Selected design parameters of a thermal management system for a lithium-ion phosphate battery

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Abstract. In order to provide a theoretical assessment of the thermal regime of the thermal regime system (SOT) of a heat-generating lithium-iron-phosphate battery, a thermal physico-mathematical model of the thermal regime system of this one has been developed, including the use of axial heat pipes, which, due to their high heat transfer capacity, transmit and divert thermal power from the device to a remote radiator. The thermal physico-mathematical model is constructed by the method of isothermal nodes (the method of concentrated parameters) and includes 4 ordinary differential equations of thermal balance, which corresponds to 4 isothermal nodes. As these nodes, the following are selected: a lithium–ion heat–generating battery, a remote radiator - radiator, as well as evaporative and condensing sections of heat pipes that work cyclically on the principle of evaporation – condensation of the coolant. This model is solved by the Runge–Kutta method of 4 orders of accuracy with linearization of the radiant-convective heat flow relative to the temperature taken from the previous time layers. Based on the developed model, the problem of optimizing the design parameters of the SRT, such as the outer diameter and the number of ATTS, is solved. The conjugate directions method was chosen as the minimization method, as the most accurate gradient method of the 1st order of optimization.

1 Introduction

When designing the thermal regime of instrumentation and aggregate equipment, special attention is paid to its design parameters, such as heat transfer capacity, dimensions of remote radiators, etc. Currently, such problems are solved as problems of finding the global optimum of such characteristics at maximum standard thermal loading. The novelty of this work lies in the preliminary assessment of the thermal regime of a lithium-ion battery, the temperature control system of which is built on axial heat pipes. Such well-known scientists as O.M. Alifanov, A.V. Nenarokomov, N.S. Kudryavtseva, A.V. Alekseev made a significant contribution to the development of the theory of thermal design. In this paper, we consider the composition of a lithium-ion lithium-iron-phosphate battery consisting of a certain number of thermal axial pipes (ATT) that transfer heat from the device to an external radiator with its subsequent radiation into the environment [1-5]. This thermal scheme is shown in Figure 1.

The task is to determine the optimum of these parameters for a given cyclogram of operation of a lithium-ion-phosphate battery, as well as to assess changes in its thermal state over time.

However, to do this, it is necessary to form a thermal physical and mathematical model of the entire system to assess its thermal regime.

2 Thermal physical and mathematical model

Thermal model of the device using N number of axial heat pipes and an external radiator. Heat balance equation for determining battery temperature:

\[ c_{\text{eff}} \rho_{\text{eff}} \frac{\partial T_{\text{lib}}(\tau)}{\partial \tau} v_{\text{lib}} = (\varepsilon a) (T_{\text{me}} - T_{\text{lib}}(\tau)) + \]
\[ + a_{\text{conv}} (T_{\text{lib}}(\tau) - T_{\text{me}}) F_{\text{rad}} + \]
\[ + \frac{\lambda_{\text{eff}} N}{\delta_{\text{lioph}} - \delta_{\text{we}}} (T_{\text{we}}(\tau) - T_{\text{lib}}(\tau)) F_{\text{cond}} + Q_{\text{heat}}(\tau); \]
\[ T_{\text{lib}}(0) = T_{\text{me}}; \]
Heat transfer coefficient for free gas connection to the wall of a lithium-iron-phosphate battery Solution from the criterion relation [6-8]:

\[ Nu = 0.348(\frac{\beta \Delta T l^3}{v^2}Pr)^{\frac{1}{3}} \]

from where it is easy to express the heat transfer coefficient for free convection:

\[ \alpha_{conv}(T_{liq}) = 0.348(\frac{\beta \Delta T l^3}{v^2}Pr)^{\frac{1}{3}} P_{air} \]

For the radiator radiator, the thermal scheme of which is shown in Figure 2 thermal balance equation:

\[ C_{rad} \rho_{rad} \frac{\partial T_{rad}(\tau)}{\partial \tau} \gamma_{rad} pradience = \]

\[ = \epsilon \sigma \eta (T_{mt}^4 - T_{rad}(\tau)) + \alpha_{conv}(T_{rad}) \lambda(T_{mt} - T_{liq}(\tau)) \gamma_{rad} + \]

\[ + \frac{\lambda_{rad} N}{\delta_{wc-rad}} (T_{wc}(\tau) - T_{rad}(\tau)) P_{cond}^{lib}; \]

\[ T_{rad}(0) = T_{mt}; \]

The heat transfer coefficient for the radiator - radiator is calculated in the same way as for the lithium-iron-phosphate battery. It is assumed that the radiation occurs at a temperature \( T_{rad} \) constant in y, the unevenness of the temperature \( T_{wc} \) is taken into account by the efficiency coefficient of the radiating surface \( n \). The expression for determining this coefficient has the form:

\[ \eta = \frac{th H}{\gamma_{m_1}} \]

\[ m_1 = 2 \frac{\epsilon \sigma T_{rad}^4}{\delta_{wc-rad} \lambda_{rad}} \]

The thermal design scheme of the radiator - radiator is shown in Figure 2.

