post structure. It follows that other factor such as ease of transportation and construction, weight of the structure, platform location, etc. are also considered when deciding between the two types of structures.

The need for adjustments in the calculation of the total ice pressure because of ice jamming between the supports is confirmed by the results of the numerical study. This jamming is particularly important when the ratio of the distance between supports to the support diameter is less than four, in which case an additional coefficient should be applied in the calculations. However, it is necessary to be careful in determining this coefficient, considering the specific conditions of each case. The results of model calculations show that the contribution of ice jamming should not exceed 10% of the total ice pressure.

The study [19] addresses the change in the main characteristics of the flat ice cover during different seasons and its impact on the ice load. As an example, the project of the hydraulic engineering structure of the floating nuclear thermal power plant in Pevek was used. Numerical modeling was carried out to determine the coefficient of ice load reduction on the perpendicular wall in the situation when it is formed due to the association of ice debris on the slope of the structure (Fig. 2).

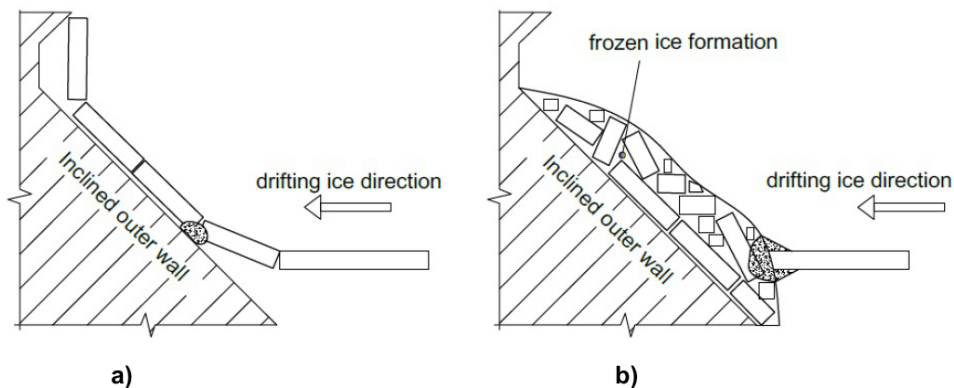


Fig. 2. Scenarios of impact of flat ice on an inclined wall: (a) when the ice collapses in bending; (b) when flat ice interacts with a vertical wall of ice on an inclined wall. *Source:* [19].

Ice pressure on the vertical wall varies with time. Under the climatic conditions of Pevek port, potentially the highest ice load can be predicted in March-April. However, statistics show that ice movement does not start until May, when the load is approximately half of the theoretical maximum for the winter. This confirms the correctness of the provision that the peak load from drifting ice should be determined for the month of the first freeze-up based on long-term observations or statistical reanalysis of ice conditions based on meteorological data (the article recommends using data for a period of at least 50-60 years). Otherwise, the ice load may exceed more than twice. Considering the non-uniformity and heterogeneity of ice formed on the inclined wall, the load reduction factor for the case of ice impact on the longitudinal structure can be determined as 0.8 (based on the results of numerical modeling).

In the article [20], a detailed analysis of the Hoek-Brown model used to simulate ice floe failure in bending in the Plaxis software package was carried out.

For this purpose, numerical experiments were carried out to determine the bending moment of an ice beam during interaction with a structure, and the results were compared with in situ data. Most of the results obtained agreed with those observed in practice.

The study showed that the Hoek-Brown model is stable and sufficiently accurate for

predicting the bending stiffness of ice under various conditions. The error of the data obtained from this model does not exceed 40%, which is acceptable within the scope of this study. However, for more accurate modeling of ice stiffness, it is necessary to consider the effect of brine penetration into the ice. Within the framework of the study, the data obtained is of practical value for the design and construction of structures.

4 Discussion

The choice of an appropriate wave-ice interaction model depends on the specific requirements and application conditions. Ice models are suitable for thorough analysis of small regions of ice accumulation, which require large computational resources. Effective medium models are less complex to use and are optimal for calculating wave attenuation characteristics, but they require calibration and are not suitable for new calculations. Finally, transport equation models provide a universal solution suitable for most applications, but they are still in the early stages of development. The classification of the models is summarized in Table 1.

