

# Construction of a project risk assessment model based on a probabilistic simulation approach

*A.I. Khorev*<sup>1\*</sup>, *M.L. Lapshina*<sup>2</sup>, *G.V. Belyaeva*<sup>1</sup>, *O.O. Lukina*<sup>1</sup>, and *D.D. Lapshin*<sup>3</sup>

<sup>1</sup> Voronezh State University of Engineering Technologies, 141701 Voronezh, Russia

<sup>2</sup> Voronezh State Forestry Engineering University named after G.F. Morozov, 394036 Voronezh, Russia

<sup>3</sup> Novousmansky Educational Center, Voronezh region, Russia

**Abstract.** Manifestations of risk in the economic sphere of human activity have entailed numerous definitions and interpretations, a significant part of which are associated with two statements: a) risk is caused by random events outside the sphere of influence of the risk subject; b) the economic consequences of these events are undesirable or unacceptable for him. An ambiguous interpretation of the original concept leads to an incorrect formulation of the problem of formalizing the process of optimizing investment projects. The paper provides an analysis of the possibility of resolving the problem using a simulation-probabilistic approach.

## 1 Introduction

Summarizing the literature, it should be stated that a typology of project risks according to the following criteria is more or less representative and has practical significance: possible results, sources, utilitarian consequences and areas of economic activity. Possible results include: 1) pure risks are risks that can only result in negative consequences, that is, their outcome can only be damage incurred as a result of failure to complete the project in whole or in part; 2) speculative risks involve the possibility of both positive and negative results. According to sources, risks can be divided into: 1) risks caused by the actions of competitors aimed at preventing the implementation of the project; 2) risks caused by natural factors, as a result of which the implementation of the project is jeopardized; 3) risks caused by the human factor, when the failure of the project is associated with the actions of managers or the project leader.

Based on utilitarian consequences, project risks in economic systems are usually divided into three categories: 1) acceptable risk is a risk as a result of which the risking entity faces loss of profit; 2) critical risk is a risk in which the risky entity faces loss of revenue; 3) catastrophic risk – a risk in which the risk subject enters a crisis zone. By type of activity, risks are divided into: two main types: commercial and financial. Commercial or trading risks arise during commercial operations and are associated with the activities of managers involved in managing these operations. Financial risks are understood as risks arising in the

---

\* Corresponding author: [oks.lukina@gmail.com](mailto:oks.lukina@gmail.com)

field of financial projects. These include [66]: the risk of decreased financial stability, the risk of insolvency, as well as investment, inflation, interest, tax and structural risks.

The risk of a decrease in financial stability is generated by imperfect capital structure, which creates an imbalance of positive and negative cash flows in terms of volumes. The risk of insolvency is generated by a decrease in the level of liquidity of current assets, which creates an imbalance of positive and negative cash flows over time. Investment risk characterizes the possibility of financial losses in the process of carrying out investment activities of economic entities and is divided into: the risk of real investment and the risk of financial investment. These types of financial risks belong to the so-called “complex risks”, which in turn are divided into their individual subtypes. Inflation risk is characterized by the possibility of depreciation of the real value of capital, as well as expected income from financial transactions in conditions of inflation.

Interest rate risk consists of an unexpected change in the interest rate in the financial market. The reason for the occurrence of this type of financial risk is a change in financial market conditions under the influence of government regulation, an increase or decrease in the supply of free monetary resources and other factors. The negative financial consequences of this type of risk are manifested in the issuing activities of economic entities, in their dividend policy, in short-term financial investments and some other financial transactions. Credit risk occurs when providing a commodity or consumer loan. The form of its manifestation is the risk of non-payment or late payment for products sold on credit, as well as exceeding the estimated budget for debt collection. Tax risk has a number of manifestations: the likelihood of introducing new types of taxes and fees on certain aspects of economic activity; the possibility of increasing the level of rates of existing taxes and fees; changing the terms and conditions for making certain tax payments; the likelihood of cancellation of existing tax benefits in the field of economic activity of the enterprise. Being unpredictable, this risk has a significant impact on the performance of entities in the real sector of the economy.

