Mass spectrum of elementary particles

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Abstract. In this study, we discuss different methods for quantifying the mass of elementary particles. The Barut's model approach gives us a more comprehensive understanding of elementary particles and their formation through magnetic interaction. Through this model, we can understand the mechanism of the formation of hundreds of unstable elementary particles whose components are interconnected through positive energy and thus quickly disintegrate into their basic components.

1 Introduction

The problem of studying the mass spectrum of elementary particles is considered one of the unsolved problems in physics, as it involves many methods and unresolved results. One of the first to mention this issue was Nambu [1], who correlated the masses of elementary particles (known at the time) with the fine structure constant. After that, Barut succeeded in developing a formula that includes leptons, based on Nambu’s idea, where he was able to obtain the mass of the tau and predict the mass of the fourth lepton [2]:

\[ m_n = m_e \left(1 + \frac{3}{2\alpha} \sum_{k=0}^{n} k^4 \right), \quad k = 0,1,2,3, \ldots \]

Later, a Japanese physicist named Yoshio Koide was able to find a relationship between the three previous leptons, and it is called the Koide formula [3]:

\[ m_e + m_\mu + m_\tau = \frac{2}{3} \left(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau} \right)^2. \]

And despite its accuracy, there is still no theoretical basis from which the Koide formula can be deduced. There is another approach in the study based on group theory SU(n) (Gell-Mann) and also a geometric approach (Polokhov-Vladimirov). Later, Varlamov introduced a different formula based on two parameters [4]:

\[ m_s = m_e \left(l + \frac{1}{2}\right) \left(i + \frac{1}{2}\right), \quad l, i \ldots \]

The mass (state energy) is determined by the "l, i - cyclic representation" of the Lorentz group, and the electron mass \( m_e \) plays the role of the "mass quantum". Cyclic representations and the number \( s = |l - i| \) are determined by the value of the particle spin.
2 Materials and methods

\[ H = \frac{1}{2m_1} \left( \vec{P}_1 - e_1 \vec{A}_1 \right)^2 + \frac{1}{2m_2} \left( \vec{P}_2 - e_2 \vec{A}_1 \right)^2 + \frac{1}{4\pi\varepsilon_0} \frac{e_1 e_2}{|\vec{r}_1 - \vec{r}_2|} + S_{12} (\vec{r}_1 - \vec{r}_2). \]

\[ \vec{A}_1(\vec{r}) = \frac{M_1 \times (\vec{r}_2 - \vec{r}_1)}{|\vec{r}_2 - \vec{r}_1|^3}, \quad \vec{A}_2(\vec{r}) = \frac{M_2 \times (\vec{r}_1 - \vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|^3}. \]

\[ H = \frac{\vec{p}^2}{2\mu} - \vec{P} \left( \frac{e_1 M_2}{c m_1} + \frac{e_2 M_1}{c m_2} \right) \times \frac{\vec{r}}{r^3} + \left( \frac{\epsilon_1^2 M_2^2}{2c^2 m_1} + \frac{\epsilon_2^2 M_1^2}{2c^2 m_2} \right) \frac{1}{r^3} + \frac{M_1 M_2}{r^3} \left( S^2 - 3(\vec{S} \cdot \vec{r}_0)^2 \right). \]

\[ S_{12} = \frac{1}{r^3} \left[ M_1 M_2 - 3(M_1 \cdot \vec{r}_0)(M_2 \cdot \vec{r}_0) \right], \quad \vec{r}_0 = \frac{\vec{r}}{|\vec{r}|}. \]

\[ \mu = \frac{m_1 m_2}{m_1 + m_2} \] is reduced mass.

\[ V(r) = \frac{a}{r^2} + \frac{b}{r^3} + \frac{c}{r^4} + \frac{d}{r^5}. \]
Fig. 1. Effective interaction potential of the electron with the proton in the Barut model.

When limited to spherically symmetric terms, the radial stationary Schrödinger equation has the form:

$$\frac{d^2X}{dr^2} + \frac{2m}{\hbar^2} [E - V(r)]X = 0.$$  \hspace{2cm} (7)

We made the replacement $X = Rr$.

To solve the previous equation, the behavior of the wave function must be discussed:

We rewrite equation (7) as:

$$\ddot{X} - \frac{\varepsilon}{r} X = -\varepsilon + V_1 + V_2 + V_3 + V_4.$$  \hspace{2cm} (8)

When $r \to 0$, the fourth term dominates the rest of the terms, equation (8) becomes:

$$\ddot{X}_0 - \frac{\varepsilon_1^2 e_2}{2 c_4^4 m_1 m_2 \mu r^4} X_0 = 0.$$  \hspace{2cm} (9)

This equation has the following solution:

$$X_0 = A e^{-\theta r}, \quad \theta = \frac{\varepsilon_1^2 e_2^2}{\sqrt{2 c_4^4 m_1 m_2 \mu}}.$$  \hspace{2cm} (10)

