

Modeling of forces perceived by a wagon when rolling down the hill

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Abstract. The article presents the results of the construction of computational models and the derivation of analytical formulas describing all active and reactive forces acting and appearing when a single car rolls down a sorting slide.

1 The relevance of the topic

It is well known [1, 2] that the sorting slide at railway stations is widely used for the formation and disbandment of rolling stock. In [1, 2], a dynamic model of rolling a car down a hill is constructed without taking into account the technology of placing (symmetrical or non-symmetrical) loads in the car, the acceleration of the movement of the car in curved sections of the track, the considered normal component of the inertia force in absolute motion, the impact force of wheel pairs on the switches on the track (about the wits, crosses and counter rails). The inaccuracies of the model also include the following.

The model does not take into account the rolling friction moments of the wheels with rails and the rolling bodies of the bearing relative to the inner and outer rings. The resistance to the movement of the car in curves depending on the type of bearing and the speed of movement are found by empirical formulas. The calculated speed of the dissolution of the car from the slide is found by the Galileo formula, as a free-falling body or according to the law of conservation of energy (the potential energy of the car turns into kinetic), which does not correspond to the physical meaning of the problem being solved, since it must be determined as a result of solving the differential equation of motion of the car. The profiles of slides with different slopes are reduced to one slope by averaging, while their calculation model should be taken into account as independent slopes. Resistance to movement when rolling the car down the hill in the form of specific resistance forces in kgfpts (as non-system units of force measurement) are represented by empirical formulas. The equality of 1 kgfpts = 1 % (ppm) does not make physical sense.

Without explaining the reasons for the occurrence of various types of resistances that appear when the car rolls down the hill, it is noted that "the determining factors are: the main resistivity w_o , resistivity from air and wind w_{rw} , resistivity from snow and frost w_{sn} ; additional – episodic resistance forces from impacts on the switches w_s , when moving in curves w_c and braking on retarders w_b ". At the same time, all these types of dimensionless resistivity (i.e.

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resistances attributed to the gravity of the car) are found by empirical formulas. In addition, the resistance to the movement of the car on the sub-assembly park is not taken into account when using the shoe.

In the formula for determining the relative uncoupling speed of v_r [1, 2], it turned out that the relative uncoupling speed of v_r depends on the average uncoupling speed on the descent section of the slide $v = v_{av}$, which is unacceptable, since the average speed of $v_{av} = (v_e + v_n)/2$, where v_n is the uncoupling speed at any time interval, and v_e is its portable speed (i.e. the uncoupling speed relative to the rail threads). If the relative speed of the cordon v_r is understood as its portable speed v_e (they cannot be otherwise), then everything is mixed up here. As a result, it turned out that v_r as v_e depends on v_e . In addition, all this does not correspond at all to the theorem on the addition of velocities in complex motion [3]. Thus, it is not at all permissible to mention the relative uncoupling velocity v_r , since the uncoupling velocity v , as the portable velocity v_e , is an unknown quantity determined as a result of solving the equations of motion that have not yet been compiled. In this problem, the cosine theorem can be used to determine the absolute velocity of air particles v_a at known values of the portable speed of the car v_e and the relative velocity of air particles v_r .

Thus, until now, researchers have completely overlooked the definition of the projection of the velocity of air particles (wind speed) relative to the movable axes associated with the car and the construction of a computational model corresponding to the forces acting on the car. In this regard, finding the forces perceived by the car when rolling down the car when rolling down the profile of the slide is still an urgent task of railway transport and transport science.

2 The formulation of the problem

It is required to determine the dependence of the longitudinal component of the projection of the relative velocity of the air flow on the wind speed relative to the ground) and the speed of the car when it rolls down the hill (the first task), to build a calculated model of the wheelset of the car (the second task), to find all the active and reactive forces acting and appearing when a single car rolls down the sorting hill (the third task).

3 Solution methods

Let's use classical concepts and provisions of theoretical mechanics, for example, such as the theorem on the addition of velocities in complex motion, connections, coupling reactions, the principle of release from bonds, the moment of a pair of forces, bringing a system of forces to a given point; Coulomb's law, portable inertia forces [3].

4 Task conditions and accepted prerequisites

Let's consider the general cases when a wagon rolls down a sorting slide progressively with a given initial speed v_0 (usually 4-5 kmph). When rolling a single car (or uncoupling) from a slide, the car will be affected mainly by external forces in the form of the gravity of the car with or without cargo – $\vec{G}$ and the force of aerodynamic air resistance – $\vec{F}_a$ (where $\vec{F}_a \in \vec{F}'_{ax}, \vec{F}'_{ay}$). The distribution of the weight of the car body with the load on the front and rear bogies depends on the technology of placing the load (symmetrically or not symmetrically relative to the axes of symmetry of the car) on the car [3-6].

Let's assume that the center of gravity of the load is spread out symmetrically relative to the axes of symmetry of the car, which allows us to consider the uniformity of the distribution of the weight of the car body with the load on the front and rear bogies.

We assume that the wheels of a wheeled pair of trolleys roll along the rail threads without sliding (pure rolling) in the absence of lateral rolling of the car, leading to contact of the wheel ridges with the sides of the rail threads, and the action of the aerodynamic drag force on the side of the car (i.e. at $F'_{ay} = 0$), as well as when the car is moving outside the brake positions (TP). According to Poinot's theorem, known from kinematics [Buchholz, 1967; Yablonsky, Nikiforova, 1998], the net rolling of the wheels of a wheeled pair of trolleys (as a movable centroid) along rail threads (as a stationary centroid) is explained by the fact that their point of contact (as a circle touching a straight line), at the considered moment t_1 , which is for them the instantaneous center of velocities (ICV), has a velocity equal to zero.

We assume that when climatic conditions change in winter, leading to a change in the density of the air environment, accompanied by snow or frost within the arrow zone of the bundles and on the sorting tracks, in the presence of lateral rolling of the car, the entry of the car on the side track through the switches, the movement of the car along the curved section of the profile of the slide, the action of force F'_{ay} , and when when the car is moving within the braking positions, the wheel pairs slide along the rail threads.

We believe that when passing a car along the length of the slide and/or park braking position (BP) wheels, wheel pairs of trolleys along the rail threads roll with sliding without rolling (pure sliding), while, depending on the magnitude of the braking forces, sliding wheels with rolling is not excluded.

It is well known [6, 7] that gravity $\vec{G}$ is a constant force and, according to the law of universal gravitation, acts on anybody that is near the earth's surface. The modulus of gravity is equal to the weight of the body (cargo). Gravity is an active force, because, starting to act on a resting body, according to Newton's second law, it can set it in motion.