Structural risk is generated by ineffective financing of current costs, causing a high share of fixed costs in their total amount.

## 2 Initial prerequisites for constructing simulation models, taking into account the peculiarities of their formalization

Let us analyze the possibility of constructing probabilistic simulation models in order to minimize risks in investment projects. Thus, a probabilistic risk assessment model is built on the basis of the apparatus of probability theory and mathematical statistics. In this implementation, the probability of the occurrence of some adverse event associated with the failure of the project is used as a measure of risk:

$$R = 1 - \prod_{i=1}^N q_i \tag{1}$$

where  $q_i$  is the probability of successful completion of the project taking into account only the  $i$ -th risk factor,  $N$  is the total number of risk factors taken into account.

In the case of independence and equivalence of risk factors, as well as under the assumption that the assessment of the  $i$ -th risk factor is a random variable  $p_i$  distributed according to a normal law, the probabilities  $q_i$  are equal

$$q_i = \frac{1}{\sqrt{2\pi}\sigma_i} \int_{m_i-\Delta_i}^{m_i+\Delta_i} e^{-\frac{(p_i-m_i)^2}{2\sigma_i^2}} dp_i \tag{2}$$

Where  $m_i, \sigma_i$  – mean value (norm) and dispersion of the value  $p_i, +\Delta_i, -\Delta_i$ , – critical deviations of the value  $p_i$  from the norm in larger and smaller directions. Substituting (2) into (1) we have:

$$R = 1 - \prod_{i=1}^N \frac{1}{\sqrt{2\pi}\sigma_i} \int_{m_i-\Delta_i}^{m_i+\Delta_i} e^{-\frac{(p_i-m_i)^2}{2\sigma_i^2}} dp_i \tag{3}$$

Let us complicate model (3), reflecting the features of project management in a competitive environment. In this case, we will proceed from the fact that competition develops according to a symmetrical multi-step “measure-countermeasure” scheme, when in response to the action of one side the action of the other side follows. Moreover, each side must have such methods of action to which the other side has the opportunity to respond with adequate actions, that is, the principle of balance of power must be fulfilled. Otherwise, one of the parties will obviously have an advantage, and the outcome of the fight becomes obvious. Therefore, in competition, gains are achieved by one side preempting the other side.

Let us introduce indicators characterizing the state of competition and corresponding risk measures:

- $P_1(t)$  – the probability that by time  $t$ , party “1” is ahead of party “2” (the risk of party “2” losing);
- $P_2(t)$  – the probability that by time  $t$  neither party manages to forestall the other party (the risk of neither party winning);
- $P_3(t)$  – the probability that by time  $t$  side “2” is ahead of side “1” (the risk of loss of side “1”).

Moreover, at any moment in time the condition is valid:

$$\sum_{i=1}^3 P_i(t) = 1$$

The task will be to build a mathematical model that reflects the specified features of competitive struggle of confrontations such as a duel, and allows one to assess the risk levels of the parties. To do this, let us imagine the simulated process in the form of an oriented symmetric periodic stochastic graph, shown in Fig.1. The vertices of this graph correspond to the probabilistic estimates of risk levels of the parties  $P_i(t)$ , and the arcs correspond to the probabilities  $p_{ij}$  of the process transition from state  $i$  to state  $j$  ( $i, j = 1, 2, 3$ ). Let the following assumptions be true:

- a)  $p_{ij} > 0$  and do not depend on  $t$ ;
- b)  $P_i(t+1)$  depends only on  $P_i(t)$  and does not depend on  $P_i(t-1), P_i(t-2), P_i(t-3)$ , etc.; transitions from one state to another occur at strictly defined moments time  $t = 1, 2, \dots, T$ , the intervals between which are determined.

