Expediency of application of approximate methods to problems of building thermal physics

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Abstract. This paper shows the advantage and possibilities of application of approximate solution methods to typical and actual problems of building thermal physics. Specific examples of successful solution of rather complicated problems are given and the accuracy of the obtained solutions is evaluated. Without detracting from the advantages of numerical solutions, the possibility of assessing their reliability in comparison with approximate solutions is shown. It is noted that approximate solutions of a large number of problems can be successfully used in engineering practice.

Keywords: Potential, non-stationarity, non-linearity, approximate methods, temperature, humidity, field.

Introduction

The main problems of building thermal physics are:

1) determination of stationary and non-stationary temperature fields in areas of different volume and shape (theory of heat resistance, thermal regime of premises, etc.);
2) consideration of convective flows in calculations;
3) problems of liquid (air) filtration with moving boundaries;
4) problems with phase transformations of the first kind - ice-water, water-vapor, (problems of determining moisture fields, thawing of soil under structures);
5) solving problems taking into account the dependence of thermophysical parameters on the potential (nonlinear problems);
6) problems of hydrodynamics (solution of the Navier-Stokes equation), joint solution of problems of hydrodynamics and thermal physics, for example, in case of fluid flow through pipes).
7) problems of diffusion of harmful impurities (mutual and self-diffusion) - problems of ecology.

The usual sequence of problem solving: a physical model of processes is considered, and then a mathematical model is proposed-writing equations and boundary and initial conditions.

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The solution of most of the above problems is reduced to the solution of the heat conduction equation [1-2], so the solutions are universal and can be used in the analysis of various physical processes obeying the law flow is proportional to the gradient of a scalar quantity (potential), that is, processes obeying the laws of Fourier, Fick, Darcy [3]. By potential \( \Pi(\vec{r}, t) \) we mean temperature, pressure, concentration.

Methods of solution used in practice: analytical methods exact and approximate solutions and numerical methods of solution - use of known programs and their correction for a particular problem, creation of new programs.

Let us dwell on the advantages of analytical methods of solution:
1) The possibility of analyzing the dependence of the desired value on various parameters and conduct evaluative engineering calculations; 2) verification of the correctness of the results obtained by numerical methods.

Obtaining exact solutions of equations is quite difficult and can be realized only in special cases, so it is often reasonable to use approximate methods - integral method, methods of Bio, Shvets, Bubnov-Galerkin, etc. [4]. Possible errors in obtaining solutions are taking the wrong physical model, lack of consideration of factors affecting the formation of the unsteady potential field, as well as inaccuracy of the parameters used in the calculations.

Let's note separately, that at definition of thermal physics parameters the exact experiment is impossible, usually the error at definition of parameters is not less than 10%, that is why aspiration to reception of exact decisions tactically is not always true, as the parameters put in them at calculations already contain an error.

Determination of parameters by the known velocity field (inverse problems) as a rule do not give good results, these problems belong to the class of incorrect.

Let us give examples of application of approximate solutions. The equation for the potential in general form

\[
\frac{\partial \varphi}{\partial t} = k \varphi - \beta (\vec{\nabla} \varphi) + q(\vec{r}, t) \tag{1}
\]

\( k \) - transfer coefficient, \( \beta \)- coefficient determining the ratio of macro- and microtransfer, \( q \) - source.

The boundary and initial conditions for each problem are written down.

**Materials and Methods**

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**Results and Tasks**

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1. Solution of the problem of potential determination in a zonally inhomogeneous medium

The equation for determining the potential (temperature, pressure, concentration) has the form

\[
\frac{\partial \varphi}{\partial t} = a_i \frac{\partial^2 \varphi}{\partial x^2} \quad l_{i-1} \leq x \leq l_i
\]

\[
\varphi_i = \varphi_{i+1} \quad k_i \frac{\partial \varphi_i}{\partial x} = k_{i+1} \frac{\partial \varphi_{i+1}}{\partial x}
\]

\[
y(x) = \begin{cases} 
  x & 0 \leq x \leq l_1 \\
  (x-l_1)\lambda_1 + l_1 & l_1 \leq x \leq l_2 \\
  (x-l_1)\lambda_1\lambda_2 + (l_2-l_1)\lambda_1 + l_1 & l_2 \leq x \leq l_3 \\
  \cdots & \cdots 
\end{cases}
\]

\[
\lambda_i = \frac{k_i}{k_{i+1}}
\]

\[
c_i = (l_i-l_{i-1})\lambda_1\ldots\lambda_{i-1} + (l_{i-1}-l_{i-2})\lambda_1\ldots\lambda_{i-2} + l_i
\]