The force of aerodynamic drag of air $\vec{F}_a$ belongs to the class of reactive force, depends on the relative velocity $\vec{v}_{ra}$ and acts on an object moving in, for example, an environment such as air. The force of the aerodynamic resistance of the air is the result of taking into account the discarded medium. Like the other reaction, it prevents movement, in this case relative to the speed of the air flow (headwind) $\vec{v}_{ra}$. At the same time, it can be attributed to the number of active forces, since, starting to act on an object, it can set it in motion if the direction of air velocity (tailwind) coincides with the direction of the speed of the car.

The force $\vec{F}_a$ is determined by the aerodynamic formula, kN [8]

$$F_a = 0.5 \times 10^{-3} c_a \rho_a A v_{ra}^2 \quad (1)$$

where c_a is the dimensionless experimental coefficient of air resistance, depending on the shape of the body and how it is oriented when moving (usually taken depending on the shape of the surface in the range from 0.55 to 1.2, for example, cylindrical bodies having a circle (tube) in cross section $c_a = 0.6$; for a flat surface $c_a = 1.1$); ρ_a is average air density (kgpm³) (usually take 1.26 - 1.29); A is maximum cross-sectional area of the plane perpendicular to the air flow (m²) ($A \in \{A_e, A_s\}$ - the area of either the end or side surfaces of the car with the load); ($\vec{v}_{ra}$ ($\vec{v}_{ra} \in \{\vec{v}_{ra,x}, \vec{v}_{ra,y}\}$) - air velocity relative to a car with a load (mps).

In (2.1), the dimensionless experimental coefficient of air resistance c_a corresponds to the coefficient of air resistance of single cars or the first car in the uncoupling described in [1, 2].

We formulate the first problem and show its solution.

5 Solution of the first problem

We will show the dependence of the projection of the relative velocity of air particles (wind speed) $\vec{v}_{rb}$ (calculated value) on the wind speed relative to the top of the hill (earth) (i.e., the absolute velocity of air particles) $\vec{v}_{a.a}$ (according to Chapter 5 of the SNiP "Construction

Climatology and Geophysics" the value set) and the speed of the car $\bar{v} = \bar{v}_b$ (the sought value). We assume that the car rolls down from the top of the slide with a portable speed $\bar{v}_e = \bar{v} = \bar{v}_a$ relative to the fixed coordinate system Oxyz. Assume that the moving coordinate system O1x1y1z1 is rigidly connected to the car, and the air particles, in turn, move at a speed $\bar{v}_{ra}$ relative to the moving coordinate system O1x1y1z1 (i.e., the car) (Fig. 1).

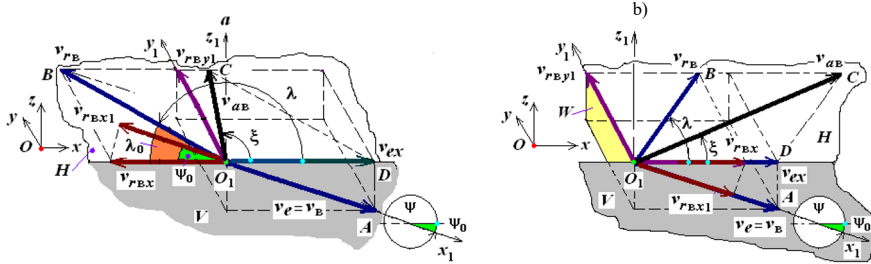


Fig. 1. Vector diagram of car speed and wind: a – headwind; b – tailwind

Fig. 1 shows: O – the beginning of the fixed coordinate system Oxyz, rigidly connected to the top of the slide (TS); O1 – the beginning of the movable coordinate system O1x1y1z1, rigidly connected to the wagon; H, V and W – horizontal, vertical and frontal planes; ψ_0 – the angle of descent (in accordance with the profile of the slide value set); $\bar{v}_{ra}$ – relative velocity of air particles (wind speed) relative to the moving reference frame O1x1y1z1 (wagon) (calculated value); λ – the guiding angle of the vector of relative velocity of air particles along the Ox axis (calculated value); $\bar{v}_{aa}$ – the absolute velocity of air particles relative to the ground (to the top of the hill) (according to Chapter 5 of the SNIP "Construction Climatology and Geophysics", the value is set); ξ – the guiding angle of the vector of the absolute velocity of air particles along the Ox axis (the value is set).

We assume that the relative velocity of air particles (wind speed) $\bar{v}_{r,b}$ is located on the horizontal plane H and is directed at an angle λ (or λ_0) to the horizon (Ox axis), and the portable velocity (car speed) $\bar{v}_e = \bar{v} = \bar{v}_a$ is on the vertical plane V and is directed at the angle of descent of the slide ψ (or ψ_0) to the horizon (Ox axis).

According to the theorem on the addition of velocities in complex motion [3, 8], we write

$$\bar{v}_{a,a} = \bar{v}_{ax} + \bar{v}_{r,a} \quad (2)$$

where $\bar{v}_{a,a}$ – absolute velocity of air particles (wind speed);

$\bar{v}_{ex} = \bar{v}_x = \bar{v}_{ax}$ – projection of the portable speed (car speed)

$\bar{v}_e = \bar{v} = \bar{v}_a$ on the axis Ox:

$$v_{ex} = v_x = v_{bx} = v_e \cos(\psi_0)$$

taking into account that ψ (or ψ_0) is the angle of descent of the slide to the horizon (Ox axis).

$\bar{v}_{r,a}$ – the relative velocity of air particles (wind speed) relative to the car.

We will keep in mind that for Fig. 1, a, where the wind direction is opposite to the direction of movement of the car (i.e. headwind), (2.2), according to the rule of vector subtraction [3], will be written as:

$$\bar{v}_{aa} = \bar{v}_{ex} + (-\bar{v}_{ra}) \quad (3)$$

where from

$$\bar{v}_{ra} = \bar{v}_{ax} - \bar{v}_{aa} \quad (4)$$

In the last expression, the modulus of the relative velocity of air particles (i.e., the wind speed relative to the car) $\bar{v}_{r.a}$ is found according to the cosine theorem [3,4]:

$$v_{r.a} = \sqrt{v_{ex}^2 + v_{a.a}^2 \pm 2v_{ex}v_{a.a} \cos(\bar{v}_{ex}, \bar{v}_{a.a})} \quad (5)$$

In (5), if the angle between the vectors $\bar{v}_{ex}$ and $\bar{v}_{a.a}$ is blunt, which corresponds to a tailwind, then $\cos(\bar{v}_{ex}, \bar{v}_{a.a}) = \cos(\xi)$ the “plus” sign, and if the angle between the vectors $\bar{v}_{ex}$ and $\bar{v}_{a.a}$ $\cos(\bar{v}_{ex}, \bar{v}_{r.a})$ is sharp, which corresponds to a headwind, then $\cos(\bar{v}_{ex}, \bar{v}_{a.a}) = \cos(\xi)$ the “minus” sign; (ξ is the angle between the vectors $\bar{v}_{ex}$ and $\bar{v}_{a.a}$) [5].