Then the mathematical representation of this graph is the following system of recurrent algebraic equations:

$$\begin{cases} P_1(t+1) = p_{11} P_1(t) + p_{12} P_2(t) + p_{13} P_3(t) \\ P_2(t+1) = p_{21} P_1(t) + p_{22} P_2(t) + p_{23} P_3(t) \\ P_3(t+1) = p_{31} P_1(t) + p_{32} P_2(t) + p_{33} P_3(t) \end{cases} \tag{4}$$

Having specified the initial (starting) states  $P_i(0)$  and the transition probabilities  $p_{ij}$ , we can determine the values of  $P_i(t)$  at any discrete time  $t$ :

$$P_i(t) = \sum_{j=1}^3 p_{ij} \left( \sum_{j=1}^3 p_{ij} \left( \dots \sum_{j=1}^3 p_{ij} \right) \right) \tag{5}$$

For example, let the initial (starting) risks be equal:  $P_1(0) = 0,3; P_2(0) = 0,3; \text{ and } P_3(0) = 0,4$  and  $\| p_{ij} \| = \begin{vmatrix} 0.1 & 0.3 & 0.4 \\ 0.3 & 0.6 & 0.2 \\ 0.6 & 0.1 & 0.4 \end{vmatrix}$ . We will determine the risk levels of the parties in three steps of competition. Using formula (5), it is easy to determine the values of quantities  $P_1, P_2$  and  $P_3$  at times  $t = 1, 2, 3$ ;

$$\begin{aligned} P_1(1) &= 0,28; P_2(1) = 0,35; P_3(1) = 0,37; \\ P_1(2) &= 0,28; P_2(2) = 0,37; P_3(2) = 0,35; \\ P_1(3) &= 0,28; P_2(3) = 0,37; P_3(3) = 0,35. \end{aligned}$$

Substantively: over three steps of competitive confrontation, the risk of the second party losing decreased by 2%, the risk of the parties remaining “within their own” increased by 7%, and the risk of the first party losing decreased by 5%. Thus, the development trend of the simulated process is to reduce the risk of the parties losing and to increase the risk of a draw. The question arises: is this trend asymptotically stable and if so, then to what level of risks it will persist. To answer this question, we rewrite (4) in vector-matrix form:

$$P(t + 1) = P(t), \tag{6}$$

where  $P(t + 1)$  and  $P(t)$  are vectors  $(P_1, P_2, P_3)$  at times  $t + 1$  and  $t$ , respectively. Then it is easy to show that, if  $\| p_{ij} \|$  is a positive matrix ( $p_{ij} > 0$ ) and  $P(t)$  satisfies the condition  $\sum_{i=1}^3 P_i(t) = 1$ , then: a)  $\lim_{t \rightarrow \infty} P(t) = P^*$ , where  $P^*$  is a probability vector; b)  $P^*$  does not depend on  $P^*(0)$ ; c)  $P^*$  is an eigenvector of the matrix  $\| p_{ij} \|$  belonging to the characteristic number 1.

This means that if the dynamics of changes in the risks of the parties during their competitive confrontation is described by a model of the form (4) and all  $p_{ij} > 0$ , then this process is asymptotically stable and has a single stationary point  $P^*$ , the coordinates of which do not depend on the initial (starting) risks. To determine the coordinates of this point, it is necessary to determine the eigenvector of the matrix  $\| p_{ij} \|$ , belonging to the characteristic number 1, that is, the problem is reduced to solving a vector-matrix equation of the form

$$\| p_{ij} \| - \| \delta_{ij} \| = 0 \text{ relative to the vector } P^*, \text{ where } \| \delta_{ij} \| = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}.$$

