Experimental study of the coefficients of the Forchheimer-Ergun equation for the forced flow of hot water in a layer of spherical particles

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Abstract. The paper presents the results of an experimental study of the coefficients of the Forchheimer-Ergun equation for the forced flow of hot water with temperatures up to 175ºC through a layer of spherical particles. The study shows that at a water filtration velocity above 25 mm/s, with the Ergun standard numerical constants in the equation coefficients, the calculation results significantly exceed the experimental data. A refinement expression is found for the coefficient at the inertial term of the equation, which ensures agreement between the calculation and experiment in the considered range of flow temperature and filtration velocity up to 300 mm/s.

1 Introduction

The tool widely used to determine pressure loss during liquid and gas filtration through porous media is the two-term Forchheimer equation with Ergun coefficients (FE) [1, 2], which has the following form for an ascending single-phase flow:

$$\frac{dp}{dz} - \rho g = \frac{k_\alpha (1-m) \mu w_0}{m^2 d_p^2} + \frac{k_\beta (1-m) \rho w_0^2}{m^4 d_p},$$

(1)

where $p$ – pressure in the flow, $z$ – vertical coordinate, $w_0$ – filtration velocity related to the total cross section, $m$ – porosity, $d_p$ – porous layer spherical particle diameter, $\rho$ – density, $\mu$ – dynamic viscosity coefficient.

The most commonly used values of the coefficients are those proposed by Ergun:

$$k_\alpha = 150, \ k_\beta = 1.75.$$  

(2)

At the same time, some studies cast doubt on the universality of the numerical constants 150 and 1.75 in expressions (2). The authors of [3], [4] and [5] propose using the following pairs of constants in (2) for layers of spherical particles, respectively, 368, 1.24; 180, 1.8; 181, 1.63. The work [6] states that the constants in the Forchheimer-Ergun equation are actually functions of the Reynolds number. Recent studies [7], [8] present the results of experiments on the pressure drop during the flow of water through monodimensional layers of spherical particles with a diameter of 2 mm and 3 mm and porosity of 0.370 and 0.383, respectively. The experiments were performed for a vertical heat-insulated cylindrical channel with an inner diameter of 39 mm, a temperature of 17°C to 175°C, and a filtration velocity $w_0$ of 5 mm/s to 300 mm/s. The pressure in the channel varied from 0.113 MPa to 4.65 MPa to ensure the single-phase liquid state of water at the indicated temperatures.

2 Experimental facility

A High-temperature circuit facility [9] was employed for experiments conducted to measure the pressure drop during the water flow through a close-packed layer of spherical steel particles with a diameter of 2 mm and 3 mm and porosity of 0.370 and 0.383, respectively. The experiments were performed for a vertical heat-insulated cylindrical channel with an inner diameter of 39 mm, a temperature of 17°C to 175°C, and a filtration velocity $w_0$ of 5 mm/s to 300 mm/s. The pressure in the channel varied from 0.113 MPa to 4.65 MPa to ensure the single-phase liquid state of water at the indicated temperatures.

Differential pressure sensors DMD-331-A-S with a measuring range of 250 kPa were installed at ring chambers at a distance of 470 mm and 370 mm. The temperature of the water entering the channel was measured with thermal resistance sensor PT100. The water flow through the circuit was determined with Coriolis flow meter EMIS-MASS 260 and, additionally, by measuring the time of filling the control volume.
3 Experimental results for \( w_0 \leq 25 \text{ mm/s} \)

Comparison of the experimental data with the calculation results obtained using the FE equations (1), (2) indicates that the use of constants (2) is valid at low filtration velocities \( w_0 \leq 25 \text{ mm/s} \). This agrees well with the results of experiments [5, 7] with water at a room temperature. To examine the influence of the variable viscosity of water during its heating, experiments were carried out at temperatures from 17°C to 175°C. The dynamic viscosity of water here changes by a factor of 7. Experimental data and results of calculation using equations (1), (2) are shown in Figure 1. The root-mean-square deviation between theory and experiment does not exceed 1%. At the same time, it can be noted that for a temperature of 175°C, equations (1), (2) give a somewhat higher value already at \( w_0 > 18 \text{ mm/s} \).