In (4), the guiding angle λ of the relative velocity of air particles (wind speed) $\bar{v}_{r.a}$ is found according to the sine theorem

$$\frac{\sin(\xi)}{\sin(\pi - \lambda)} = \frac{v_{ra}}{v_{aa}}$$

or, given that $\sin(\pi - \lambda) = \sin(\lambda)$,

$$\sin(\lambda) = \frac{v_{aa}}{v_{ra}} \sin(\xi) \quad (6)$$

For Fig. 1, a, where the wind direction is opposite to the direction of movement of the car (i.e. headwind), the projection (2.4) on the Ox axis has the form:

$$v_{ra.x} = v_e \cos(\psi_0) - v_{aa} \cos(\xi) \quad (7)$$

where ξ is the angle between the resulting vector $\bar{v}_{aa}$ (absolute velocity of air particles (wind speed)) and the longitudinal axis Ox, rad.

For Fig. 1, b, where the wind direction coincides with the direction of movement of the car (i.e. tailwind), the projection (2.2) on the Ox axis has the form:

$$v_{aa} \cos(\xi) = v_e \cos(\psi_0) + v_{ra.x}$$

from

$$v_{ra.x} = v_{aa} \cos(\xi) - v_e \cos(\psi_0) \quad (8)$$

In accordance with (7) and (8), the aerodynamic drag force of the air $\bar{F}_a$ is determined, kN:

for a headwind on the Ox axis (Fig. 1, a)

$$F'_{ax} = 0.5 \times 10^{-3} c_a \rho_a A_T (v_e \cos(\psi_0) - v_{aa} \cos(\xi))^2 \quad (9)$$

for a tailwind on the Ox axis (Fig. 1, b)

$$F'_{ax} = 0.5 \times 10^{-3} c_a \rho_a A_T (v_{aa} \cos(\xi) - v_e \cos(\psi_0))^2 \quad (10)$$

for a headwind on the Oy axis (Fig. 1, a)

$$F'_{a.y} = 0.5 \times 10^{-3} c_a \rho_a A_b (v_{ra} \sin(\lambda_0))^2 \quad (11)$$

for a tailwind on the Oy axis (Fig. 1, b)

$$F'_{a.y} = 0.5 \times 10^{-3} c_a \rho_a A_b (v_{ra} \sin(\lambda))^2 \quad (12)$$

In the last formulas: A_e – the area of the end surface of the car with cargo, m²: $A_e = 2H \times 2W$ (where $2W$ and $2H$ is the width and height of the windward surfaces of the car with cargo, m); A_b – the area of the side surface of the car with cargo: $A_b = 2L \times 2H$ (where is the length of the side windward surfaces of the car with cargo, m), m².

Let's imagine a physical model of rolling a car down a hill, as shown in Fig. 2, a, b, B.

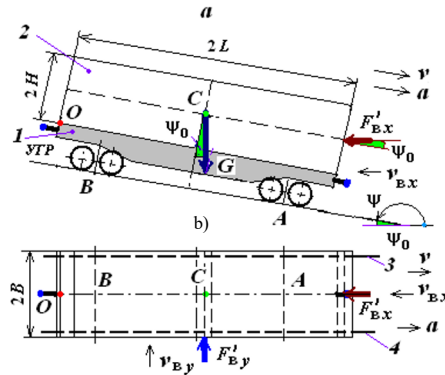


Fig. 2, a. Physical model of rolling the car down the hill. Here and further are indicated: 1 – wagon; 2 – cargo; 3 and 4 – external and internal rail threads; a – side view; b – top view

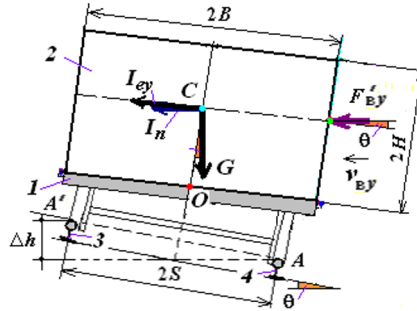


Fig. 2, b. Physical model of the car entering the side track when it rolls down the hill (view from the end)

Fig. 2, a and b indicate: $\bar{G}$ – the gravity of a car with a load (or without a load), kN; F'_{ax} and F'_{ay} – the projections of the aerodynamic drag force on the longitudinal and transverse axes of the car, kN; I_{ey} – the transverse portable force of inertia directed across the car (axis Oy) when the car is moving as on a straight line, so on a curved section of the track, and also appearing when the wheelsets hit the switches, kN (there is a possibility that when the wheelsets hit the switches, the transverse portable accelerations of the car reach up to $0.46g$ [6, 7]); I_n is the normal component of the inertia force in absolute motion directed horizontally across the car (axis Oy), kN; $2L$, $2W$ and $2H$ are, respectively, the length, width and height of the load, m; ψ (or ψ_0) is the slope of the slide profile relative to the horizontal, rad. In Fig. 2, b are indicated: I_n is the normal component of the inertia force, kN [6, 7]; Δh is the elevation of the outer rail thread 3 relative to the inner 4, m; θ is the angle characterizing the elevation of the outer rail thread 3, rad.; $2S$ is the distance between the points of contact of the wheels with the rails, i.e. the distance between the axes of the rails, m (usually 1.58 m).

The normal component $\bar{I}_n$ of the inertia force in absolute motion does not arise and does not appear, but only takes into account the acceleration of the absolute motion of the body along the curve, i.e. no force $\bar{I}_n$ is actually applied to the body [6, 7]. For example, when a train is moving along a curved section of the track to a carriage with a rigidly fixed load (crew), in fact, no force $\bar{I}_n$ is applied.

In accordance with this, we emphasize that the normal inertia force $\bar{I}_n$ on the physical and mathematical model of a car with a load and, in particular, a wheelset (see Fig. 2, c) is given only in order to take into account the acceleration of the movement of rolling stock along a curved section of the track, although there is simply no such force in absolute motion [6, 7].

6 Solution of the second problem

To build a computational model of the movement of the car from the hill, the principle of release from bonds is used [3]. First, they choose an object in the form of a car rolling down a hill. We will keep in mind that for the wheelset of the bogies of the car A' (or B'), and A (or B), the rail threads 3 and 4 are the main links that keep it from moving in the transverse direction, i.e. along the sub-rail base (sleepers). Otherwise, the main purpose of the rail threads 3 and 4, as external links, is the direction of the wheels of the car bogies when moving on straight and curved sections of the track.

In this regard, the wheelset of the car bogies is first released from the rail threads 3 and 4, replacing their influence with the reactions of the bonds $\bar{R}_A$ and $\bar{R}_B$; $\bar{R}_{A'H}$ and $\bar{R}_{B'H}$ (Fig. 3, a, b, c).