Having performed the indicated operation for the accepted initial data, we obtain:  $p^* = (0.28; 0.37; 0.35)$ , that is, the previously identified trend is stable, and the maximum risk levels of the parties will be equal: risk of loss of party “2” –28%;- risk of non-win by either party – 37%;- risk of loss for side “1” – 35%. A situational model, when as a measure of risk we will use the magnitude of the deviation of the real situation from some reference, which is taken to be a situation with a zero level of risk. Thus, with a situational approach to risk assessment, the task of correctly identifying the developing situation and assessing the errors that arise due to underestimation of subtle signs or hidden factors comes to the fore. Several methods can be used to minimize identification errors: 1) look for additional signs that allow you to more accurately separate classes of situations; 2) recognize the inevitability of errors, introduce penalties for risk and construct a separating rule in such a way as to minimize the mathematical expectation of the penalty; 3) move to a fuzzy division of classes of situations, use the so-called membership functions, which assess the degree of confidence in classifying a situation into a particular class. Since in real conditions the characteristics of situations are specified, as a rule, in linguistic (non-numerical) terms, the third method has the greatest practical importance. In this case, as a fuzzy measure of risk, we will use the Hamming distance [4], which defines the “distance” between the assessed situation  $P_1$  and the reference situation  $P_2$ , specified by their membership functions  $\eta_{P_1}(P_1)$  and  $\eta_{P_2}(P_2)$ , reflecting the possible income of the risky subject in a given situation.

Using this measure, situational risk is assessed as follows:

$$R(P_1/P_2) = \frac{1}{n} \sum_{i=1}^n |\eta_{P_1}(x_i) - \eta_{P_2}(y_i)|, \tag{7}$$

where  $x_i, y_i$  are successive discrete values of the membership functions  $\eta_{P_1}$  and  $\eta_{P_2}$  on the  $P_1$  and  $P_2$  axes. Let us define, for example, the situational risk for the current (assessed) situation  $P_1$ , given by the membership function  $\eta_{P_1}(P_1) = \{10/0.2; 15/0.4; 20/0.9; 25/0.7\}$ , for the desired (reference) situation  $P_2$ , specified by the membership function  $\eta_{P_2}(P_2) = \{10/0.4; 15/0.6; 20/0.4; 25/0.3; 30/0.6\}$ . Using the above formula we get: Substantively: the risk that the actual situation will correspond to the desired one is 38%. Let's consider the possibility of using a game model, in which it is initially assumed that the optimal behavior strategy of competitors is one that, when repeating an economic operation many times, ensures the maximum possible average gain [6]. When choosing such a strategy, the basis of reasoning is the assumption that the players are doing everything possible to prevent each other from achieving their goals. At the same time, to ensure the possibility of mathematical analysis of the game, the parties must know the rules of the game and the system of conditions regulating the possible options for the players' actions, the amount of information each side has about the behavior of the other, and the result of the game that each given set of moves leads to.

As a measure of gaming risk, we will use the deviation of the current winnings of the parties from the winnings obtained by implementing guaranteeing strategies. Guaranteeing strategies are usually those that are adopted based on the use of the so-called maximin criterion, the essence of which can be expressed in the phrase: “The best strategy will be the one that, under the worst conditions, will allow you to get the greatest win.” Let us clarify this

criterion using a game-theoretic approach [3]. Let managers of two competing enterprises select strategies  $x_i, y_i$ , belonging to admissible sets  $X$  and  $Y, x_i \in X, y_i \in Y$ . Let the parties be interconnected in such a way that the gain of each of them depends both on its own strategies  $x_i \in X$  and on the strategies of the other party  $y_i \in Y$ . In addition, we will take into account the external conditions  $v_i \in V$ , which do not depend on the actions of the competing parties, but either otherwise influence the choice of strategies. Let us assume that the estimated gain of each party is described by a non-negative function  $E_k = E_k(x_i, y_i, v_i)$ , ( $k = 1, 2$ ) and each of them strives to increase it, that is, the behavior of competitors is consistent with the rationality hypothesis. We will also assume that when choosing its strategy, each side knows only the sets  $X, Y$  and  $V$ . Then for each side, the guaranteeing strategies will be  $x^G, y^G$  those that satisfy the following criterion:

$$\begin{aligned} x^G &\in \text{Arg max}_{x_i \in X} \min_{y_i \in Y} \min_{v_i \in V} E_1(x_i, y_i, v_i); \\ y^G &\in \text{Arg max}_{y_i \in Y} \min_{x_i \in X} \min_{v_i \in V} E_2(x_i, y_i, v_i). \end{aligned} \tag{8}$$

