Towards the substantiation of methods for optimizing the transportation work of locomotives under railway operating conditions

O. Ablyalimov

Tashkent state transport university (Tashkent, Uzbekistan)

Abstract. An analysis of existing methods for optimizing the transportation work of locomotives in operation is presented, based on a refined formulation and criteria for the problem of optimal control of train movement by locomotives of different types of traction. The advantage of the dynamic step-by-step programming method proposed by the author, based on the principle of parametric optimization, is substantiated, which makes it possible to calculate optimal train driving modes in areas of locomotive circulation and locomotive crew work with savings in natural diesel fuel consumption of up to eight to ten percent per trip. The analytical dependence of the stepwise optimization of the process, which determines the sequence of solving the optimization problem, is recommended to be used in the development of optimized train driving modes maps on sections of the Uzbek railways of varying difficulty. Key words: Dynamic programming, freight train, diesel locomotive, calculus of variations, maximum principle, optimal control, optimization task, train driving mode.

1 Introduction

Currently, optimization, as a matter of course, seems to be a powerful tool in the future development of any complex engineering systems associated with the operation of all types of transport, including, especially, railway.

The constantly growing demand for high precision in manufacturing and increasing the operational reliability of structures, taking into account the search for promising optimal technological solutions, creates additional difficulties that can only be realized through the development of special optimization models.

For railway transport, one of these models are models of optimal control of train movements, which take into account all artificial and natural restrictions imposed on the rolling stock by operating conditions.

At the Department of Locomotives and Locomotive Facilities of TSTU (Tashkent State Transport University), research is very actively being carried out related to solving problems of choosing the optimal mode for controlling train traffic on sections of railways of varying degrees of difficulty (complexity), including of Uzbek ones.

* Corresponding author: o.ablyalimov@gmail.com

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2 Objects and methods of research

In this regard, for many years, issues related to solving problems of optimal control of train traffic with various types of locomotive traction have been an active topic of research by many foreign scientists [1-32] and concern a wide range of cargo and passenger transportation, not only mainline [1-18, 30-32], suburban [19] and high-speed [20, 21] railway transport, but also using the metro [22-25] and involving public urban rail transport [26-29].

As a result of the analysis of a considerable part of these studies, carried out by the author of the work [33], a classification of methods for calculating the energy-optimal trajectory of a train was justified and proposed, which is shown in fig. 1. And, in essence, he comes to the conclusion that all the optimization methods he analyzes can be grouped into two main categories - these are analytical and numerical methods for solving optimization problems. Basically, research [33] is devoted to energy-optimal calculation methods for locomotives of electric traction and they do not at all consider specific optimization issues for locomotives of diesel traction, the influence on optimal control of the characteristics of the track profile of railway sections and of the organization of freight and passenger types of movement in real operating conditions.

The purpose of the study is to analyze existing optimization methods for selecting optimal train movement control modes on railway sections with different types of track profiles in real and virtual operating conditions of railways, including Uzbek ones.

The object of study is the tasks of optimizing the transportation work of locomotives and mathematical methods of the theory of optimal control of motion processes in space.

The subject of the study is the criteria for setting and methods for solving the problem of optimal control of train movement using the example of locomotives of diesel tractions.

The starting point of the modern theory of optimal control is a system of equations of the state of an object that describe the behavior of a dynamic system in space.

In a continuous process, the equation of state of an object can be a system of ordinary differential equations of the first order, namely:
\[ \dot{x}^1 = f^1(x^1, x^2, ..., x^n, u^1, u^2, ..., u^n) \]
\[ \dot{x}^2 = f^2(x^1, x^2, ..., x^n, u^1, u^2, ..., u^n) \]
\[ \dot{x}^n = f^n(x^1, x^2, ..., x^n, u^1, u^2, ..., u^n) \]

where \( x \) – object state vector including phase coordinates \( x^1, x^2, ..., x^n \), that is \( x = (x^1, x^2, ..., x^n) \);
\( u \) – control vector including control parameters \( u^1, u^2, ..., u^n \), that is \( u = u^1, u^2, ..., u^n \);
\( f \) – vector is a function whose coordinates are the right-hand sides of the system of equations (1).

The mathematical formulation of the optimal control problem is as follows. The equations of state of the object are: boundary conditions imposed on object state variables; restrictions placed on state and control variables; quality indicator (optimality criterion) of the control object. It is necessary to determine such admissible control under which the optimality criterion (quality indicator) will be minimal or maximum.

Let us specifically describe the formulation of the optimization problem for the technological process of the transportation work of a locomotive, based on Chapter II of the monograph [38], and introduce a conditional concept about the signs of the optimization problem.