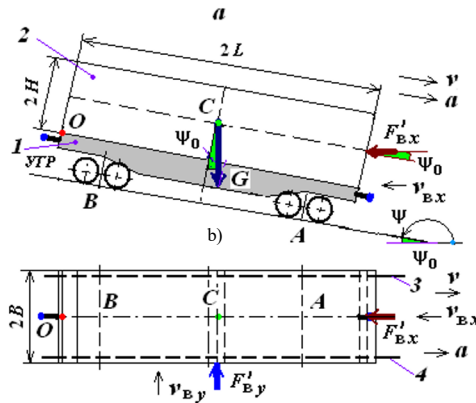


Fig. 3, a. Calculation model of rolling the car down the hill

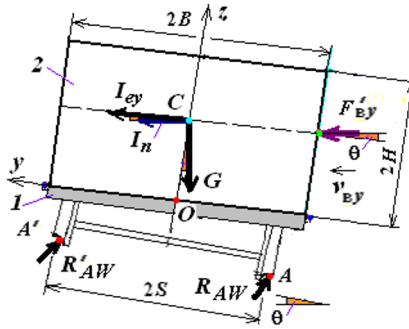


Fig. 3, b. Calculation model of the car entering the side track when it rolls down the hill (view from the end)

Here it is assumed that the indicated reactions of the connections of the wheel pairs of the front and rear bogies of the car A' (or B'), and A (or B) applied to the wheels, i.e. $\bar{R}_A \in \{\bar{R}_{A1}, \bar{R}_{A2}\}$, $\bar{R}_{A'} \in \{\bar{R}_{A'1}, \bar{R}_{A'2}\}$ and $\bar{R}_B \in \{\bar{R}_{B1}, \bar{R}_{B2}\}$, $\bar{R}_{B'} \in \{\bar{R}_{B'1}, \bar{R}_{B'2}\}$; $\bar{R}_{A'1H} \in \{\bar{R}_{A'1H}, \bar{R}_{A'2H}\}$, $\bar{R}_{B'1H} \in \{\bar{R}_{B'1H}, \bar{R}_{B'2H}\}$, and also $\bar{R}_{A1W} \in \{\bar{R}_{A1W}, \bar{R}_{A2W}\}$, $\bar{R}_{A'1W} \in \{\bar{R}_{A'1W}, \bar{R}_{A'2W}\}$, $\bar{R}_{B1W} \in \{\bar{R}_{B1W}, \bar{R}_{B2W}\}$, $\bar{R}_{B'1W} \in \{\bar{R}_{B'1W}, \bar{R}_{B'2W}\}$ (see fig. 3, b). Note that in Fig. 3, the reaction of the coupling and the rear trolley of car B to the frontal plane W is not shown.

It should be particularly noted that the appearance of a bond reaction $\bar{R}_A$ and $\bar{R}_B$ in the plane of rolling the car down the hill is associated with the impact on the car with the load of vertical components of gravity $\bar{G}_z$, aerodynamic drag force $\bar{F}_{az}$ (both from F'_{ax} and F'_{ay}) and

the normal force of inertia $\bar{I}_{nz}$ in absolute motion. The appearance of the bond reaction $\bar{R}_{A'H}$ and $\bar{R}_{B'H}$ in the horizontal plane lying along the level of the heads of the rail threads 3 and 4 (see Fig. 2.2) and the outer wheel sets of bogies applied to the wheel ridges is associated with the impact on the side surface of the carriage with the load of the transverse component of the aerodynamic drag force F'_{ay} .

It is believed that the active forces in the form of $\bar{G}$, F'_{ax} , F'_{ay} and $\bar{I}_n$ and have already been applied to the computational model. In the computational model, the active forces are directed away from the object. All reactive forces $\bar{G}$ and $\bar{I}_n$ (coupling reactions) $\bar{R}_A$, $\bar{R}_B$, $\bar{R}_{A'H}$ and $\bar{R}_{B'H}$, $\bar{R}_{AW}$ (and $\bar{R}_{A'W}$) and $\bar{R}_{BW}$ (and $\bar{R}_{B'W}$), including F'_{ax} and F'_{ay} , are directed towards the object. The axes of the Oxyz coordinates are shown as shown in Fig. 3, a, b, v.

Further, given that the movement of the wheelset relative to the rail threads is hindered by the friction forces between their contacting surfaces, the reactions of the bonds $\bar{R}_A$ and $\bar{R}_B$, $\bar{R}_{A'H}$ and $\bar{R}_{B'H}$, $\bar{R}_{AW}$ (and $\bar{R}_{A'W}$) and $\bar{R}_{BW}$ (and $\bar{R}_{B'W}$) are directed opposite to the movement of the wheels at the friction angle of the ϕ_{fr} to the normal $n - n$ (not shown in the figures). This is done because, although the application points $\bar{R}_A$ and $\bar{R}_B$, $\bar{R}_{A'H}$ and $\bar{R}_{B'H}$, $\bar{R}_{AW}$ (and $\bar{R}_{A'W}$) and $\bar{R}_{BW}$ (and $\bar{R}_{B'W}$), as vector quantities, are known, but their directions and magnitude are unknown.

It is known from the theory of friction [3] that a cylindrical roller (wheels of a wagon wheelset) contacting on a horizontal plane (the surface of the rail), in the case of application to the center of the roller of the sum of all active forces $F = G_z - F'_{ax} \cos \psi_0$ (where $G_z = G \sin \psi_0$), experience a reaction of bonds (rail threads) in the form of a coupling friction force (in magnitude greater than friction sliding) and rolling friction pairs, depending on which section of the slide profile the car is moving.

In addition, at the point of contact of the roller (the wheels of the wagon wheel pair) with the plane (the surface of the rail), a normal coupling reaction $\bar{N}$ ($\bar{N} \in \{\bar{N}_A, \bar{N}_B\}$) of this plane, opposite to the weight of the roller, and the friction force of the coupling (sliding) $\bar{F}_{fr}$ ($\bar{F}_{fr} \in \{\bar{F}_{frA}, \bar{F}_{frB}\}$), preventing the roller from sliding along the rail threads and, only in the case of equilibrium, equal in modulus to the force $\bar{F}$, but directed in the opposite direction.

It is assumed that in sections of the slide where there are no random (or so-called "episodic" resistances) wheels with a radius of r_{wh} (for a freight car of 0.475 m) roll without slipping with a rolling friction coefficient of the wheel along the rail f_{wh} , the inner radius of the inner ring of the rolling bearing rotates in with a rolling friction coefficient f_{wh0} .