Accordingly, the gaming risk of the parties is determined by the expressions:

$$R_1 = \frac{1}{E_1^G} |E_1^G - E_1| 100\%; \quad R_2 = \frac{1}{E_2^G} |E_2^G - E_2| 100\%, \tag{9}$$

where  $E_1^G, E_2^G$  and  $E_1, E_2$  are the guaranteed and estimated gains of the parties.

It is obvious that strategies  $x^G$  and  $y^G$  satisfying this criterion correspond to the limiting case when competing parties refuse all interaction, believing that the other party will take all possible measures to win the competition. In reality, such “antagonism” is not always useful. Therefore, competitors go to negotiations, believing that it is possible to find some kind of compromise that can bring each of them greater profits than in the absence of a negotiation process. The use of a reflexive risk assessment model reflects the desire of competitors implementing a project to anticipate each other’s actions and the ability to impose on the competitor the desired method of action [6, 7]. The point of this approach is that by organizing one’s actions in accordance with assumptions about what a competitor might do, there is a risk of making a mistake if the other party acts in a different way than those assumptions. Competitors predict possible options for a competitor’s actions, but there is never any certainty that the set of options has been completely exhausted. The greatest risk will be if the other party finds out the plan. In this case, risk is measured by establishing a preference between different options for action, for example, in the following form: if a competitor acts on option “ $X$ ”, then our action “ $A$ ” has less risk compared to action “ $B$ ”. The difficulty is how to “force” a competitor to act on option “ $X$ ”. As a result, when assessing such risks, we arrive at models of mutual reflexive management [7].

For an expert assessment of risks in conditions of mutual reflection for each combination of strategies, we compare the chart  $\langle P_{i,j}^A, \dot{P}_{i,j}^B, E_{i,j}(A), E_{i,j}(B), \dot{E}_{i,j}(A), \dot{E}_{i,j}(B) \rangle$ , where  $P_{i,j}^A$  is the subjective probability (given in the assessment by party “ $A$ ”) of the use by party “ $B$ ” of strategy  $j$ ;  $\dot{P}_{i,j}^B$  – reflexive assessment by party “ $B$ ” of the subjective probability of party “ $A$ ” using strategy  $i$ ;  $E_{i,j}(A), E_{i,j}(B)$  – payoffs “ $A$ ” and “ $B$ ” when combining strategies  $i$  and  $j$ ;

$\dot{E}_{i,j}(A), \dot{E}_{i,j}(B)$  reflective assessments by one party of the other party's gain. A similar six can be written for the other side. Then the process under consideration can be characterized by the reflexive matrices  $RM(A)$  and  $RM(B)$ . Each element of this matrix  $M_{i,j}^k$  is a submatrix, which for side "A" looks like this

$$M_{i,j}^k(A) = \begin{vmatrix} E_{i,j}(A) & E_{i,j}(B) \\ \dot{E}_{i,j}(A) & \dot{E}_{i,j}(B) \end{vmatrix}.$$

If it is possible to find unifying functions, for example, of the form  $\Phi_{i,j}^k(A) = E_{i,j}(A) - E_{i,j}(B); \dot{\Phi}_{i,j}^k(A) = \dot{E}_{i,j}(A) - \dot{E}_{i,j}(B)$ , then the submatrix takes

the following form  $M_{i,j}^k(A) = \begin{vmatrix} \Phi_{i,j}^k(A) \\ \dot{\Phi}_{i,j}^k(A) \end{vmatrix}.$

Now, by specifying various combinations of probabilities, it is possible, using the matrices  $P_{i,j}^A$  and  $P_{i,j}^B$ , to give quantitative estimates of the risks of the parties and choose strategies for their risk behavior, taking into account the probable behavior of the competitor.