When $r \to \infty$, the wave function $X \to X_\infty$, and we can write the following equation:

$$\ddot{X}_\infty + \frac{2 \mu E}{k^2} X_\infty = 0.$$  \hspace{2cm} (11)

The solution to the previous function is given in the following form:

$$X_\infty = A e^{-kr}, \quad k = \sqrt{-\frac{2 \mu E}{k^2}}.$$  \hspace{2cm} (12)

Hence, the final solution is given in the following form:

$$X(r) = f(r) r^\alpha X_\infty X_0.$$  \hspace{2cm} (13)

Where

$$f(r) = \begin{cases} \prod_{i=1}^{N} (r - a_i^{(N)}), & N > 0; \\ 1, & N = 0. \end{cases}$$

$$\varepsilon = -k^2, \quad a = -2ak, \quad b = a(\alpha - 1) - 2k\theta, \quad c = 2\theta(\alpha - 1), \quad d = \theta^2.$$  \hspace{2cm} (14)

The corresponding energy levels are given by the following formula:

$$E_3 = \frac{e_1^2 e_2^2}{4d} + \frac{c}{2\sqrt{d}} + \frac{2ad}{c+2\sqrt{d}} - b = 0.$$  \hspace{2cm} (15)
\[ \varepsilon_0^\pm = \frac{-1}{16d} \left( b \pm \sqrt{b^2 - 2ca} \right)^2 \]

When \( N = 0 \),

\[ X^{(0)}(r) = B^0 r^\alpha e^{-kr} e^{\frac{q}{r}} \]

\( B^0 \) is normalization constant.

\( N = 1 \),

\[ f(r) = r - a_1^{(1)} \]

The previous energy levels, when \( N = 0 \), is composed of several other levels that depend on \( \ell \), correspond to the following wave function:

\[ X_0(r) = B_0 r \alpha e^{-k r} e^{-\theta r} \]

Where \( B_0 \) is normalization constant.

For \( N = 1 \),

\[ f(r) = r - a_1^{(1)} \]

we find the next level of energy:

\[ \varepsilon_1^\pm(15) \]

\[ \varepsilon_1^\pm = -\left[ \frac{b \pm \sqrt{b^2 - 4d(\frac{1}{2}\sqrt{1 + \frac{c}{2\sqrt{d}}})}}{4(\sqrt{1 + \frac{c}{2\sqrt{d}}})} \right]^2 \]

... \( \varepsilon_N^\pm \)

3 Results and discussions

\[ \varepsilon_0^\pm, \varepsilon_1^\pm, ..., \varepsilon_N^\pm \]

\( m = m_1 + m_2 + \varepsilon_i^\pm \), where \( \varepsilon_i^\pm < 0 \), \( i = 1, 2, ..., N \)

\[ N \rightarrow N_{final} \]

\[ m_1 + m_2 \geq -\varepsilon_i^\pm \]

\[ m_d - m_p - m_n = 1876\text{MeV} - 938.27\text{MeV} - 939.57\text{MeV} = -1.72\text{MeV} \]

\[ E = \varepsilon_i^\pm = T + V(r) \]

\[ T = \varepsilon_i^\pm - V(r) \geq 0 \]

\[ V_0 < \varepsilon_i^\pm < V_{\text{max}} \leq 0 \]

\[ \text{Same for the hydrogen atom and stable chemical compounds.} \]

For elementary particles, we notice two things:

1. The mass of any particle is much greater than the mass of its components.
2. Elementary particles (except the proton, electron, and neutrino) quickly disintegrate, meaning that they do not form a stable physical system.

The previous two observations assure us that the bonding between elementary particles occurs through positive energy. The total energy \( E = \varepsilon_i^\pm = T + V(r) \), where \( T = \varepsilon_i^\pm - V(r) \geq 0 \).

\[ \text{Nature tends to be in the lowest energy state possible. Therefore, stable bound states must have negative bonding energy.} \]

As an example, deuterium is stable against breaking up into a proton and a neutron, since \( \varepsilon_i^\pm - V_0 < 0 \).
Fig. 2. Bonding through negative energy and forming stable outputs

\[ 0 \leq V_0 < \varepsilon_i^+ < V_{\text{max}}, \]

the physical system is not stable; see figure (3). This state corresponds to elementary particles like the muon, neutron, and so on. The bonding energy is positive, which means that the mass of the formed particle is greater than the sum of the masses of the particles forming it.

Fig. 3. Bonding through positive energy and forming unstable particle.

In this case:

\[ m > m_1 + m_2, \quad \varepsilon_i^+ > 0 \]

\[ m = m_1 + m_2 + \varepsilon_i^+ \]

4 Conclusion

The nuclei of atoms, atoms, and other compounds bond through negative energy and form stable physical systems that do not disintegrate except with an external influence. On the other hand, elementary particles bond through positive energy to form an unstable physical system that disintegrates into its elementary components (which do not disintegrate), such as the proton, electron, and neutrino.

References


4. V. V. Varlamov, Mass quantization and lorentz group Mathematical Structures and Modeling, 2(42), 11–28 (2017)


