Crack resistance of bar reinforced concrete elements during construction and reconstruction

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Abstract. The consequence of the well-established idea of the crack resistance of reinforced concrete structures are two types of engineering calculations: for the formation of cracks and for limiting their opening width. Such a view is based on certain assumptions, simplifying the calculation to the possibility of its implementation "manually" and neglecting some real physical processes occurring in reinforced concrete under load. Seven of the most significant assumptions are considered in the article. For each of them, an analysis was carried out to determine whether they correspond to reality throughout the entire life cycle of the structure. It was proposed to use only one single calculation at the level of standards to assess the crack resistance of structures - according to the crack opening width, $a_{cr}$. At the same time, the calculations already available in the design codes for limiting permeability and the safety of reinforcement will still remain in demand. The compressed algorithm is proposed for the possible consideration of the effect of cracks at all scale levels of the concrete structure, the key for which is the normalization of the statistical parameters of the distribution of discontinuities by diameters, lengths, openings, depths, directions, distances between discontinuities, etc.

1 Introduction

The consequence of the well-established idea of the crack resistance of reinforced concrete structures are two types of engineering calculations: for the formation of cracks and for limiting their opening width (the calculation for crack closure was also practiced earlier). This view is based on the following assumptions, which have been subjected to justified criticism in recent years:

1) the stress-strain state (SSS) of reinforced concrete bar elements is considered linear (uniaxial); if we talk about modern Russian design codes, then they provide an assessment of crack resistance only for sections that are in conditions of direct pure or longitudinal

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bending (eccentric compression), and for other loading conditions it is absent, that is, the stressed state of the bar is simplified as much as possible: \( \sigma_{ij} = \sigma_{xx} = \sigma_{1}, \ \varepsilon_{ij} = \varepsilon_{xx} = \varepsilon_{1} \), all other components of the stress and strain tensors are equated to zero;

2) before the appearance of cracks, reinforced concrete is considered as a solid deformable body without any initial defects and damage, which does not correspond to reality;

3) as a condition for the initiation of cracks, a force or deformation criterion is used, respectively: \( M = M_{cr}, \ \varepsilon_{bt} = \varepsilon_{bt,ult} \);

4) after the appearance of cracks, they are taken into account by partial or complete exclusion of the tensioned concrete zone from work by subtracting their area from the initial area of tensioned concrete;

5) plastic deformations of concrete and reinforcement are taken into account using mathematical dependencies of the "\( \sigma-\varepsilon \)" type, obtained by approximating the experimental data of standard samples tests for uniaxial tension or compression;

6) experimentally confirmed by methods of fracture mechanics such effects as: a) mutual influence of closely spaced cracks, b) the presence of a zone of stress relief along the crack trajectory, c) the appearance of stress concentration at the crack tip, etc. are not taken into account;

7) the consequence of such assumptions is yet another forced simplification, this is the refusal to consider the SSS at a point (infinitely small volume) of a solid body and the tensor analysis used in this case as applied to problems of mechanics (in which relationships are established between stresses, strains, strain moduli and other differential quantities), instead integral (generalized) quantities are considered: forces, bar axis curvature, section stiffness, etc.

All of the above assumptions generate a total error that is acceptable only for a narrow range of tasks to be solved, and practice dictates their diversity. In this regard, the purpose of this publication was to analyze the existing theoretical foundations of the essential approach for assessing the crack resistance of reinforced concrete bar elements and to outline ways to improve the methods and techniques for their calculation.

2 Materials and methods

The listed assumptions can be sorted into two groups: a) physical assumptions are the primary assumptions associated with the simplification or complete neglect of some real physical processes occurring in reinforced concrete under load: 2, 4, 5 and 6; b) model assumptions are the secondary assumptions related to the mathematical description of the work of reinforced concrete, taking into account the simplifications from the previous section: 1, 3 and 7.

Let's start with the analysis of assumption No. 2 of the first group. To do this, we turn to the structure of ordinary heavy concrete and the physical properties associated with it [1]. The most general classification is accepted, which distinguishes three main types of structure: microstructure - the structure of cement stone, mesostructure - the structure of a cement-sand mortar in concrete, macrostructure - a two-component system (mortar and coarse aggregate).

The microstructure of cement stone is characterized by such components as a crystalline intergrowth, tobermorite gel, not completely hydrated cement grains, and a space of discontinuities. The mesostructure of a cement-sand mortar is considered as a conglomerate structure with non-additive properties, in which the matrix is cement stone, and the filler is sand. The macrostructure has much in common with the mesostructure, since for it a cement-sand mortar can be considered as a matrix, in which coarse aggregate is distributed.
One of the most important characteristics of the structure of concrete, as a capillary-porous material, at all levels are the parameters of its space of discontinuities (pores, capillaries, cracks, etc.). It is known that the volume of discontinuities in the steamed cement stone is on average from 15 to 50%. The volume of discontinuities in mortars and concretes, in which the entire volume between the grains is filled with cement stone, is somewhat smaller, but still quite large.