External (both active and reactive) forces act on the mechanical system: projections of gravity on the coordinate axes $G_x = G \sin \psi_0$ and $G_z = G \cos \psi_0$, projections of the aerodynamic drag force $F'_{ax} \cos \psi_0$, $F'_{ax} \sin \psi_0$ and $F'_{ay} \cos \theta$, $F'_{ay} \sin \theta$ on the coordinate axis; the normal reaction of N rail threads, the coupling force of the wheel $F_c = F_{fr}$ with a rail, the rolling friction moment of the wheel on the rail $M_{fr} = f_{wh}N$ ($M_{frA} \in \{M_{frA1}, M_{frA2}, M_{frA'1}, M_{frA'2}\}$; $M_{frB} \in \{M_{frB1}, M_{frB2}, M_{frB'1}, M_{frB'2}\}$) where f_{wh} is the coefficient of rolling friction of the wheel on the rail (usually take $0.005 \cdot 10^{-3}$), m [3].

Rolling friction is the resistance that occurs when one body rolls over another. In reality, the reference plane (rail threads) is not absolutely solid and under the influence of the pressure of the roller (vertical load) is always, albeit slightly, deformed, because due to the curvature of the roller, the contact area is extremely small and contact stresses (specific pressure) have very high values. As a result of displacement and unevenness of the pressure plot $\bar{N}$, the normal reaction of the support surface, as a resultant of reactive pressure (contact stresses), is shifted by a certain amount, for example, by the value of the rolling friction coefficient f_{wh} , mm, since this coefficient is equivalent to the shoulder of the rolling friction pair (wheel on rail $f_{wh} = 0.005$, steel hardened on steel $f_{wh} = 0.001$), in the direction of the shear force $\bar{G}_z$ (see Figure 2.3, a).

The resulting pair of forces $(\vec{G}_z, \vec{N})$, called the rolling friction pair, is opposite in the direction of rotation to the pair $(\vec{F}, \vec{F}_{fr})$ and for the case in question of rolling the car down the hill cannot balance it. It is for this reason that the rolling bearings in the axle box and the wheels of the wheelset of the car experience a rolling friction moment.

In accordance with this, the model shown in Fig. 4 is taken as a simplified calculation model of rolling a car down a hill, taking into account rolling friction (pure rolling).

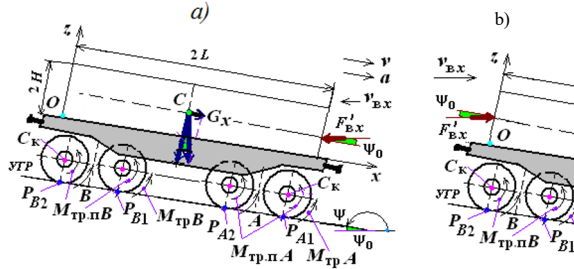


Fig. 4. Simplified calculation model of rolling the car down the hill: a) – with a headwind; b) – with a tailwind

Thus, the calculated models of the car are obtained when rolling down the hill.

7 Solution of the third problem

For the wheels of a wheelset of a car rolling down a hill, due to the fact that the condition is always met $\vec{F} \geq \vec{F}_{fr}$, i.e. $F > (f_{wh}/r_{wh})F_z$ (where F_z is the sum of the projection of all active forces on the vertical, kN; r_{wh} is the radius of the wheel). Here, the ratio f_{wh}/r_{wh} for most materials is significantly less than the sliding friction coefficient f , which is 0.25 between the contacting surfaces of the wheels of freight cars and rail threads [9].

Hence it is clear that rolling friction, if necessary, can be replaced by conditional sliding friction in pure rolling

$$F_{fr.wh} = \frac{n_{wh} f_{wh}}{r_{wh}} F_z \quad (13)$$

where n_{wh} – number of wheels in trolleys, pcs. ($n_{wh} = 8$); f_{wh} – rolling friction coefficient, m, since this coefficient is equivalent to the shoulder of the rolling friction pair (wheel on rail $f_{wh} = 5 \times 10^{-6}$, steel hardened by steel $f_{wh} = 1 \times 10^{-6}$), r_{wh} – wheel radius equal to 0.475 m for a freight car;

F_z – the sum of the projection of all active forces on the vertical axis, falling on each axle box node, kN:

$$F_z = G \cos \psi_0 + (I_b + F'_{ax}) \sin \psi_0 + F'_{ay} \sin \theta \quad (13, a)$$

or

$$F_{fr.wh} = \frac{n_{wh} f_{wh}}{r_{wh}} (G \cos \psi_0 + (I_b + F'_{ax}) \sin \psi_0 + F'_{ay} \sin \theta) \quad (13, b)$$

The mechanical system is also affected by internal forces in the form of rolling friction moments of the MfrbA ($M_{frbA} \in \{M_{frbA1}, M_{frbA2}, M_{frbA'1}, M_{frbA'2}\}$) and MfrbB ($M_{frbB} \in \{M_{frbB1}, M_{frbB2}, M_{frbB'1}, M_{frbB'2}\}$) in the bearings of the axle boxes of the front A and rear B bogies of the car, and $M_{frb} = M_{frbA} + M_{frbB}$.

At the points of contact with the rolling bodies of the inner diameter of the inner ring of the bearing, internal forces N_{rb} appear – the normal reaction of the bearing and at the same

point, the same modulus, but oppositely directed reaction N_o acts on the rolling bodies from the inner ring of the bearing [7]. Here, as before,

$$N = \sum_{i=1}^2 (N_{Ai} + N_{A'i} + N_{Bi} + N_{B'i})$$

(i – the number of wheels on one axle of the cart);

$$\bar{F}_{fr} = \bar{F}_{fr.A} + \bar{F}_{fr.A'} + \bar{F}_{fr.B} + \bar{F}_{fr.B'} + \bar{F}_{fr.AW} + \bar{F}_{fr.A'W} + \bar{F}_{fr.BW} + \bar{F}_{fr.B'W}$$

The rolling friction moment in the bearings of the axle boxes of the front and rear bogies of the car

$$M_{frb} = n_b f_{wh0} N_b \quad (14)$$

where $n_b = 8$ – number of axle box assembly in trolleys, pcs.; f_{wh0} – the coefficient of friction of rolling bodies along the bearing rings (usually taken $0,001 \cdot 10^{-3}$), m; N_b – the normal reaction occurring on one rolling bearing, or the force acting on the most loaded rolling body and determined by the formula, kN [11]:

$$N_b = \frac{k F_z}{n_b n_{re}} \quad (14, a)$$

n_{re} – the total number of rolling elements receiving the load in each bearing, pcs.; k – a constant coefficient adopted depending on the row and type of rolling bearings (for single-row bearings $k = 4$, for roller bearings with $n_b = 10 \dots 20$, the average value is $k \approx 4$. Taking into account the influence of the gap in rolling bearings, $k = 4.6$ is taken for calculation).