![Fig. 2. Thermal diagram of the radiator.](image)

When designing the thermal regime of axial heat pipes, the supplied thermal capacities should be taken into account to prevent drying out of the capillary-porous structure of the product. To solve this problem, it is required to calculate the heat transfer inside the axial heat pipe [1-5]. The thermal design scheme of the heat pipe is shown in Figure 3.

![Fig. 3. Thermal diagram of an axial heat pipe.](image)

To compile a thermal model of the ATT, a thermal circuit with thermal resistances between its individual elements has been compiled. This scheme is shown in Figure 4.

![Fig. 4. Thermal typical ATT circuit with thermal resistances.](image)

Along the length of the heat pipe, it is possible to distinguish evaporative, adiabatic and condenser sections having lengths \( L_e, L_a, \) and \( L_c \), respectively. The evaporative section of the ATT is connected to a structural element or equipment from which heat is removed [5]. Due to the supplied heat flow \( Q_{heat} \), part of the working fluid filling the wick evaporates. Thus, due to the pressure difference, the steam in the evaporator and the condenser is transferred to the condenser. The condensation site is associated with a structural element whose temperature is lower than the temperature of the evaporation site. The heat flow \( Q \) from the condensing working fluid is transferred to the condensation site. Figure 4 shows the thermal circuit of the TT. From the element of the structure heated to the temperature \( T_{liq} \), heat is transferred to the outer surface of the wall of the section having a temperature and thermal resistance of the contact \( R_i \). Then part of the heat flow passes to the inner surface of the wall of the evaporator having a resistance \( R_{we} \), with a temperature \( T_{we} \), and through the wick (resistance \( R_{f}^{w} \)) into the evaporation zone, and the part is transferred along the TT wall to the condensation site (resistance \( R_{w} \)). From the surface of the wick, which has a temperature \( T_{wc} \), the working fluid evaporates [6-9]. The resistance of the phase transition is \( R_{f}^{g} \), then the steam transfers heat to the condensation...
zone. The steam has temperatures in the evaporation and condensation zone $T_{ev}$ and $T_{con}$, respectively, the resistance of the transport section is $R_g$. In the condensation zone, heat is transferred to the wick, and then through the wick, the wall and the contact passes to the structural element with a temperature of $T_{rad}$. The chain of thermal resistances in this section:

$$\frac{1}{R_g} + \frac{1}{R_w} + \frac{1}{R_{cond}}.$$ 

In practical calculations, the values $R_e$, $R_w$, $R_g$ can be neglected.

Thus, taking into account the accepted assumptions, the ATT thermal model can consist of two ordinary differential equations constructed by the method of thermal balances for the evaporation zone and the condensation zone [6]:

for the evaporation zone:

$$C(T_{we})\rho \frac{dT_{we}(\tau)}{d\tau} V_{we} = \left[\frac{1}{R_{we} + R_{f}^e + R_{c}^e + R_{wc}}\right] \times$$

$$\times [T_{we}(\tau) - T_{we}(\tau)]F_{we} + \frac{\lambda_{ef}}{\delta_{lwc-wc}} (T_{lwc}(\tau) - T_{we}(\tau)) F_{lwc}^{con}, \quad T_{we}(0) = T_{mt};$$

for the condensation zone:

$$C(T_{wc})\rho \frac{dT_{wc}(\tau)}{d\tau} V_{wc} = \left[\frac{1}{R_{wc} + R_{f}^c + R_{c}^c + R_{wc}}\right] \times$$

$$\times [T_{wc}(\tau) - T_{wc}(\tau)]F_{wc} + \frac{\lambda_{ef}}{\delta_{lwc-wc}} (T_{wc}(\tau) - T_{rad}(\tau)) F_{wc}^{cond}, \quad T_{wc}(0) = T_{tm};$$

where

$$R_{we} = \frac{1}{R_{wc} + R_{f}^e + R_{c}^e + R_{wc}}; \quad R_{wc} = \frac{1}{2\lambda(T)}; \quad R_{we} = \frac{L_{wc}}{2\lambda(T)};$$

The following designations have been introduced in the formulas presented earlier:

- $C$ is specific heat capacity, J/kg;
- $\rho$ is density, kg/m$^3$;
- $V$ is volume of the isothermal unit, m$^3$;
- $F$ is contact area, m$^2$;
- $\delta$ is thickness, m;
- $\varepsilon$ is degree of blackness of the surface;
- $\lambda$ is coefficient of thermal conductivity, W/(mK);
- $\sigma$ is Stefan–Boltzmann constant W/(m$^2$K$^4$);

The resistances $R_{con}$ and $R_{vap}$ are especially due to the high thermal conductivity of the wick. The expression for the definition of $\lambda_{ph}$ has the following form:

$$\lambda_{ph} = \frac{1}{\lambda(T)}\left[\frac{\lambda_{solid}(T)}{1 + \frac{\lambda_{liquid}(T)}{\lambda_{solid}(T)}}\right];$$

Here $\lambda_{solid}$ is the thermal conductivity of the solid phase, W/(mK); $T_{mt}$ is the ambient temperature, K; N is the number of axial heat pipes; $V_{we} = (\pi d_2^2)/4 L_e$ is volume of the evaporation zone, m$^3$; $V_{wc} = (\pi d_2^2)/4 L_e$ is volume of the condensation zone, m$^3$; $F_{rad}$, $F_{lwc}$ are area of the radiator-radiator and lithium-iron-phosphate battery, $m^2$; $Q_{heat}(\tau)$ is cyclogram heat dissipation of lithium-iron-phosphate battery, W.

The above system of heat balance equations is solved by the Runge–Kutta method of the 4th order [9-12].

To determine the design parameters, namely the required number of heat pipes and the area of the external radiator, it is necessary to solve a two-parameter optimization problem. The critical limit temperature of the lithium-iron-phosphate battery is selected as a stop criterion. Thus, it can be formulated in the form of a root-mean-square error, which will be minimized:

$$S(N, d_2) = \frac{1}{2} (T_{lwc}(N, d_2, \tau) - T_{crit})^2;$$

The method of conjugate directions is chosen as the optimization method, as the most accurate method of the first order of convergence. His typical algorithm [13] on the example of optimizing the number of ATTS is presented below:

$$N^{n+1} = N^n + \Delta N^{n+1}$$

where

$$\Delta N^{n+1} = -\beta_k P^{(n)};$$

The direction of descent is determined from:

$$\beta = 0, P^{(0)} = 0, \; \frac{\lambda_{eff}}{(\lambda_{eff})^{N^{n+1}}};$$

$$\beta = \frac{|\lambda_{eff}|}{(\lambda_{eff})^{N^{n+1}}};$$

The criterion for stopping an iterative process is the expression:

$$T_{lwc} < T_{crit};$$

The gradient of the target functional is presented below:

$$\nabla S(N, d_2) = \frac{\lambda_{eff}}{\lambda_{eff}^N} \times$$

$$\times \left[\frac{\delta S(N, d_2)}{\partial N} + \frac{\delta S(N, d_2)}{\partial d_2}\right];$$

As can be seen from the presented algorithm, one of the necessary tasks is to find the gradient components of the target minimization functional. To do this, it is simply necessary to differentiate the desired system of thermal balances for each of the components. We obtain a conjugate system of equations when differentiating by $N$. For lithium-iron-phosphate battery:

$$c_{eff} \rho_{eff} \frac{\partial^2 T_{lwc}(\tau)}{\partial \tau^2} V_{lwc} = -4\varepsilon \sigma T_{lwc}^3 \left(\frac{\partial T_{lwc}(\tau)}{\partial N} F_{rad} + \frac{\partial \alpha_{conv}(T_{lwc})(T_{mt} - T_{lwc}(\tau))}{\partial \tau} \frac{\partial T_{lwc}(\tau)}{\partial N} - \frac{\partial \alpha_{conv}(T_{rad} T_{lwc}(\tau)) F_{conv}^{con}}{\partial N} + \frac{\lambda_{eff}}{\lambda_{lwc-ve}} \left(T_{lwc}(\tau) - T_{lwc}(\tau)\right) + N \left(\frac{\partial T_{lwc}(\tau)}{\partial N} - \frac{\partial T_{lwc}(\tau)}{\partial N}\right) F_{lwc}^{cond} \frac{\partial T_{lwc}(\tau)}{\partial N} = 0;$$