According to some researchers, for example, prof. Kolchunov Vl.I., the process of destruction of a material under load is based on two interdependent and mutually competing mechanisms: 1) the formation of plastic deformations by shear along certain crystallographic planes; 2) formation and development of cracks with subsequent rupture. Several directly adjacent dislocations are already a microcrack, which, as tensile stresses increase, can begin to expand. To describe these mechanisms, a general model in the form of a ball was proposed in [2], for which the summation of volume sectors, levels-radii from the matrix of slip planes was written. In this case, an alternative to the theory of plasticity is used in the form of an energy interpretation on the surface of a sphere and the determination of the integral of the root-mean-square value of shear stresses.

Delving further into the causes of discontinuities, we single out a few of the most significant:

1) reasons due to the chemical composition of the concrete mix (the amount of mixing water has a special effect here) and the process of cement hydration, for example: a) the difference in the rate of crystallization contacts formation of the intergrowth of various crystalline phases [2], [3], which leads to the appearance of internal stresses $\sigma_0$ exceeding the tensile strength of the crystalline intergrowth and cement stone grains: $\sigma_0 > R_t$; b) weakening in some areas of the strength characteristics of the same components of the cement stone structure due to a decrease in the surface energy of particles during the adsorption of liquids and gases [4] to such a level that $\sigma_0 > R_t$; c) the capillary pressure gradient exceeds the strength of the cement stone, which occurs between pores with different radii [1], [4], [5]; d) the occurrence of osmotic pressure due to the formation of shielding shells around the particles [1], which also leads to $\sigma_0 > R_t$ in some areas; and others - all of the above manifests itself at the stage of manufacturing reinforced concrete structures (hardening of the concrete mix);

2) reasons due to the stochastic nature of the cement stone crystals growth, which in turn leads to a distortion of the crystal lattice in the form of: a) dislocations [6], [7], [8], point, surface and volume defects, etc. b) contact interaction of low- and high-angle crystal boundaries [8], [9], [10], etc.; the listed distortions lead to the fact that in some areas of the inhomogeneous structure of the cement stone $\sigma_0 > R_t$;

3) reasons associated with shrinkage processes (reduction in the size and volume of concrete due to moisture loss, compaction, hardening as a result of physical and chemical processes) and the mode of hardening of the concrete mixture and the concrete itself [1], [4], [11], [12]: b) plastic (moisture) shrinkage due to surface evaporation of water; c) chemical shrinkage due to cement hydration (the products of chemical reactions have a higher density than the initial reagents and, accordingly, occupy a smaller volume due to the law of mass conservation); d) in addition, water is involved in chemical reactions, its consumption leads to the appearance of voids and, as it were, to self-drying of concrete or the so-called autogenous shrinkage (deformation of a heat-insulated non-drying sample); e) hydraulic (moisture) shrinkage which is the loss of water of hardened concrete in an environment with insufficient humidity; f) carbonization shrinkage of concrete ($CO_2 + Ca(OH)_2 = CaCO_3 + H_2O$) which develops over many years, leads to reinforcement corrosion;

4) temperature effects of the environment, causing concrete deformations exceeding the limiting tensile: $\varepsilon_T > \varepsilon_{bt,ult}$;

5) mechanical load causing concrete deformations exceeding the limit: $\varepsilon > \varepsilon_{bt,ult}$.
As it can be seen, there are a lot of reasons for the formation and subsequent development of discontinuities in concrete, and the list above is far from complete. But it is important to understand the following here:

- microcracks in concrete are always present;
- they arise from the very beginning of mixing the concrete mixture with water, then their number and size only increase;
- moreover, this happens long before the application of a mechanical load, according to which, in the theory of the force resistance of reinforced concrete, the crack resistance is calculated;
- at the same time, the cracks that have arisen by the time the load is applied are quite comparable in their geometric parameters (opening, deepening, etc.) with those that are subject to the normative calculation according to SR 63.13330, that is, the boundaries between the initial, including shrinkage cracks, power cracks at a load of $M < M_{\text{crc}}$ and cracks at $M \geq M_{\text{crc}}$ physically do not exist; this conclusion has far-reaching consequences, as will be discussed below.