### 3 Result and conclusions

When implementing commercial projects, it is advisable to consider risk as an economic category associated, on the one hand, with the desire to obtain maximum profit, and, on the other hand, with the desire to ensure the necessary level of economic sustainability of the company in a competitive environment. The paper presents probabilistic, situational, game and reflexive risk assessment models. The advantages of the probabilistic model: simplicity and the presence of a well-developed mathematical apparatus that does not require too much mathematical training from the manager. Weaknesses: purposeful actions of competitors that impede the achievement of goals remain "behind the scenes," as well as assumptions about the independence of risk factors. The advantages of the situational model include the correspondence of the risk measure to its intuitive understanding: the more the current situation deviates from the reference one, the higher the risk. The weaknesses of this model are as follows: for its practical implementation it is necessary to have multidimensional descriptions of the reference and current situations; as with the probabilistic approach, the actions of competitors that impede the achievement of project goals are not sufficiently taken into account; it is required to correctly specify the type of numerous and diverse membership functions that characterize the characteristics of situations, which is far from a trivial task.

The main disadvantage of the game risk assessment model is that it does not take into account the miscalculations and mistakes of competitors, which are inevitable in the real sector of the economy. In addition, the most important limitation of the gaming approach is that the winnings are artificially reduced to one single number. A strategy of economic behavior that is optimal according to one indicator will not necessarily be optimal according to other indicators. In addition, the difficulties in implementing the game model are associated with finding the so-called "saddle point", that is, such a set of strategies of competitors, deviation from which is unprofitable for any of them. The reflexive risk assessment model is most adequate to the substantive essence of the risks that occur in the real sector of the economy. However, in its practical application it requires the use of a rather complex mathematical apparatus, which is not yet available to mid-skilled managers. Considering the

above, when analyzing and assessing specific project risks, it seems advisable to use the above models in a comprehensive manner.

## References

1. A.M. Anokhin, V.A. Glotov, V.V. Pavelev, A.M. Cherkashin, Complex assessment: the binary principle and its applications (Moscow, IPU RAS, 1994)
2. I. Ansoff, Strategic management (Moscow, Economics, 1986)
3. Yu. N. Arsenyev, V. S. Minaev, Risk management (Moscow, Vyssh.school, 1997)
4. V.N.Burkov, D.A.Novikov How to manage projects (Moscow, SINTEG-GEO, 1997)
5. A.A. Voronin, M.V. Gubko, S.P. Mishin, D.A. Novikov, Mathematical models of organizations (Moscow, Lenand, 2008)
6. R. Beard, Risk theory. The stochastic basis of insurance. Third edition (London, 1984)
7. F. Knight, Risk, Uncertainty and Profit (Boston, Houghton Mifflin, 1921)
8. M.L. Lapshina, et.al, Bulletin of the Voronezh State University of Engineering Technologies **84(3)**, 345-351 (2023)
9. M. I. Korolev et al, *Using the possibilities of digitalization tools in building a dynamic model of competition in conditions of economic security*, 2nd International Conference on Computer Applications for Management and Sustainable Development of Production and Industry (CMSD-II-2022) **12564**, 273-279 (2023)
10. M. I. Korolev et al, *Justification of the Use of Mathematical Analogues in the Construction of an Optimization Model of the Company's Functioning, Taking into Account Its Economic Security*, International Scientific and Practical Conference Digital and Information Technologies in Economics and Management. Cham: Springer Nature Switzerland, pp. 23-31 (2022)