1st sign. The process of a moving train is described by a system of differential equations that characterize the feature of the mathematical description (MD), that is, the patterns of changes in state coordinates:

\[ \frac{dx}{ds} = f(x, u) \]

where \( x = V \) – the state vector of a moving train, including the phase coordinate of the speed \( V \) km/h; the remaining integral coordinates - time \( t \) min, energy consumption \( E \) kg and so on, which are determined uniquely;
\( u \) - control vector containing the \( i \) th positions of the driver controller:

\[ u \in \{1, 2, ..., m\} \]
\[ u \in \{0\} \]
\[ u \in \{-1\} \]

boundary conditions of the state of the process of a moving train (set by the phase coordinates of the path \( S \) km and speed \( V \) km/h) and restrictions imposed on the state and control variables characterizing the sign of boundary conditions (\( BC \)) and restrictions (\( R \)), that is

\[ (S_n, V_n) \]
\[ (S_k, V_k) \]

\[ V \leq V_{\text{max}} \]
\[ F_k \leq F_{kU} \]
\[ 0 \leq N \leq N_{\text{max}} \]

where \( F \) - locomotive traction force;
\( N \) - locomotive power.

3th sign. Optimality criterion (OC), characterizing the effect of the process (can be in the type min of reduced costs \( \mathcal{E} \) rub., max profit \( \mathcal{P} \) rub. or min energy consumption \( E \) kg):

\[ \mathcal{E} (x, u) \rightarrow \text{min} \]
\[ E (x, u) \rightarrow \text{min} \]

The problem in the general case is formulated as follows: for conditions characterized by equations (3) - (6), find such an optimal control \( u(S) \) for which the optimality criterion (7), (8) is the smallest (minimal).

4th sign. An additional sign of optimization is a sign that characterizes the goal of the process.

Comparison of problem statements based on the specified optimization criteria allows us to highlight the general properties of the characteristics and their features (table 1), which were not sufficiently taken into account by a number of researchers [4, 6, 16, 30-32 and others] and were noted in work [39].

When solving problems of optimal control of train movement, the basic methods of the mathematical theory of optimal processes are used, which include dynamic programming (DP) [34, 35], the maximum principle (MP) [36, 37] and methods of the calculus of variations (CA) [40, 41].

<table>
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<th>Signs of setting an optimization task</th>
<th>General properties and characteristics</th>
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<td>First</td>
<td>The required functions are that the trajectory of the process is continuous throughout the entire section. The independent variable is the path (( S )) or time (( t )).</td>
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<td>Second</td>
<td>It corresponds to the formulation of the problem in the general case.</td>
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<td>Third</td>
<td>Different in view: a) ( E \rightarrow \text{min} ) energy consumption. b) ( E \rightarrow \text{min} ) transportation costs.</td>
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<td>Fourth</td>
<td>The overall goal of the process is set.</td>
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Table 1. Statement of the problem of optimal control of train movement

References [4, 6, 16, 30-32] and others.
Methods of the calculus of variations are applicable to problems in which there are no restrictions on state and control variables, optimality criteria are presented in the form of functionals, and unknown functions serve as solutions. The method of calculus of variations allows in such cases to reduce the solution of the optimization problem to the integration of Euler differential equations, each of which is a second-order nonlinear differential equation with boundary conditions specified at both ends of the integration. This leads to a two-point boundary value problem, the analytical solution of which, with the exception of some very simple cases, is associated with great difficulties. Obtaining a numerical solution to a two-point boundary value problem is also quite difficult.

Dynamic programming is the most universal method for solving multi-stage optimization problems, for which the general optimality criterion is described by an additive function of the optimality criteria of individual stages. In principle, this method is an algorithm for selecting optimal control at all stages of the process.

The peculiarity of the method is that in order to find the optimal control (train movement mode), the section is divided into a number of successive segments, called variation steps. Optimal control at each step is selected based on R. Bellman’s principle of optimality [34], which is formulated as follows: the optimal strategy has the property that, whatever the initial state of the multi-stage process and the initial decision made, subsequent decisions must constitute an optimal strategy with respect to condition resulting from the original decision.

The main disadvantage of dynamic programming is the huge volume of the computational procedure and the excessive requirements for the machine's memory to store promising calculation options.

The maximum principle is used to solve problems of optimization of processes described by systems of differential equations, and restrictions can be imposed on the range of changes in variables. Finding the optimal solution comes down to the problem of integrating the system of differential equations of the process and the adjoint system for auxiliary variables under boundary conditions specified at both ends of the integration interval.

The maximum principle is valid for a class of problems in which the mathematical dependencies describing the process have continuous derivatives of the first and second order.

The task of optimal control of train movement was solved by a number of authors using dynamic programming of DP [34, 35], variational calculus of VC [40, 41], and the principle of maximum PM [36, 37].

The shortcomings and simplifications identified during the analysis of methods for solving optimization problems, admitted and accepted in the studies [4, 6, 16, 18-21, 30-32 and others], are presented in table 2.

Based on a comparison of formulations of optimization problems and analysis of methods for solving them, the following intermediate conclusions can be drawn:

- the problem of optimal control of train movement has a number of features of the process of a moving train, which were not taken into account by researchers and led to an insufficiently convenient and accurate formulation of the problem of optimizing the transportation work of locomotives and her formulations;
- when solving the problem, only various expressions of optimality conditions were given, and the solutions themselves were carried out in separate, small areas using well-known methods of optimal control theory;
- the calculation results could not have practical application;
- it is obvious that there is a need to clarify the formulation of the problem of optimal control of train movement and to develop a method for solving it.