Let us continue consideration of the assumptions No. 4 and 6 of the first group. The easiest way to take into account the influence of cracks is to subtract their total volume (or area) from the concrete volume (or cross-sectional area), as a solid body. Such an approach at the micro- and meso-levels of consideration is adopted in the theory of damage accumulation, in which the so-called damage parameter is introduced, for example [13], [14]:

$$D_V = \frac{\delta V_D}{\delta V}, \quad D_S = \frac{\delta S_D}{\delta S},$$

where $\delta V$ and $\delta S$ are respectively the volume and cross-sectional area of a representative volumetric element (RPE), within which the properties of the material are considered homogeneous; $\delta V_D$ and $\delta S_D$ are respectively, the increment of the total volume and area of all damages in the RPE.

This parameter is used when calculating the effective macrostresses associated with the nominal ones by the following relationship:

$$\sigma_{\text{eff}} = \frac{\sigma}{1-D}, \quad 0 \leq D \leq 1.$$  

In order to use stresses $\sigma_{\text{eff}}$ in practical calculations, it is necessary to construct a damage kinetic equation [15] as a function of many variables $D = f(\sigma, x_i)$, where $x_i$ is a set of geometric, physical, and kinematic parameters that describe the operation of a structure under load under given operating conditions, taking into account the history. At this stage in the development of science, a single approach to reliably obtain such an equation for reinforced concrete has not yet been developed. It is assumed that it can be obtained experimentally, for example, by mercury porosimetry [1], ultrasonic scanning [16], [17], etc.

The author's [18] monograph presents modern stochastic models of damage at the junction with the physical chemistry of the processes occurring in the structure of concrete under the influence of various factors: temperature, humidity, corrosion, etc. This direction of research related to the issue of crack formation seems to be the most promising.

Another class of modern concrete and reinforced concrete damage models is also being automatically developed by researchers [19], [20] and others, they are called the Softened Damage-Plasticity Model. Their essence is generally such that the behavior of concrete up to the vertex stresses on the material deformation diagram is described by the relationship
between stresses, relative deformations, damage parameter and other necessary parameters 
\((\sigma = f(\varepsilon, D, x))\), and with further loading of concrete (on the descending branch of the 
diagram), the relationship between stresses and the width of the macrocrack opening is 
already used \((\sigma = f(a_{cr}, x))\).

The introduction of the damage parameter, although somewhat clarifies the physics of 
the concrete deformation process, taking into account cracks, nevertheless, such phenomena 
as the initiation of cracks, their elongation and opening, the interaction of closely spaced 
cracks, their merging, curvature of tractors, the occurrence of stress concentration at the 
vertices, etc., remain without consideration (all this is the subject of study of another 
particular scientific discipline - fracture mechanics). That is, the approach of the damage 
accumulation theory retains a phenomenological character.

Similar conclusions apply to the macro level, with the only difference being that in the 
educational literature on reinforced concrete and in design codes (in the force and 
deforation variants of the calculation of reinforced concrete structures), an integral 
approach is adopted (in the theory of damage accumulation, as can be seen from formula (1)), 
a differential approach is used: a dangerous reinforced concrete section is considered, the 
stiffness of which decreases with loading due to the subtraction of the area occupied by a 
single macrocrack from the area of its tensioned concrete zone.

The way to eliminate the identified shortcomings is seen as follows:

1 - experimentally or on the basis of computer simulation, obtain a set of statistics for 
the geometric parameters of the initial discontinuities in concrete (formed by the time the 
mechanical load is applied) at all three scale levels in the form of distribution functions of 
discontinuities in diameters, lengths, openings, depths, directions, distances between 
discontinuities, etc.;

2 - to select for these statistics the theoretical distribution functions of a random variable 
(for the geometric parameters given in the previous paragraph) with the calculation of the 
moments of the random variable and other integral parameters necessary for further 
calculation - all this together should become the subject of normalization at the level of the 
corresponding sets of rules;

3 – to identify, according to certain criteria, sets of geometrically similar discontinuities 
for which one or another model of nonlinear fracture mechanics is applicable [21], [22], [23], 
[24] (according to the models of J. Irwin, D. Dugdale, A. Hilerborg, G.I. Barenblat, S. Sha, 
V.V. Panasyuk, M.Ya. Leonov, etc.), allowing to take into account a) the mutual influence 
of closely spaced discontinuities (hereinafter, specifically, cracks), b) the presence of a zone 
of unloading of forces along the trajectory of a particular crack, c) the appearance of stress 
concentration at the crack tip, etc.;

4 - based on p.p. 2 and 3 to develop such a kinetic equation of damage accumulation with 
a parameter of type (1), which would take into account both the statistical heterogeneity of 
discontinuities in concrete and the effects of nonlinear fracture mechanics.