Following [10] and [11], we rewrite equation (1) in a dimensionless form in terms of the Reynolds number and the Galileo number:

\[
\psi \Gamma_a = 150 R_e_p + 1.75 R_e_p^2.
\]  

(3)

Here

\[
R_e_p = \frac{\rho w_0 d_p}{\mu (1 - m)} \quad \Gamma_a = \frac{\psi d_p (m d_p)}{\mu (1 - m)}^3, \quad \psi = \frac{d_p}{d d} \frac{d \rho_g}{\rho_g}.
\]  

(4)

Figure 2 shows the change in the product of the dimensionless pressure gradient and the Galileo number as a function of the Reynolds number. It is worthwhile to note that the experimental results in the form of dimensionless \( R_e_p \) and \( \psi \Gamma_a \) fall on one curve set by relation (3) for all considered particle diameters and temperatures.

Fig. 1. Comparison of experimental and calculated values of pressure gradient for \( w_0 \leq 25 \text{ mm/s} \). Marks – experiment, lines – calculation according to the FE equations (1), (2).

Fig. 2. The Reynolds number dependence of the product of the dimensionless pressure gradient and the Galileo number for two particle diameters at \( w_0 \leq 25 \text{ mm/s} \). Marks – experiment, line – calculation according to the FE equations (1), (2).
4 Experimental results for $w_0 > 25$ mm/s

With an increase in the flow velocity, the use of the pair of equations (1) and (2) makes the calculation results significantly higher than the experimental data. Since the contribution of the inertial term dominates in the region of increased flow velocity, it was decided to preserve the numerical coefficient $k_\alpha = 150$ in the entire region of $w_0$ variation for the viscous term, and obtain a new expression for the coefficient $k_\beta$ for the inertial term.

Values of coefficient $k_\beta$ were found by the least squares method for each series of experimental data with constant values of particle diameter and temperature. It was established that the effect of particle diameter is not significant, and the effect of temperature can be expressed in the terms of the water density as follows:

$$k_\beta = 150; \quad k_\beta' = 1.33 - 2.39 \frac{\rho(17\,^\circ\text{C}) - \rho(T)}{\rho(17\,^\circ\text{C})}. \quad (5)$$

In order to minimize the error, we propose using the initial Ergun coefficients (2) for velocities below 25 mm/s, the coefficients determined by equation (5) for velocities above 100 mm/s, and interpolation $k_\beta$ for velocities in the range of 25–100 mm/s. The calculated values of pressure loss in the layer according to the proposed method are consistent with the experimental data with an average error of 7.5% (see Fig. 3).

The expression obtained for $k_\beta$ was used to test the hypothesis of a significant predominance of the inertial term of the FE equation in the range of high velocities. Figure 4 shows the values of the relative contributions of the viscous and inertial terms, $A_\mu$ and $A_\rho$, given by the following relations:

$$A_\mu = k_\mu \frac{(1-m)2 \mu w_0}{m^2 d_p} \left( \frac{k_\mu (1-m)2 \mu w_0}{m^2 d_p} + k_\rho (1-m) \rho w_0^2}{m^3 d_p} \right); \quad (6)$$

$$A_\rho = k_\rho (1-m) \rho w_0^2 \left( \frac{k_\mu (1-m)2 \mu w_0}{m^2 d_p} + k_\rho (1-m) \rho w_0^2}{m^3 d_p} \right). \quad (7)$$

As seen in Figure 4, the predominance of the inertial component of the pressure gradient is more pronounced for high temperatures and large particle sizes of the packed bed. In this case, the maximum value $A_\rho$ at the upper limit of the transitional range of velocities, at $w_0 = 100$ mm/s, is 0.3 for spheres with a diameter of 2 mm, and 0.2 for spheres with a diameter of 3 mm.

Analysis of experimental results for the velocities above 25 mm/s in the plane of dimensionless complexes $Re_p$ and $\psi Gd_p$ suggest (see Fig. 5) that the points lie on a set of closely spaced curves defined by formula (3),

Fig. 3. Comparison of experimental and calculated values of pressure gradient for $w_0 > 25$ mm/s. Marks - experiment, gray dotted lines - calculation using the FE equations (1), (2) at four temperatures, black solid lines - calculation using the equation with modified coefficients (5).

Fig. 4. The relative contribution of the viscous and inertial terms at $w_0 > 25$ mm/s. Calculation using equations (5), (6), and (7).
where expression (5) is used for the temperature-dependent coefficient $k_\beta$ instead of constant 1.75.

5 Conclusions

New experimental data on the pressure drop during water flow through a packed bed of steel balls with a diameter of 2 and 3 mm were obtained for a filtration velocity of 5-300 mm/s and a temperature of 17-175°C. The value of the Ergun inertial coefficient has been specified depending on the water velocity and temperature. Given the new expression for the inertial coefficient, the root-mean-square error of generalization of experimental data was 7.5% over the entire considered range of water flow conditions.

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References

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