The system of equations is reduced to the need to solve a single equation, which is not very difficult.

\[
\frac{\partial^2 \varphi}{\partial y^2} = \beta(y) \frac{\partial \varphi}{\partial t}
\]

\[
\beta(y) = \begin{cases} 
  1 & 0 \leq y \leq c_1 \\
  \frac{1}{a_i \lambda_i} & c_1 \leq x \leq c_2 \\
  \cdots & \cdots \\
  \frac{1}{a_n \lambda_1^2 \ldots \lambda_{n-1}^2} & c_{n-1} \leq x \leq c_n
\end{cases}
\]

2. Solution of problems taking into account the dependence of the transport parameter on the potential in the stationary case.

1. Here is an example of moisture potential determination. This example is taken due to the complexity of dependence of the moisture conductivity coefficient on the potential [5].

The dependence of the diffusivity coefficient on temperature is less complicated and the solution is quite simple.

Let us denote the moisture potential by the letter \(u(x, t)\).
When wet air filtration is taken into account, the humidity field changes, but its character remains the same. In this case, it is possible to introduce an effective moisture conductivity coefficient that takes filtration into account. Calculations have shown that introduction of the effective coefficient is possible up to
filtration rate values equal to 0.1 m/s.


Necessity to take into account the dependence of transport parameters on the potential \( \Pi(r,t) \) leading to the need to solve the nonlinear heat conduction equation [5-6]:

\[
\frac{\partial \varphi}{\partial t} = \frac{\partial}{\partial x} \left[ k(\varphi) \frac{\partial \varphi}{\partial x} \right]
\]  

(3)

Let us make the replacement

\[
\Omega(\varphi) = \int k(\varphi) d\varphi
\]

Then equation (3) has the form:

\[
\frac{\partial \Omega}{\partial t} = k(\varphi) \frac{\partial^2 \Omega}{\partial x^2}
\]  

(4)

The boundary and initial conditions will be written as follows:

\[
\Omega|_{x=0} = c_1, \quad \Omega|_{x=l} = c_2, \quad \Omega|_{t=0} = c_0.
\]

According to the comparison theorem the required potential will be in the interval

\[
\varphi_{\text{min}}(x,t) \leq \varphi(x,t) \leq \varphi_{\text{max}}(x,t).
\]

Solving equation (4) by the Bubnov-Galerkin method, we obtain (first approximation) for the transformed function:

\[
\Omega_{\text{min(max)}} = 1.068 \cdot 10^{-5} - 3.86 \cdot 10^{-5} x - 2.602 \cdot 10^{-5} \frac{x}{l} \left(1 - \frac{x}{l}\right) \exp \left(-\frac{10k_{\text{min(max)}}}{l^2} \right)
\]

Doing the inverse transformation, we obtain the desired expression for the potential.
Fig. 3. Comparison of moisture field calculation results with experimental data.

Fig. 3 shows the dependence of relative humidity potential on distance at a certain value of $Fo_2$, $Fo_2 = k_{av} t/l^2$, modified Fourier coefficient.

The introduction of effective physical quantities is useful for analytical and numerical solutions.


This problem differs from the classical Stefan problem with phase transformations, since the freezing front does not represent the interface surface. That is why the solution of this problem presents great difficulties, making it nonlinear.

The effective heat capacity is represented as a sum of two summands.

The first is the specific heat capacity in the solid phase, $c_0$

The second is the consideration of phase transformations, taking into account that the melting centers are located according to the Gaussian distribution.

$$c_{eff} = c_0 + \frac{\lambda_{melt}}{T_H \sqrt{\pi}} e^{\left(\frac{T - T_{mel}}{T_H}\right)},$$

where $T_H$ – is the half-width of the Gaussian function, $T$ – is the changing temperature at a given point of the substance experiencing the phase transition, $T_{melt}$ – the melting temperature.
5. Temperature distribution under time-varying boundary condition.