We will not dwell on assumption No. 5 in detail, its analysis is a topic for a separate 
voluminous article. We only note that from the literature we have established about 30 
different geometric, physical and kinematic significant factors (they are also called scale 
fa\c-tors) that affect the nature of the work of the material under load and, accordingly, the 
recording of approximating dependencies of the “\(\sigma-\varepsilon\)” type of concrete and reinforcement. 
Taking into account or not taking into account one or another factor significantly affects the 
final result, including the assessment of the crack resistance of the structure. At the same 
time, only 4 of them are taken into account in the standards for concrete in the most simplified 
form: the concrete strength class, the increase in brittleness for high-strength concretes, the 
duration of the load, the humidity of the environment, and for reinforcement only one factor 
is taken into account: the strength class of steel.
Let’s move on to the assumptions of the second group. In part, they have already been discussed above, now we will analyze some issues in detail. In particular, according to the assumption number 1.

Consideration of the stress-strain state of reinforced concrete bar structures and their elements in the form of a linear (uniaxial) was justified in the era of "manual" calculation. Since the late 1980s, when computer modeling, computer-aided calculation, etc. began to be actively used in routine engineering design, it became possible to calculate any building structures, including taking into account the physical nonlinearity of reinforced concrete, in a three-dimensional formulation. Fortunately, the theoretical basis by this time already existed and was widely introduced into various software systems. However, due to some inertia, up to the present moment, the design codes for the reinforced concrete structures continued to develop along the path of the maximum simplification of calculations, so that they could be made “manually”. The appearance in 2003 in SNiP 52-01-2003 of the so-called non-linear deformation model (NDM) somewhat changed the situation: it became possible to perform calculations of reinforced concrete sections normal to the longitudinal axis of the bar, for both groups of limit states at all stages of loading from a single position and taking into account the physical nonlinearity of concrete and reinforcement (based on the corresponding deformation diagrams for uniaxial tension and compression). And this, in general, is a significant achievement of that time. However, this innovation came at the expense of the ability to calculate structures “manually” according to this model: it is necessary to solve the problem in several iterations, and within the calculated section, which is divided into small areas, it is necessary to perform multiple calculations of the SSS parameters for each area. At the same time, the model, unfortunately, retained the “birthmarks” of the era of “manual” calculation: the stress-strain state of reinforced concrete bar structures and their elements is still considered as linear (uniaxial). In connection with this, the science of the present day is faced with the task of introducing into the design codes methods for calculating reinforced concrete structures in a volumetric setting based on tensor analysis of the stress-strain state of a solid at a point (small volume), taking into account the physical nonlinearity of materials, which can be formalized within the framework of the concept of machine-readable norms.

By assumption No. 3. For the first time, a formula for estimating the moment of crack formation in reinforced concrete elements was proposed by A.A. Gvozdev and S.A. Dmitriev in [25] as part of the consideration of reinforced concrete as a rigid-plastic body (based on a rectangular diagram of normal stresses in the tensile zone of concrete) and then entered the Soviet, later Russian design codes for the design of reinforced concrete structures almost unchanged:

\[
M_{ec} = R_{bn} W_{pl},
\]  

(3)

where \( R_{bn} \) is the design strength of concrete in axial tension for calculations for the second group of limit states; \( W_{pl} \) is elastic-plastic modulus of the reinforced concrete section, equal to

\[
W_{pl} = \gamma W_{red},
\]  

(4)

where \( W_{red} \) is the elastic moment of resistance of the reduced section; \( \gamma \) is the coefficient of plasticity.

In passing, we note that the introduction of the theory of a rigid-plastic body, which maximally simplifies the nonlinear behavior of concrete and reinforcement under load, was at one time a necessary measure due to the lack of then the possibility to widely apply in
practice the automation of computer calculations: buildings and structures were calculated “manually”. In general, this echoes what was stated above for assumption No. 1.

Earlier, the first in order author of this article investigated a noticeable discrepancy of 35% in determining the coefficient $\gamma$ according to the force approach in SNiP 2.03.01-84* ($\gamma = 1.75$) and in SR 63.13330.2012 ($\gamma = 1.3$) [26]. It has been established that the technique of the previous standard gives, on the whole, a more adequate assessment of the influence of plastic deformations on the moment of crack formation. Nevertheless, not taking into account the strength of concrete and the presence of reinforcement in the section when determining the coefficient $\gamma$, as shown by comparison with the experiment, leads to an error from +64% to -17%. To eliminate this discrepancy between the calculation of reality, a formula was proposed, obtained using a nonlinear deformation model:

$$
\gamma = \begin{cases} 
1.6 + \frac{1}{100\mu_s}, & \text{if } \mu_s \geq 0.003 \\
1.6, & \text{if } \mu_s < 0.003
\end{cases}
$$

(5)

which in the range of concrete strength from B15 to B35 gives almost complete agreement with the experiment.

