On the stability of supersonic boundary layer in interaction with weak shock waves

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Abstract. Numerical simulation of the problem of interaction of the supersonic ($M_\infty=2$) boundary layer on the plate with weak shock waves was performed. The shock wave was generated using a wedge located above the plate. The problem was solved using the computational technology, combining the ANSYS Fluent gasdynamic package, which calculates the main flow, and the LOTRAN 3.0 software package, which calculates not only the stability of no-break flow, but also the laminar flow failure. The intensity of the incident shock wave was varied by varying the wedge inclination angle. The $N$-factor envelopes of the Tollmien-Schlichting wave instability for different wedge tilt angles, including those in the local breakaway zones, were obtained. It is demonstrated that the proposed approach using the ANSYS Fluent and LOTRAN 3.0 combination computational technology allows one to calculate the stability of flows with the shockwave, including the presence of local breakaway zones.

1 Introduction

It is known that the characteristics of an aircraft largely depend on the position of the laminar-turbulent transition region (LTT) and its length. A correct prediction of the LTT position is very important for the design problems of promising aircraft. When determining the position of the LTT in the boundary layer (BL), it is necessary to consider various specific features of the flow that degrade the characteristics of individual elements and the aircraft as a whole. One aspect of the problem of ensuring laminar flow in aircraft and turbomachine blades is the need to predict the effect of weak pressure gradients caused by shock waves (SW) on the LTT. Despite the small value of pressure gradient, such shockwaves significantly change the parameters in BL and can cause local detachments. Scheme of such flow (Fig. 1) was given in [1].

![Flow diagram in the problem of interaction of SW with BL on the plate](image)

Fig. 1. Flow diagram in the problem of interaction of SW with BL on the plate [1].

Because of its importance for practical applications, this phenomenon of interaction of SW with BL of bodies has been studied and investigated [1-6] both experimentally, for example, [1,5,7-9], and in numerical simulation [10]. Because of the presence of local tearing zones in such currents, traditional methods for predicting the transition position based on calculating the non-viscous flow around the body, solving the BL equations, and then analyzing its stability do not allow us to predict the transition in such currents. Methods based on the calculation of the RANS flow using transient turbulence models are not sufficiently verified for such flows [10]. In this case, an approach that combines the merits of RANS with a physically based transition prediction method based on the e$N$-method seems promising [11]. Since this approach makes it possible to calculate the perturbation growth coefficients both in the no-break and breakaway flows, it seems possible to perform a parametric study of the effect of SW on the transition in a sufficiently wide range of shockwave intensities.

The purpose of this work is to study the stability of supersonic ($M_\infty = 2$) BL on a plate during its interaction with weak shock waves. This problem is solved numerically using the method of integration of the LOTRAN 3.0 software package created in ITAM SB RAS and the ANSYS Fluent gas dynamic package [12]. The LOTRAN 3.0 software package is based on the physically justified N-factor method (e$N$-method) implemented for predicting LTT in viscous compressible flows using original specialized matrix algorithms [11]. The advantage of the LOTRAN 3.0 software module is the possibility to investigate the stability not only of a no-break flows but also of laminar break of BL.

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2 Problem definition

Two tasks are solved in the framework of this computational technology [12]: 1) calculation of the laminar flow on the model using the ANSYS Fluent software package (obtaining the characteristics of the main flow), and 2) analysis of the stability of the main flow profiles using the LOTRAN 3.0 software package. In present work, the BL on a 0.31m long flat plate with a sharp leading edge was considered. The computational domain (in the xy plane) for the problem of flow over the plate is shown in Fig.2, where the bottom surface (1) coincides with the plate, the top surface has a ledge (2), designed to drain the SW coming from the front edge of the plate, and a wedge (3) with a slope angle θ. The wedge generates incident SW, and by changing the angle of the wedge varied the intensity of the SW.

![Fig. 2. Schematic representation of the computational area.](image)

The origin of the coordinate system corresponds to the front edge of the plate. The height and location of the ledge (2) of the computational domain were chosen so that the head SW from the leading edge of the plate interacted with the boundary (2) to avoid SW reflection. Coordinate of wedge leading edge \( x = 0.09 \text{m}, y = 0.045 \text{m} \). The upper location of the wedge was chosen so that the reflected waves, formed by the interaction of the falling from the wedge SW with the surface of the plate, when reflected from the upper boundary of the calculated area returned to the surface of the plate closer to its end. The coordinate of the wedge end was equal to \( x = 0.21 \text{m} \) and was chosen so that the rarefaction wave fan interacted with the plate closer to the end of the computational domain, i.e., far from the area of interaction between the BL and the incident SW.

Earlier, it was described that to solve the problem of SW interaction with the plate, the computational technology of combining the ANSYS Fluent gas-dynamic package and the LOTRAN 3.0 software package, created to calculate the stability of basic three-dimensional flow profiles, was used. Since the LOTRAN 3.0 software package operates with three-dimensional BL, the computational domain size in the cross-direction \( z \) was set to 0.02 m. On side surfaces of such pseudo-two-dimensional computational domain the symmetry boundary conditions were set. This approach allows to use the LOTRAN 3.0 for two-dimensional flows stability study.

On Fig. 2, the color also indicates the type of used boundary conditions. On the left inlet boundary, the incoming flow condition was set (blue line). On the right output boundary and on the top ledge of the computational domain, the outflow condition was used (red lines). On the plate (1) and wedge (3) surface, nonslip and adiabatic wall condition was set (black lines). On the area in front of the plate and on the upper boundary of the ledge (2), the symmetry condition was set (green lines)).