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<th>Table 2. Research results and comparison of methods for solving an optimization problem in the transportation of locomotives</th>
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<td>Disadvantages and simplifications</td>
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Source: «Compiled by the author». 
\[ E[S_k] = \sum_{i=1}^{N} \sum_{j \in \Lambda} S_{jk} \int \varphi_j(s \{\bar{u}_j\}) ds \]

where \( j = a, b, c, \ldots \) are process stages with corresponding controls and continuously differentiable functions \( f_j \) and \( \varphi_j \).

\( \lambda \) is the number of optimization steps in a given area \( S_h \in Sk \), which is revealed during the calculation process.

\( \{u_j\} \)

\( \rightarrow \Pi_n \Lambda \)

\( \rightarrow \Pi_n \gamma \)

sequence \( \rightarrow \Pi \Lambda \) controls \( \{u_j\} \) at the corresponding \( i \)-th optimization step, while the choice of the sequence \( \rightarrow \Pi \gamma \) controls \( \{u_j\} \) is based on the parameters \( \Pi \Lambda j \) or \( \Pi \gamma \) (a similar notation for other parameters \( \Pi j \) and \( \Pi c \)).

3 Results and their discussion

controller of driver’s, since the optimal mode ensures accurate fulfillment of the travel time movement for the hauls and for the entire section, and the amount of consumed diesel fuel is within the calculated values, which indicates possible savings in natural diesel fuel.

So, depending on the selected movement mode of the train on individual trips, deviations in energy consumption from average values, under the same conditions of transportation work of diesel locomotives on railway sections, are approximately \( 10-15 \) percent, which indicates huge reserves in savings energy due to the using optimal modes.

Available works on the issue of driving trains with locomotives [2-19, 31-33] do not provide a sufficiently substantiated theoretical basis for choosing optimal control modes for diesel locomotives, which makes their practical use difficult. Some authors have made attempts to use methods of the mathematical theory of optimal control for this purpose. However, the use of the dynamic programming method based on R. Bellman's optimality principle [5, 33 and others], as well as of the maximum principle [7, 31 and others], did not give practically satisfactory results.

As a result of the research carried out by the author of this article in this area [38], a method of dynamic step-by-step programming (DSP) has been proposed, the use of which allows one to obtain satisfactory results at a relatively small amount of time.

To implement the above, first, the values of the "desirable" speeds of movement along two at the beginning of the stage, which, by analogy with the above, are the "desirable"...
Fig. 2. Actual and calculated modes of driving a train on the section Dj-C:

Actual mode according to the speed measure tape, - - - - -
By mode idle movement

Thus, calculations lead from the two indicated “desirable” conditionally optimal speeds of movement in the direction to two final, also “desirable” speeds of movement, which allows us to identify four total locally optimal trajectories (TLOT) on the stretch, of which two are left for further calculations relatively optimal trajectories (ROT) with the best performance one in the direction of the “desired” speed of movement \( V_{ni+1} \), and the other \( V_{ni+1}^0 \).

If the initial or final speeds on the stretch have been specified (for example, there is a train stop), then respectively, two total locally optimal trajectories (TLOT) are identified, which will also be relatively optimal trajectories (ROT) on this stretch. Throughout the entire section of the account, the total sums relative to the optimal trajectories of ROT in the direction of \( V_{ni+1} \) and \( V_{ni+1}^0 \) are compared and the best, optimal option for the entire section of the account, but according to the minimum value of the total gain.

At each stage, the identification of the locally optimal trajectory (LOT) and the corresponding mode of driving the train in operation is carried out according to the condition of realizing the highest efficiency of power transformations [38].

By changing the value of the smallest and largest positions of the controller of the working stroke driver (range of positions being sorted out), the coordinates of the transition points from the working stroke to the idle stroke are adjusted. The mode of driving the train at idle, as well as during braking, is adopted as single-position.

For a given train travel time along the stretch, which is the final value of the time coordinate, in the general case, one or another mode of driving the train \( n_k(S) \) can be adopted based on the appropriate method for selecting control mode of the rolling stock.

4 Conclusion

1. The above characterizes the possible practical application of the dynamic step-by-step programming method for solving the complex and important problem of optimal control of train movement on various, including elongated, traction shoulders of circulation of locomotive.

2. The use of the principle of parametric optimization and the method of dynamic step-by-step programming eliminates uncertainty in the choice of the number and length of calculation steps, the need to enumerate options for all possible states at the boundaries of these steps, sharply reduces the amount of computational work, and makes it possible to...
Calculate optimal train driving modes for real operating conditions on areas of work of the locomotive crew.

3. The calculated optimal modes of movement trains, according to a preliminary comparative analysis, can provide (for certain areas) savings in natural diesel fuel on average up to eight to ten percent.

4. To solve the problem of optimizing the transportation operation of locomotives by comparing the method of dynamic step-by-step programming proposed by the author of this article with classical methods of optimal control theory, based on the optimality principle of R. Bellman (Dynamic programming) and the maximum principle (of L. Pontryagin).

References