However, in the course of research, two contradictions were established between the theory of a non-linear deformation model and experience:

1 - in theory, the coefficient $\gamma$ is practically constant as the strength class of concrete increases over the entire range of change, in experiments - for concretes of low and medium strength, this pattern is confirmed, but after class B40, the considered coefficient begins to noticeably decrease;

2 - in theory, the coefficient $\gamma$ also increases with an increase in the reinforcement ratio, in experiments, on the contrary, it decreases.

Rethinking the results obtained led to the formulation of the following questions and an attempt to solve them within the framework of this article:

1 - how relevant are the studies of the force criterion for the appearance of cracks such as formulas (3), (4) today, what else can be clarified in them in order to bring theory closer to experiment, what factors should be taken into account?

2 - under what load do cracks actually appear and what would be the most correct to take as a criterion for their appearance, what will it depend on?

Answering the first question, let’s first turn to Eurocode-2. An analysis of the provisions of this regulatory document shows that there is no such concept as the moment of crack formation $M_{cr}$ in it. However, paragraph 7.1 (2) states: “When determining stresses and strains, the sections are considered as sections without cracks, provided that the bending stresses do not exceed $f_{ct,eff}$ (the average value of the tensile strength of concrete at the time when cracking can first occur). The values $f_{ct,eff}$ can be taken as $f_{ctm}$ (average tensile strength of concrete in axial tension) or $f_{ctm,fi}$, provided that the calculation of the minimum amount of longitudinal reinforcement is also based on this value. This remark is followed by provisions for calculating the width of the crack opening. From the above quotation, using the well-known formula from strength of materials $\sigma = \frac{M}{W} \leq R$, one can obtain an expression like (1): $M_{cr} = R_{cr,ef} W$, as a consequence, however, Eurocode-2 itself does not stipulate which stress diagram should be taken in the tensile zone of concrete before the appearance of a crack, how to take into account plastic deformations (in general, information is rather scarce). At the same time, we note that English BS 8110 and French BAEL 91 design codes do not even have this.
A more detailed cracking moment is presented in American standards ACI 318:

\[ M_{cr} = \frac{f_y I_g}{y_t}, \quad (6) \]

where \( f_y = 7.5 \lambda \sqrt{f_{ct}'} \) is tensile modulus of concrete (flexural tensile strength of concrete), \( f_{ct}' \) is the given concrete compressive strength, \( \lambda \) is the coefficient of transition from the properties of heavy concrete to the properties of lightweight concrete; \( y_t \) is distance from the center of gravity of the section, excluding reinforcement, to the tensioned concrete face (otherwise, the height of the tensioned concrete zone); \( I_g \) is moment of inertia relative to the main central axes of the section without regard to reinforcement.

As can be seen from the explanations to the formula (6), in ACI 318 everything is taken rather simplified.

This state of affairs suggests: either determining the moment of cracking is an insignificant, completely irrelevant task, or the task is still so significant and relevant, but so non-trivial and complex that foreign developers of standards were unable to do it and they either bypassed it (BS 81, BAEL 91), or made very rough assumptions (Eurocode-2, ACI 318). And Russian scientists more or less successfully coped with it, and for a long time (1957) [25].

According to SR 63.13330, the moment of cracking is necessary for:

1 - in fact, to answer the question of whether cracks appear at a given level of design load, which is important for non-reinforced concrete elements (centrally loaded foundations, floors on the ground, etc.), since the stage of cracking for them is at the same time the stage of destruction, as well as for reinforced concrete elements in which the appearance of cracks is not allowed (structures that are under the pressure of liquids and gases, i.e., which must ensure impermeability and tightness - tanks, pressure pipes, etc., structures with increased durability (for example, protective reinforced concrete shells of operating nuclear installations), structures exposed to strong aggressive environmental influences (for example, sea berths));

2 - establishing by what formulas of the method of limiting forces to consider deflections of bent structures: deflections are with or without cracks;

3 - to calculate the coefficient \( \psi \), using the limit force method, which takes into account the work of tensile concrete in the area between two adjacent cracks - assuming a triangular diagram of normal stresses in the compressed zone of concrete and complete shutdown of the tensile concrete zone from work, including above the top of the appeared crack.