Let us consider the solution of the problem of determining the temperature field in a cylinder when the temperature at the boundary increases according to the linear law [7].

The heat conduction equation has the form:

$$\frac{\partial}{\partial t}(rT) = a \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right)$$

The boundary and initial conditions are of the form:

$$r = 0 \quad \frac{\partial T}{\partial r} = 0$$

$$r = R \quad T = T_0 + kt$$

$$T \Big|_{r=0} = T_0$$

Let's write down the approximate solution (first approximation)
\[ T_{\text{apr}} = T_0 + kt + \varphi(t)\left(r^2 - R^2\right), \]

\[ \left(r^2 - R^2\right)^k - \text{coordinate function} \]

Condition for minimization of the non-convexity:

\[ \int_0^R \Phi^1(r,t)\left(r^2 - R^2\right)dr = 0 \]

We obtain the solution in the form:

\[ T_{\text{apr}} = T_0 + kt + Ce^{\frac{6at}{R^2}}\left(r^2 - R^2\right) \]

where \( C \) – is an unknown quantity, which can be found from the condition of minimizing the mismatch of initial conditions:

\[ \int_0^R \Phi^1(r,t)\left(r^2 - R^2\right)dr = 0 \]

The final solution is obtained as:

\[ T_{\text{apr}} = T_0 + kt + \frac{k}{4a} e^{\frac{6at}{R^2}}\left(r^2 - R^2\right) \]

The calculations show that with \( Fo \geq 0.07 \) the solution error does not exceed 10%.

6. Stefan’s problem - determination of the temperature field in phase transformations (nonlinear problem).

One of the main tasks in solving the Stefan problem [8] is to determine the velocity of the phase boundary, i.e., the freezing boundary.

For this purpose, we solve the system of equations:
\[ \frac{\partial T_1}{\partial \tau} = a_1 \frac{\partial^2 T_1}{\partial x^2}, \quad 0 \leq x \leq \delta(\tau) \]

\[ \frac{\partial T_2}{\partial \tau} = a_2 \frac{\partial^2 T_2}{\partial x^2}, \quad \delta(\tau) \leq x \leq \delta_{cm} \]

\[ \left( \frac{\partial T_1}{\partial x} \right)_{0,\tau} = 0; \quad T_2(\delta_{cm}, \tau) = T_a; \quad T_1(\delta, \tau) = T_2(\delta, \tau) = T_{melt} \]

\[ \pm \omega p_2 r_{melt} \frac{\partial \delta}{\partial \tau} = \left( \lambda \frac{\partial T_1}{\partial x} \right)_{x=\delta-0} - \left( \lambda \frac{\partial T_2}{\partial x} \right)_{x=\delta+0} \]

\( \delta(\tau) \) – freezing boundary, \( T_1 \) and \( T_2 \) - temperature in frozen and unfrozen zones.

The nonlinearity of the problem gives the boundary condition at the phase transition boundary.

The problem is solved using the Bubnov-Galerkin method, and an approximate solution is written based on the boundary conditions in each zone.

The expression for the time dependence of the boundary is as follows:

\[ \delta = \sqrt{\frac{k\lambda T_H \tau}{\omega p r_{melt}}}, \text{ где } k = \frac{2}{1 - c t_H / 2 \omega r_{melt}} \]

**Fig. 5.** Dependence of frozen layer thickness (meters) on time (hour) for the layer of polystyrene foam insulation (line 1) and brick wall (line 2).

Dependence of frozen layer thickness (m) on time (h) for two types of insulation.

**The main results**

1. Approximate solution methods provide an opportunity to evaluate the correctness of
solutions obtained by numerical methods.

2. Solutions obtained by approximate methods are more accurate than simplification of exact solutions.

3. Approximate solutions allow us to analyze the dependencies of physical quantities on parameters and time.

4. The accuracy of approximate solutions is often not less than the accuracy of the parameters put into the solution.

5. The proposed solution methods allow us to analyze and understand nonlinear processes.

6. It is possible to obtain approximate solutions of problems with phase transformations (Stefan's problem, the problem of vapor condensation front advancement).

7. Approximate solutions allow faster numerical engineering calculations.

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