Similarly, (13) rolling friction can be replaced by conditional sliding friction with pure rolling of rolling bodies in bearings

$$F_{r.fr} = \frac{n_b f_{r0}}{r_b} N_b \quad (14, b)$$

where n_b is the number of bearings in the axle boxes of trolleys, pcs. ($n_b = 16$); r_b is the outer radius of the inner ring of the rolling bearing, m,

Let's rewrite the last formula taking into account (13, a) and (14, a):

$$F_{r.fr0} = \frac{n_b f_{wh0}}{r_b} \frac{k}{n_b n_{re}} (G \cos \psi_0 + (I_b + F'_{ax}) \sin \psi_0 + F'_{ay} \sin \theta) \quad (15)$$

By combining (13) and (15), the rolling friction of the wheels can be replaced by conditional sliding friction with pure rolling of the wheels and rolling bodies in the bearings of axle boxes [Komarov, Yashin, 2004]:

$$F_{fr}^r = F_{r.fr} + F_{r.fr0}$$

or

$$F_{fr}^r = \left(\frac{n_{wh} f_{wh}}{r_{wh}} + \frac{n_{nb} f_{wh0}}{r_b} \frac{k}{n_b n_{re}} \right) (G \cos \psi_0 + (I_b + F'_{ax}) \sin \psi_0 + F'_{ay} \sin \theta)$$

or

$$F_{fr}^r = f_0 (G \cos \psi_0 + (I_b + F'_{ax}) \sin \psi_0 + F'_{ay} \sin \theta) \quad (16)$$

where f_0 is some conditional (or reduced) coefficient of sliding friction:

$$f_0 = \frac{n_{wh} f_{wh}}{r_{wh}} + \frac{n_b f_{wh0}}{r_b} \frac{k}{n_b n_{re}} \quad (16, a)$$

In accordance with this, all these bond reactions are decomposed into normal $\bar{N}_A$, $\bar{N}_B$, $\bar{N}_{A'H}$, $\bar{N}_{B'H}$, $\bar{N}_{A'W}$, $\bar{N}_{B'W}$, and tangent $\bar{F}_{\tau A}$, $\bar{F}_{\tau B}$, $\bar{F}_{\tau A'H}$, $\bar{F}_{\tau B'H}$, $\bar{F}_{\tau A'W}$, $\bar{F}_{\tau B'W}$ components as shown in Fig. 5, a, b, c.

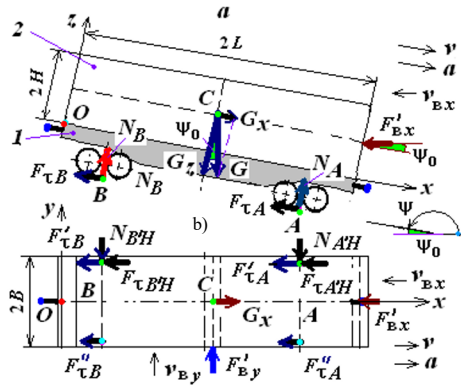


Fig. 5, a, b. A kind of calculation model of rolling a car down a hill

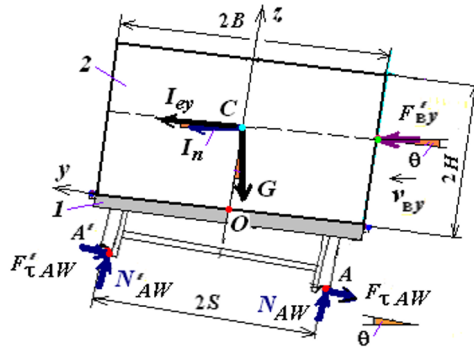


Fig. 5, b. A variety of the calculated model of the entry of the car on the side path when it rolls down the hill (view from the end)

In Fig. 5, a, b, c take into account that $\bar{N}_A \in \{\bar{N}_{A1}, \bar{N}_{A2}\}$, $\bar{N}_{A'} \in \{\bar{N}_{A'1}, \bar{N}_{A'2}\}$, $\bar{N}_B \in \{\bar{N}_{B1}, \bar{N}_{B2}\}$, $\bar{N}_{B'} \in \{\bar{N}_{B'1}, \bar{N}_{B'2}\}$; $\bar{F}_{\tau A} \in \{F'_{\tau A}, F''_{\tau A}\}$, $\bar{F}_{\tau A'} \in \{F'_{\tau A'}, F''_{\tau A'}\}$, $\bar{F}_{\tau B} \in \{F'_{\tau B}, F''_{\tau B}\}$, $\bar{F}_{\tau B'} \in \{F'_{\tau B'}, F''_{\tau B'}\}$

It is also taken into account that the tangential components $\bar{F}_{\tau A}$, $\bar{F}_{\tau A'}$, $\bar{F}_{\tau B}$, $\bar{F}_{\tau B'}$, $\bar{F}_{\tau A'W}$, $\bar{F}_{\tau B'W}$ represent the friction forces of the coupling between the contacting surfaces of the wheels and the rail threads, i.e. $\bar{F}_{\tau A} = \bar{F}_{\tau PA}$, $\bar{F}_{\tau A'} = \bar{F}_{\tau PA'}$, $\bar{F}_{\tau B} = \bar{F}_{\tau PB}$, $\bar{F}_{\tau B'} = \bar{F}_{\tau PB'}$, $\bar{F}_{\tau A'W} = \bar{F}_{\tau A'W}$, $\bar{F}_{\tau B'W} = \bar{F}_{\tau B'W}$. In this case $\bar{F}_{\tau}$, the tangential component directed along the surface of the rail threads is called the friction force $\bar{F}_{\tau} = \bar{F}_{\tau}$.

Generalizing the concepts of friction forces, it can be especially noted that in addition to other friction forces, there are random or so-called "episodic" forces, which include the following reactive forces:

- the friction forces of sliding wheels on the rail (without rolling) $F_{\text{fr}}^{\text{tp}}$ when passing the car on the length of the slide or park brake position (BP):

$$F_{\text{fr}}^{\text{bp}} = F_{\tau} + f_{\text{bp}} F_{\text{re}}$$

where F_t is the tangent component of the coupling reaction (rail threads), which is equal according to Coulomb's law $F_t = f_{\text{тп}}N$, taking into account that f_{bp} is the coefficient of friction of sliding wheels on rail threads ("metal on metal" – $f_{\text{bp}} = 0.15...0.25$), N is the normal component of the coupling reaction;

F_{re} – the force of pressing the brake pads of the car's retarder beams [20].