Table 1. Classification of models of wave-ice interaction

Characteristics of models	Ice floe models	Effective medium models	Transport equation models
Modeling method	base elements - individual ice floes, calculation is based on wave diffusion in two-dimensional and three-dimensional spaces	the ocean surface layer as a single substance, calculated based on the law of wave dispersion	based on the transport equation to describe the propagation of wave energy density
Complexity of use	require significant computational resources, especially when the number of ice floes is large	do not require large computing resources	do not require large computing resources
Accounting for natural phenomena	adapt to different phenomena	only consider sea ice in conjunction with water	operate at a low density of ice floes in the ocean
Predictive accuracy	high (due to modeling of each individual ice floe)	low (requires calibration for each individual calculation case)	high (by integrating the effect of wave scattering by individual ice floes into the transport equation)
Advantages	ability to consider a variety of phenomena	good results in the calculation of wave attenuation	high accuracy of results at low computational resource costs
Disadvantages	are suitable only for the calculation of stationary processes	inapplicability to new conditions	do not work when ice floes are close to each other

Source: Compiled by the authors.

Based on the results of the analysis of articles on the interaction between ice and structures, the following conclusions were made numerical modeling effectively estimates

the magnitude of ice pressure on different structures. An important factor in modeling the interaction between ice and multi-pillar structures is ice jamming between the pillars on which the total ice pressure depends. When analyzing the time-dependent effect of ice loading on a structure, peak loads should be expected in March-April, but the onset of ice movement is in May. The Hoek-Brown model can be used to model the failure of a constant thickness ice floe, expecting results of a sufficient degree of accuracy. The classification of models is given in Table 2.

Table 2. Classification of models of interaction between ice and structures

Characteristics of models	Level ice interactions with multi-legged offshore structures	Level ice forces on hydraulic structures during ice season	Hoek-Brown model for ice breaking simulation
Modeling method	three-dimensional numerical analysis using ANSYS program	three-dimensional numerical analysis using ANSYS program	two-dimensional numerical analysis using the Plaxis program
Complexity of use	does not require large computing resources	does not require large computing resources	does not require large computing resources
Accounting for natural phenomena	various, including ice thickness, distance between supports, angle of ice drift relative to the structure, and ice jam between supports	changes in the main parameters of the flat ice cover during the ice season	individual ice floes of constant thickness without taking into account the influence of the brine solution
Predictive accuracy	>90%	>80%	>60%
Advantages	detailed and accurate methodology for determining ice load on three- and four-post structures	good results for determining the coefficient of ice load reduction on a vertical wall	Ability to simulate the bending of an ice floe before it breaks up
Disadvantages	there is no analysis of the influence of geographical location of the structure and changes in ice structure with time	may be less accurate if there is insufficient or poor-quality data on seasonal changes in ice cover	can lead to inaccurate calculations if there is a high percentage of brine in the ice floe

Source: Compiled by the authors.

5 Conclusion

The study categorized the models in the field of wave, ice and structure interaction studies and the following results were obtained.

The merits of the current models are:

1. Increasing comprehensiveness. Modern models can include an increasing number of different variables, allowing them to determine with greater accuracy the values of those or other required parameters.
2. Increasing efficiency. The use of numerical modeling and remote observation can improve the efficiency of design and maintenance procedures.

Limitations of current models:

1. Limited accuracy. The dynamics of the interaction between waves, ice and structures is a complex process. Developing accurate numerical models that consider all relevant factors is a challenging task, which, combined with the limitation on computational resources, leads to the introduction of certain assumptions that affect the validity of the results.

2. Limited applicability. Some of the proposed modeling techniques can only be applied to specific types of structures or environments, which reduces their effectiveness in other different conditions.

A principal vector for future research could be the integration of work studying wave and ice dynamics and modeling studies of ice effects on structures. A combined model addresses a broader range of applications, including accounting for the mechanical changes that ice acquires as it interacts with waves. Such a model can help to determine the wave-ice load more quickly and accurately on hydraulic structures.

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