An error in the definition of \( M_{cr} \) in each of the above cases can lead to the following consequences:

1 - When you need to find out: will cracks appear or not? If \( M_{cr}^{calc} > M_{cr}^{fact} \), then there will be cracks in structures in which their appearance is not permissible, which is fraught with additional costs for repair and restoration measures at the stage of operation of incorrectly designed structures; if \( M_{cr}^{calc} < M_{cr}^{fact} \), then due to the margin of cracking, no cracks will occur, but a noticeable overrun of materials is possible.

2 - When you need to calculate deflections. If \( M_{cr}^{calc} > M_{cr}^{fact} \), then in calculations of structural deflections without taking into account cracks, they will be significantly underestimated - from authors experience in calculations and design - by more than two times, which in some cases will require strengthening at the stage of operation of incorrectly designed structures; if \( M_{cr}^{calc} < M_{cr}^{fact} \), when calculating deflections, taking into account...
cracks that will not actually occur, we proceed to re-evaluate them - also by at least two times, which, especially for structures with large spans, will lead to a noticeable waste of materials.

3 - When you need to determine the width of crack opening. If \( M_{\text{calc}} > M_{\text{fact}} \), then the cracks that appear will have an actual opening width that is less than the calculated value, which in some cases will lead to an overrun of materials; if \( M_{\text{calc}} < M_{\text{fact}} \), then the cracks that have appeared will have an actual opening width greater than the calculated value, which is fraught with additional costs for repair and restoration measures at the stage of operation of incorrectly designed structures.

The consequences listed above will be more tangible, the larger the ratio \( M_{\text{calc}} / M_{\text{fact}} \) is. Thus, the magnitude of the cracking moment \( M_{\text{crc}} \) is a very significant parameter for practice in the framework of calculations using the limit force method.

As for the calculations for the nonlinear deformation model (NDM) by the diagram method:

1 - The question is, do cracks appear at a given level of design load? This question is solved on the basis of the deformation criterion of the following form:

\[
\varepsilon_{\text{bt}}^{\text{max}} = \varepsilon_{\text{bt,ult}}
\]

(7)

where \( \varepsilon_{\text{bt}}^{\text{max}} \) are the relative deformations of concrete on the most tensioned face of the section; \( \varepsilon_{\text{bt,ult}} \) are the ultimate relative strains of tensile concrete.

2 - Due to the unity of the algorithm of the diagram method for both groups of limit states at all stages of loading - from zero to the flesh to failure, it does not matter what deflections we consider: with or without cracks - the formula for determining the stiffness of the section (the main characteristic that determines the resistance of the reinforced concrete structure to deflections) at all stages of the SSS remains the same. In this case, the condition for the growth of a normal crack along the height (inward) of the section of a structural element has a form similar to (7):

\[
\varepsilon_{\text{bt,i}} = \varepsilon_{\text{bt,ult}}
\]

(8)

where \( \varepsilon_{\text{bt,i}} \) are the relative deformations in the \( i \)-th stretched area (strip) of the concrete part of the section into which it is divided, with a load acting on the element above the level of cracking.

In that part of the section where the crack has grown, the tensile concrete deformation modulus is simply reset to zero, that is, if \( \varepsilon_{\text{bt,j}} > \varepsilon_{\text{bt,ult}} \), then \( E_{\text{bt,j}} = 0 \), which entails the nulling of stresses in this area: \( \sigma_{\text{bt,i}} = E_{\text{bt,i}} \varepsilon_{\text{bt,i}} = 0 \).

3 - When determining the coefficient \( \psi \) in the NDM, the moment \( M_{\text{crc}} \) is not used (see formula (8.161) SR 63.13330), it is calculated through the relative deformations in the reinforcement: a) arising from a given load - these are the averaged relative deformations of the tensile reinforcement crossing cracks, in the considered stage of calculation - \( \varepsilon_{b} \), and b) arising from the load immediately the appearance of the first crack - \( \varepsilon_{s,crc} \).

Thus, the knowledge of \( M_{\text{crc}} \) for NDM calculations by the diagram method is no longer in demand. Nevertheless, \( M_{\text{crc}} \), as a derivative (secondary) quantity, is easily determined by numerical integration (summation) of the products of normal stresses \( \times \) the areas on which
they act, × the corresponding arms of the pair of forces, over all components of the element section: 

\[ M_{crc} = \sum_{i=1}^{n-1} \sigma_i A_i h_i = \sum_{i=1}^{n-1} E_i \varepsilon_i A_i h_i. \]

The calculation model of the section for the case of oblique bending is shown in Fig. 1.