For radiator:

$$c_{rad} \rho_{rad} \frac{\partial^2 T_{rad}(\tau)}{\partial t^2} V_{rad} = -4\varepsilon \sigma T_{rad}^3 \times$$
For the evaporation zone:
\[ S_{\text{rad}}(N, d_2) = \frac{1}{2} \left( T_{\text{rio}}(N, r) - a_d(N) \frac{dT}{dN} \text{grad}S - T_{\text{crit}} \right)^2 + \frac{1}{2} \left( T_{\text{rio}}(d_2, r) - a_d(d_2) \frac{dT}{dN} \text{grad}S - T_{\text{crit}} \right)^2 \]

To find the optima of the descent steps, it is simply necessary to differentiate this variation by two descent steps and equate it to zero, from where you can easily get expressions for these steps:

For the descent step at gradient method of optimization when iteratively searching for the number of ATT:
\[ a_d(N) = \frac{T_{\text{crit}} - T_{\text{rio}}(N, r)}{\frac{dT}{dN} \text{grad}S} \]

For the descent step during the iterative search for the effective area of the remote radiator radiator:
\[ a_d(d_2) = \frac{T_{\text{crit}} - T_{\text{rio}}(d_2, r)}{\frac{dT}{dN} \text{grad}S} \]

Thus, this algorithm includes the following sequence of actions:
1. Setting initial approximations for the number of ATTS and the effective area of the radiator-radiator;
2. Solving the "direct" problem of heating the structure by the method of thermal balances, modeling the conditions of regular operation of the structure;
3. Compilation of the RMS error between the theoretical and critical temperature field of a lithium-iron-phosphate battery;
4. Solution of two conjugate problems for finding the gradient components of the target functional of the discrepancy between the theoretical and experimental temperature field;
5. Calculation of the descent step in the conjugate directions method based on the iterative regularization method;
6. Obtaining the following iterated approximations of the design parameters of the;
7. Checking the criterion for stopping the iterative process. If it is executed, the parameterized values are considered to be the desired ones, otherwise it is necessary to repeat steps 1-6.

To test the effectiveness of the developed algorithm, a program for the thermal regime of a lithium-ion lithium-iron-phosphate battery with a volumetric heat release of 60 W has been compiled. The design parameters are presented in Table 1.

### Table 1. Design parameters of a lithium-iron-phosphate battery and a thermal regime system.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimensions of lithium-iron-phosphate battery</td>
<td>0.04×0.13×0.4 m³</td>
</tr>
<tr>
<td>Radiator dimensions</td>
<td>0.1×0.13×0.4 m³</td>
</tr>
<tr>
<td>The degree of blackness of the surface of the lithium-iron-phosphate battery</td>
<td>0.8</td>
</tr>
<tr>
<td>Radiator material</td>
<td>Aluminium</td>
</tr>
</tbody>
</table>
The change in the temperature of a lithium-iron-
phosphate battery depending on the number of heat
pipes obtained during the solution of the optimization
problem by their number is shown in Figure 5.

![Figure 5](image)

The change in the average volume temperature from
time to time with a variation in the amount of heat pipe.

The change in the temperature of the average
volume temperature of a lithium-iron-phosphate battery
depending on the outer diameter of the ATT at 6 ATT is
shown in Figure 6.

![Figure 6](image)

The nonlinear dependences of the unsteady
temperature field on the temperature, shown in Figures
5 and 6, in the corresponding isothermal nodes are due
to the dependence that the inertia of heat pipes depends
squared on their diameter, as well as the presence of
radiant-convective heat exchange with the environment.

The iterative convergence of the parameters under
study should be estimated by the standard deviation
between the critical and standard temperatures,
depending on the iteration number. This convergence is
shown in Figure 7.

![Figure 7](image)

The obtained estimated state of the SRT for the
lithium-iron-phosphate battery and the external radiator
is shown in Figure 8.

![Figure 8](image)

As can be seen from Figures 6 and 8, with 6 thermal
ATTs and an external diameter of 12 mm, the remaining
design parameters are given in Table 1.

3 Conclusions:

1. A thermal physico-mathematical model of a
lithium-iron-phosphate battery consisting of N axial
heat pipes and an external radiator radiator has been
developed. Based on this model, the optimization
problem of finding the design parameters of the SRT,
such as the outer diameter of the ATT and their number,
is solved.

2. A numerical analysis of the thermal state of the
device is carried out when solving the formed thermal
model by the method of thermal balances. The system
of ordinary differential equations formed by this method
was solved by the Runge–Kutta method of 4 orders of
accuracy.

3. The results of calculations showed that it is
recommended to use 6 ATTS with a minimum external
minimum diameter of 12 mm.

References

1. Vershinin S.V., Maidanik Yu.F., Thermal
Processes in Engineering 12, 559 (2012)


10. Ignatiev S.A. Problems of Mechanical Engineering and Automation 2, 27 (2009)