Let's rewrite the last expression in the form:

$$F_{\text{fr}}^{\text{bp}} = f_{\text{bp}}(N + F_{\text{re}})$$

Here, the normal component N of the coupling reaction modulo is equal to the sum of the projection of all active forces on the vertical axis falling on each axle box node, F_z (see (13, a)). In accordance with this, we will give the last relation the form:

$$F_{\text{fr}}^{\text{bp}} = f_{\text{bp}}[G \cos \psi_0 + (I_b + F'_{ax}) \sin \psi_0 + F'_{ay} \sin \theta + F_{\text{re}}] \quad (17)$$

Considering that each wheel of the trolley of the car enters the section of the braking position of the slide sequentially, i.e. with a delay of some time τ , (17) we rewrite as:

$$F_{\text{fr}}^{\text{bp}} = f_{\text{bp}}[G \cos \psi_0 + (I_b + F'_{ax}) \sin \psi_0 + F'_{ay} \sin \theta + F_{\text{re}}] \times \\ \times (\sigma_0(t) + \sigma_0(t - \tau_1) + \sigma_0(t - \tau_2) + \sigma_0(t - \tau_3)), \quad (17, a)$$

where $\sigma_0(t)$, $\sigma_0(t - \tau_1)$, $\sigma_0(t - \tau_2)$ and $\sigma_0(t - \tau_3)$ are dimensionless lagging Heaviside functions [Pchelin, 1973]; allowing to represent time t_i by one analytical expression, suitable for any value of t in the interval $0 \leq t \leq \tau_i$, and $\sigma_0(t - \tau_i) = 0$ at $t < \tau_i$; t is the current time, s ; τ_1 , τ_2 and τ_3 – the delay time of the appearance of the friction force during the passage of the braking position of the second wheel of the front trolley (A'), the first (B'_1) and second (B'_2) wheels of the rear trolley compared with the first wheel of the front trolley (see Fig. 2.5, a), c:

$$\tau_1 = \frac{2l_t}{v}, \quad \tau_2 = \frac{2l_c}{v}, \quad \tau_3 = \frac{2l_c + 2l_t}{v}$$

taking into account the fact that l_t and l_c are half of the base of the trolley and the car, m ; v is the speed of the car, mps (unfortunately, the value is being sought);

- the sliding friction forces of the wheel ridges along the side faces of the thrust rail $F_{\text{fr}}^{\text{ay}}$, which appear when the wheel is rolling along the sliding rail while moving in curves taken into account by the normal component I_b of the inertia force in absolute motion [6, 7, 19], from the effect of the transverse portable inertia force I_{ey} and the projection of the aerodynamic drag force on the side surface of the wagon with the load F'_{ay} on the transverse axis. Here it is assumed that the wheels of wheel sets of trolleys located on the side of the thrust rail are completely pressed against the thrust rail I_{ey} and $F_{\text{fr}}^{\text{ay}}$. These forces, according to the rule of reduction of forces, known from the course of theoretical mechanics [Turanov, Bondarenko, 2006], are brought to the wheels of the wheel pairs of the front and rear bogies of the car A' (or B'), i. e. $\bar{R}_{A'} \in \{\bar{R}_{A'1}, \bar{R}_{A'2}\}$ and $\bar{R}_{B'} \in \{\bar{R}_{B'1}, \bar{R}_{B'2}\}$ (Fig. 7).

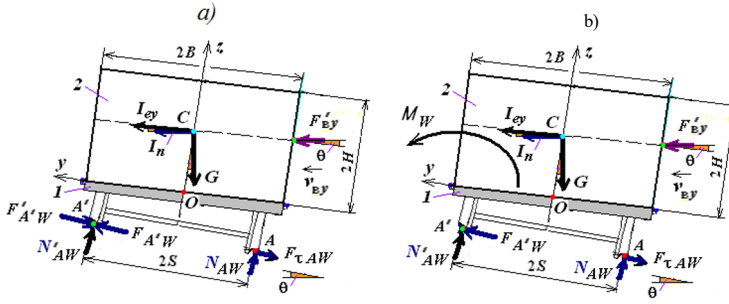


Fig. 7. Bringing the force F_{fr}^{ay} to a point A' (and B')

Figure 7 shows: $F_{A'W} = I_{ey} + (I_b + F_{ay}') \cos(\theta)$ – the sum of the forces I_{ey} and the projection of the forces I_b and F_{ay}' on the transverse y axis (index W means that the forces act in the frontal plane); $F_{A'W}' = -F_{A'W}$ – the transverse forces brought to the points A' and B'_1 (the point B'_1 in Figure 2.5 is not shown); M_W – the moment from the pair of forces $F_{A'W}'$ (which is applied at the points A' and B'_1) and (which is attached to the center of mass of the car and cargo), and $F_{By}' \cos(\theta)$ (which is attached to the sides of the car and/or cargo). The moment from a pair of M_W forces will not be taken into account in the future due to the lack of need to solve the problem under consideration. Recall that M_W is necessary to solve the problem of overturning a wagon with cargo [Turanov, 2007].

Force F_{ip}^{by} is found according to Coulomb's law:

$$F_{fr}^{ay} = f_{whr} [I_{ey} + (I_b + F_{ay}') \cos(\theta)] \quad (18)$$

where f_{whr} is the reduced sliding friction coefficient for freight cars (i.e. some conditional value) [8]:

$$f_{whr} = \frac{F_z}{F_z + G_{wh}} \frac{r_b}{r_{wh}} f \quad (18, a)$$

Here F_z is the projection of all active forces on the vertical axis that falls on each axle box unit, kN (see (2.13. a)); G_{wh} is the gravity of the wheelset of the car, equal to 19 kN; f is the coefficient of sliding friction between the contacting surfaces of wheels and rail threads (0.25 is accepted for freight cars); r_b – internal radius of rolling bearings in axle boxes, m; r_{wh} – wheel radius equal to 0.475 m for a freight car;

- the friction forces of sliding wheels (with rolling) along the rail F_{fr}^{ac} , which appear in winter conditions when the density of the air environment changes, accompanied by snow or frost within the arrow zone of the bundles and on the sorting tracks. Here it must be borne in mind that according to Coulomb

$$F_{fr}^{ac} = faeN \quad (19)$$

where fae is the coefficient of resistance of the rail to the movement of the wheels (taken according to field experiments, or $fae = f_{whr}$ (see (18, a))), N is, as before, the normal component of the bond reaction (N_A and N_B);

- friction forces $\bar{F}_{fr.s}$ when sliding wheels without rolling (taking into account the short duration of the impact), appearing from the impact force $\bar{F}_s$ (equivalent to the transverse portable inertia force I_{ey}) directed across the car (axis Oy) and appearing when the wheel pairs hit the switches (on the wits, crosses and counter rails) [3, 6, 7, 18]. Here it should be borne in mind that the force $\bar{F}_{fr.s}$ is found according to Coulomb's law, taking into account the sequence of contact of the wheel ridges with the arrow translation:

$$F_{fr.s} = f_s I_{ey} (\sigma_0(t) + \sigma_0(t - \tau_1) + \sigma_0(t - \tau_2) + \sigma_0(t - \tau_3)) \quad (20)$$

where f_s is the coefficient of sliding friction of the ridges of the impacted wheels on the rail threads (for wagon wheels, $f_s = 0.25$ can be taken);