![Fig. 1. Design scheme of a reinforced concrete element with oblique bending: a - diagram of forces in cross section; c - design diagram of the cross section of the element for constructing a nonlinear deformed model (stresses and strains are conventionally not shown)](image)

In addition, it follows from the previous considerations that in all cases related to the influence of normal cracks on the stress-strain state of a reinforced concrete element, according to the NDM, only relative deformations appear, and when they are calculated, plastic deformations of materials are fully taken into account - through the corresponding physical ratios for concrete and reinforcing steel in the form of “σ-ε” dependencies. That is, the problem of accurately determining the coefficient γ disappears by itself - it becomes simply not needed.

However, it is still possible to determine this coefficient γ according to NDM if two independent calculations are performed with criterion (7), while using identical initial data on design, loading, etc., but with the exception of data on the law of deformation of materials, there will be a difference: in the first case, the dependencies “σ-ε” should be applied, taking into account the physical nonlinearity of concrete and reinforcing steel, in the second case, the linear Hooke’s law should be applied. Then, as a result, we obtain, respectively, two moments of cracking: plastic - \( M_{crc}^{pl} \), and elastic - \( M_{crc}^{el} \). Their ratio will give the desired coefficient:

\[ \gamma = \frac{M_{crc}^{pl}}{M_{crc}^{el}}. \]

The coefficient γ obtained by this method in [22] had not only a quantitative, but, importantly, a qualitative discrepancy with the experimental data, as discussed above.
Further, in order to clarify this discrepancy, we must move on to answering the second question posed above: “at what load do cracks actually appear and what would be the most correct to accept as a criterion for their appearance, why will it depend?” In general, the answer to the second part of the question has already been given when analyzing the causes of non-continuities in concrete in the form of the following conclusion: “the boundaries between the initial, including shrinkage cracks, power cracks at a load of $M < M_{crc}$ and cracks at $M \geq M_{crc}$ do not physically exist.”

It follows from this that the calculation for the formation of cracks does not have any strict physical meaning. This is an abstraction in its purest form, a convention based on the accepted design model of the limit force method, or the diagram method, etc.

To fill this abstraction with specifics, it is necessary:

1 – to analyze the most complete list of causes of non-continuities (including cracks) in concrete and reinforced concrete, arrange them according to the degree of influence and mutual nesting of causes and effects;

2 - for each reason, to obtain a set of statistics for the geometric parameters of discontinuities in the form of discontinuity distribution functions for diameters, lengths, crack openings, depths, directions, distances between discontinuities, etc., this can be done experimentally or on the basis of modeling;

3 - to select for these statistics the theoretical distribution functions of a random variable (for the geometric parameters given in the previous paragraph) with the calculation of the moments of the random variable and other integral parameters necessary for further calculation - all this together should become the subject of normalization at the level of the corresponding sets of rules;

4 – to take as criteria for the onset of a particular limit state the corresponding integral parameters (such as the mathematical expectation, dispersion, etc. of the statistics described in the previous paragraph), characterizing the kinetics of the development of a space of discontinuities in concrete, reinforcement, in the zone of their contact, etc.;

5 - also for each cause (factor) to develop a separate calculation model of the "factor-impact-result" type, containing a set of integral parameters from the previous paragraph; to link them into a single general model, at the output of the calculation, by which you can get the most complete picture of the defects in the structure of concrete at all stages of the life cycle of structures, depending on the specified conditions.

3 Results and discussion

In the first approximation, it is possible to solve these problems using the finite element method, for example, in the Mechanical APDL of the ANSYS software, where concrete can be modeled with finite elements of the Solid-65 type with the William-Warnake strength criterion based on the theory of plastic flow; specify the longitudinal and transverse reinforcement with bar finite elements of the Beam 188 type with the von Mises strength criterion. Fig. 2 shows some finite element models of reinforced concrete beams in oblique bending, for which the moment of cracking was determined (5 factors of influence were varied).
Fig. 2. Computer models of some reinforced concrete beams, with varying parameters according to the plan of a multifactorial numerical experiment.
The results of the numerical experiment are analyzed. In particular, a Paretto Map of standardized effects was obtained (Fig. 3).

Fig. 3. Pareto map of standardized effects

The boundary between initial shrinkage cracks, force cracks under load $M < M_{crk}$ and cracks at $M \geq M_{crk}$ will become clear, quantitatively measurable and qualitatively perceptible. But in this case, the need to count on crack formation will disappear by itself. The key and only assessment of the crack resistance of a structure then becomes the calculation for crack opening, which can be determined at all stages of loading - from zero to failure.