![Fig. 3. Computational grid (every 16th cell)..](image)

The computational domain was covered by a regular hexahedral computational grid with refinement towards the plate surface and its leading edge (see Fig. 3). The total number of cells was 584.7 thousand.

The main flow was calculated using ANSYS Fluent based on the solution of Navier–Stokes equations. To solve these equations, a density-based solver, an implicit scheme of 2nd order accuracy in space with Roe-FDS method of splitting convective flows were used. The thermal conductivity of the working gas (air) was given by a formula from kinetic theory, viscosity by Sutherland's law, heat capacity \( c_p = 1006.43 \text{ J/(kg-K)} \).

After the main flow calculation, the calculated data including the detachment area flow are transferred to the LOTRAN 3.0 module. In the process, the LOTRAN 3.0 module determines the characteristics of the BL (displacement and momentum loss thicknesses, etc.) to qualitatively evaluate the result of assimilating the data obtained from ANSYS Fluent, performs flow stability analysis, draws the start and end points of temporal instability areas, N-factor curves and their envelope.

![Fig. 4. N-factors of the Tollmien–Schlichting wave instability with different computational grids.](image)

In order to choose the optimal computational grid, computations on a sequence of converging grids were performed. Computations were performed on a grid with twofold increased size of cells in all directions 0mesh (144 000 cells) and twofold decreased size 2mesh (2 361 000 cells). Fig. 4 shows that the computational grid 0mesh is not optimum, as difference in N-factors for this grid, exceeds 15% (at \( x = 0.15 \text{ m} \) at comparison with the data obtained on 1mesh grid. Difference in N-factors, obtained on 1mesh and 2mesh grids does not exceed 1% in area of interaction BL with incident SW (\( x \sim 0.17 \text{ m} \)). Thus, the computational grid 1mesh (584.7 thousand cells) is optimal for this problem.
3 Results and Discussion

Figure 5a shows the static pressure field at $\theta = 0.25^\circ$. It can be seen that the SW from the leading edge of the plate interacts with the ledge (2) of the computational domain and does not give any reflected SW. The incident SW (4) generated by the wedge (3) interacts with the BL on the plate surface.

Figure 5b shows the field of the longitudinal velocity component near the interaction region between the BL and the incident SW. The black solid line schematically shows the incident WS (4), the black dashed line - the reflected wave (5), and the gray line - the isoline $u_x = 0$, which shows the boundary of the detached zone formed during the interaction of the incident shock wave with the surface of the plate.

This is confirmed by Fig. 6, which shows profiles of longitudinal component of velocity at different angles of wedge inclination in section $x = 0.16$ m. It can be seen that at small angles ($\theta = 0.25$ and $0.5^\circ$) profiles of longitudinal component of velocity correspond to laminar flow regime, and at large - to the pre-detached regime.

To see the size of the local detachment zone, Figure 8 shows the longitudinal components of shear stresses on the plate surface in the region of interaction of the BL with the incident SW at different angles of the wedge tilt $\theta$. It is also seen that at $\theta > 0.75^\circ$ we observe the area with negative values of the longitudinal component of shear stresses, which also indicates the presence of a local detachment region. Moreover, the greater the angle of inclination of the wedge, the larger the size of the local detachment zone.

It should be noted that the cross section $x = 0.16$ m approximately corresponds to the interaction location of the SW with the BL of the plate. Fig. 7 shows the profiles of the longitudinal velocity component at different $x$-coordinates. It can be seen that the profile corresponding to the detachment flow is observed exactly in the section $x = 0.16$ m. At the same time in sections $x = 0.12$ m and $x = 0.2$ m the profiles of longitudinal component of velocity correspond to laminar regime.

Fig. 5. The static pressure field (a) and (b) the field of the longitudinal velocity component near the BL interaction area with the incident shock wave (not to scale): $\theta = 1.25^\circ$.

Fig. 7. Profiles of the longitudinal velocity component at different $x$-coordinates: $\theta = 1.25^\circ$.

Fig. 6. Profiles of the longitudinal velocity component at different wedge angles for $x = 0.16$ m.

Fig. 8. Distribution of the shear stresses longitudinal component on the plate surface in the area of interaction of the BL with the incident SW at different angles of inclination of the wedge $\theta$. 
Figure 9 shows the $N$-factor envelopes for the Tollmien–Schlichting ($\chi = 57^\circ$) wave instability for different wedge angles $\theta$. It can be seen that in the region of interaction of the BL with the falling SW ($x \approx 0.16m$) there is a marked increase in the values of $N$-factors. At the same time, the degree of increase of $N$-factors increases with increase of the wedge tilt angle $\theta$.

4 Conclusion

Numerical simulation of the problem of interaction of a supersonic ($M_\infty = 2$) boundary layer on a plate with weak shock waves is performed. The computational technology that combines the ANSYS Fluent gas-dynamic package, which calculates the main flow, and the LOTRAN 3.0 software package, which calculates the stability of not only a gapless flow, but also the laminar detachment of the BL, is used for the solution.

The results of the influence of the incident shock wave on the boundary layer of the plate in a wide range of its intensity for both the main flow, including local tearing zones, and its stability are presented. Using the LOTRAN 3.0 software package, the $N$-factor envelopes of the Tollmien–Schlichting wave instability for various angles of the wedge generating a shock wave including local breakaway zones were obtained. It is shown that in the region of interaction of the boundary layer with the incident shock wave there is a marked increase in the values of $N$-factors of instability of Tollmien–Schlichting waves, and with increasing intensity of the shock wave the rate of growth of $N$-factors in this region increases.

It is demonstrated that the proposed approach using the computing technology of combination of ANSYS Fluent and LOTRAN 3.0 allows calculating the stability of flows with shockwave, including the presence of local tearing zones.

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References