$\sigma_0(t)$, $\sigma_0(t - \tau_1)$, $\sigma_0(t - \tau_2)$ and $\sigma_0(t - \tau_3)$ are dimensionless lagging functions of Heaviside [Pchelin]; allowing to represent time t by one analytical expression, suitable for any value of t in the interval $0 \leq t \leq \tau_i$, and $\sigma_0(t - \tau_i) = 0$ at $\tau_i < t$; t is the current time, s;

τ_1 , τ_2 and τ_3 are the delay time of the appearance of the friction force during the passage of the switch of the second wheel of the front trolley (A'), the first (B'_1) and the second (B'_2) wheels of the rear trolley compared with the first wheel of the front trolley (see Fig. 7), s:

$$\tau_1 = \frac{2l_t}{v}; \quad \tau_2 = \frac{2l_c}{v}; \quad \tau_3 = \frac{2l_c + 2l_t}{v}$$

taking into account the fact that l_t and l_c are half of the base of the trolley and the car, m; v is the speed of the car, m/s (unfortunately, the value is being sought);

- the sliding friction forces of the wheels of the first wheelset of the front trolley of the car (pure sliding) in combination with the rolling of the remaining six wheels of the trolleys when the car is moving on the undercarriage park when using a shoe.

The force $\bar{F}_{fr.b}$ is found by the formula:

$$\bar{F}_{fr.b} = F_{fr.b}^{wh} + F_{fr}^{swhb} \quad (21)$$

where $F_{fr.b}^{wh}$ is the conditional sliding friction with pure rolling of wheels and rolling bodies in the bearings of axle boxes (see (16) and (16, a)):

$$F_{fr.b}^{wh} = f'_0 (G \cos \psi_0 + (I_b + F'_{ax}) \sin \psi_0 + F'_{ay} \sin \theta) \quad (22)$$

taking into account the fact that f'_0 – some conditional (or reduced) coefficient of sliding friction:

$$f'_0 = \frac{n_{whb} f_{wh}}{r_{wh}} + \frac{n_{bb} f_{whb}}{r_b} \frac{k}{n_b n_{re}} \quad (22, a)$$

Here n_{wh} is the number of wheels in the trolleys, pcs. ($n_{wh} = 6$); n_{bb} is the number of bearings in the axle boxes of the trolleys, pcs. ($n_{bb} = 12$);

F_{fr}^{sb} – the sliding friction force of the wheels of the first wheelset of the front trolley of the car (clean sliding):

$$F_{fr}^{sb} = f_{sb} N_1 \quad (23)$$

where f_{sb} is the coefficient of friction of sliding wheels on the rail ("metal on metal" – $f_{sb} = f_{bp} = 0.15 \dots 0.25$); $N_1 = N / n_{whb}$ (N is, as before, the normal component of the bond reaction (NA and NB)).

Substituting (22) and (23) into (21), we get:

$$F_{fr.b} = f'_0 (G \cos \psi_0 + (I_b + F'_{ax}) \sin \psi_0 + F'_{ay} \sin \theta) + f_{sb} N_1,$$

or, given that $N_1 = N / n_{whb}$,

$$F_{fr.b} = \left(f'_0 + \frac{f_{sb}}{n_{whb}} \right) (G \cos \psi_0 + (I_b + F'_{ax}) \sin \psi_0 + F'_{ay} \sin \theta),$$

Given that the sub-assembly park in the foundations is designed on a straight section, where $I_b = 0$, since the curvature of the trajectory is $\rho = 0$, then the last equality is represented as:

$$F_{fr.b} = f_{fr.b} (G \cos \psi_0 + F'_{ax} \sin \psi_0 + F'_{ay} \sin \theta) \quad (24)$$

or, given that for small angles $\cos \psi_0 = 1$ $\sin \psi_0 = 0$ $\sin \theta = 0$,

$$F_{fr.b} = f_{fr.b} G \quad (24, a)$$

where $f_{fr.b}$ is the coefficient of rolling friction of wheels with slipping shoe under the wheels:

$$f_{fr.b} = f'_0 + \frac{f_{sb}}{n_{whb}} \quad (24, b)$$

It is well known that friction forces, as resistance forces, are always directed in the direction opposite to rolling the car down the hill (see Fig. 5, a, b, c).

The friction force is a reactive force. In the case of rolling the car (with and/or without cargo) down the hill, the friction force is the holding force. Therefore, it is "harmful" for the compilers of the train, preventing the accelerated formation of the composition.

Since the normal and tangential (friction force) components of the bond reaction, as parallel forces, are directed in one direction, then we can write:

$$N = \sum_{i=1}^2 (N_{Ai} + N_{A'i} + N_{Bi} + N_{B'i}) \quad (25)$$

where i is the number of wheels on one axle of the cart, which is equivalent to (13, a):

$$\begin{aligned} F_z &= G \cos \psi_0 + (I_b + F'_{ax}) \sin \psi_0 + F'_{ay} \sin \theta; \\ \bar{F}_t &= \bar{F}_{fr} = \bar{F}_{fr.A} + \bar{F}_{fr.A'} + \bar{F}_{fr.B} + \bar{F}_{fr.B'} + \bar{F}_{fr.AW} + \bar{F}_{fr.A'W} + \\ &+ \bar{F}_{fr.BW} + \bar{F}_{fr.B'W} + \bar{F}_{fr}^{bp} + \bar{F}_{fr}^{ac} + \bar{F}_{fr.s}, \end{aligned}$$

or

$$\bar{F}_t = \bar{F}_{fr} = \bar{F}_{fr}^{wh} + \bar{F}_{fr}^{bp} + \bar{F}_{fr}^{ac} + \bar{F}_{fr.s}, \quad (26)$$

where

$$\bar{F}_{fr}^{wh} = \bar{F}_{fr.A} + \bar{F}_{fr.A'} + \bar{F}_{fr.B} + \bar{F}_{fr.B'} + \bar{F}_{fr.AW} + \bar{F}_{fr.A'W} + \bar{F}_{fr.BW} + \bar{F}_{fr.B'W},$$

Its equal to (16):

$$F_{fr}^{wh} = f_0 (G \cos \psi_0 + (I_b + F'_{ax}) \sin \psi_0 + F'_{ay} \sin \theta)$$

At the same time, we will keep in mind that the resulting friction force F_{fr} does not have any effect on the movement of the center of mass of material systems (wagon with cargo), since it does not depend on how this force is distributed between the wheel pairs of both bogies [16, 17].

Thus, the formulas of the forces acting on the car when rolling down the hill are obtained.

8 Conclusion

Based on the classical provisions of theoretical and applied mechanics, calculation models are constructed and all active and reactive forces acting and appearing when a single car rolls down a hill are found, which will subsequently allow mathematically modeling the speed of movement and the traveled path of the car depending on the time of movement.

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