With such a statement of the question, the requirements for the calculation can be as follows:

- as in the traditional calculation according to modern standards, the operating conditions of the structure are first established, what loads and effects are exerted on it, from the design experience, the necessary values of geometric, physical and design parameters are assigned - $x_j$;
- for the considered limit state, the normalized value of the corresponding integral parameter $P_{i,ult}$ is set, which characterizes the space of discontinuities in concrete and reinforcement;
- a calculation is performed, as a result of which either the direct problem of selecting the required reinforcement $A_{ult} = F(x_1, x_2, \ldots, x_{n-1}, P_{i,ult})$ is solved, or the inverse problem is solved based on the type check $P_i < P_{i,ult}$.

According to the foregoing, for example, the following problem is possible: for structures that are under the pressure of liquids and gases, i.e. which must ensure tightness and tightness: tanks, pressure pipes, etc.:

- the crack width is calculated,
- this width is used to determine the depth of cracks,
- if the crack opening is within the acceptable range (obviously less than 0.3 mm), then the structure will be impermeable,
- otherwise, we will need to change the design: increase reinforcement, increase the class of concrete, etc.

However, at the current stage of development of the science of reinforced concrete, it is still impossible to solve the above tasks. This article outlines only a few steps towards this.

4 Conclusion

1. The article considers the seven most significant assumptions of the traditional approach to calculating the crack resistance of reinforced concrete structures, explicitly or implicitly adopted at the level of modern design standards and simplifying the calculation to the possibility of its implementation "manually". For each of them, an analysis was carried out for compliance with real physical phenomena occurring in reinforced concrete throughout the entire life cycle of the structure: from mixing the concrete mixture with water, ending with destruction.
2. It has been established that there are a huge number of reasons for the formation and subsequent development of discontinuities in concrete; in this article, five main groups are singled out: due to the chemical composition of the concrete mixture, the stochastic nature of the growth of cement stone crystals, shrinkage processes, temperature effects of the external environment, and the action of mechanical load.
3. It is noted that the cracks that have arisen by the time the load is applied are quite comparable in their geometric parameters (opening, deepening, etc.) with those that are subject to the normative calculation according to SR 63.13330, that is, the boundaries between the initial, including shrinkage cracks, power cracks at a load of $M < M_{crc}$ and cracks at $M \geq M_{crc}$ do not physically exist. It follows from this that the calculation for the formation of cracks does not have any strict physical meaning. This is an abstraction in its purest form, a convention based on the accepted design model of the limit force method or the diagram method, etc.
4. In this regard, it is proposed to use only one calculation at the level of standards to assess the crack resistance of structures - according to the crack opening width, $a_{crc}$. So, at a certain value of $a_{crc}$, the structure will still remain impermeable (the cracks will be non-through), and if this value is exceeded, they will not. At the same time, the calculations for the limitation of permeability and the safety of reinforcement, which are already available in the design codes, will still remain in demand.
5. At the junction of the theory of damage accumulation and nonlinear fracture mechanics, a compressed algorithm is proposed for the possible consideration of the effect of cracks at all scale levels of the concrete structure, the key for which is the normalization of the statistical parameters of the distribution of discontinuities by diameters, lengths, openings, depths, directions, distances between discontinuities, etc.

References

1. A.E. Sheikin, Yu.V. Chekhovsky, M.I. Bresser, Structure and properties of cement concretes (Stroyizdat, Moscow, 1979)
2. V.I. Kolchunov, Construction and reconstruction 4, 15-33 (2022)
3. T.V. Kuznetsova, M.V. Kudryashov, V.V. Timashev, Physical chemistry of binding materials (Higher School, Moscow, 1989)
4. K. G. Krasilnikov, L.V. Nikitin, N.N. Skoblinskaya, Physico-chemistry of proper deformations of cement stone (Stroyizdat, Moscow, 1980)
5. G. I. Gorchakov, Building materials (Higher School, Moscow, 1981)
8. A.N. Orlov, Introduction to the theory of defects in crystals (Higher School, Moscow, 1983)
10. W. Boolmann, Crystal Defects and Crystallite Interfaces (Sprimser-Verlag, Berlin, 1970)
12. Z.N. Tsilosani, Shrinkage and creep of concrete (Metsniereba, Tbilisi, 1979)
13. L.M. Kachanov, Izv. AN USSR, REL. 8, 26-31 (1958)
15. F.H. Akhmetzyanov, To assess the strength and durability of damaged concrete and reinforced concrete elements (Novoe Znanie, Kazan, 1997)
17. O.Ya. Berg, E.N. Sheherbakov, G.N. Pisanko, High-strength concrete (Stroyizdat, Moscow, 1971)
18. G. De Schutter, Damage to Concrete Structures (CRC Press, Taylor & Francis Group, Boca Raton, 2013)
25. A. A. Gvozdev, S. A. Dmitriev, Concrete and reinforced concrete 5, 205-211 (1957)