Mathematical model of wagon wheels rolling along the hump profile

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Abstract. The article clarified the cause of the moment of rolling friction due to the impact of the rail on the wheel (or reaction in-ideal constant on the wheel) and identified a condition of the lack of wheel rolling along the rail. In her final analytical formulas and examples of calculations proved that in the areas of deceleration at the stations the brake position hump happen to slip of the wheels of the wheel pairs of the car. For this reason is erroneous determination of sliding speed of the car at the stations the brake position according to the formula of free fall of bodies, taking into account the inertia of the rotating parts.

1 Introduction

1.1 Relevance of the problem

In this article, as in [1-8], the theoretical provisions of the existing methods of hump yards [9-17] will be analytically evaluated regarding the possibility of rolling the wheels of the car wheelsets in the braking zones in the areas of braking positions. Mathematical expressions and calculation examples will prove that the rolling friction moment arises due to the impact of the rail on the wheel and / or the reaction of the non-ideal connection on the wheel.

1.2 Purpose of this article

Based on the principle of classical provisions of theoretical mechanics on the theory of sliding and rolling friction [18 - 20], try to explain in detail the reason for the rolling of wheels with sliding, if such movement is possible, and the pure sliding of the wheel along the rail threads in the zones of braking of the car in the areas of braking positions.

2 Task Formulation

Based on the provisions of the geometric statics of the wheel rolling of theoretical mechanics, to substantiate the possibility and / or impossibility of the car moving along the slope of the

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marshalling yard, in contrast to [16], with the rolling of the wheel with simultaneous sliding and pure sliding of the wheel relative to the rail.

3 Research results

VII.3. Reasoning about the analytical statics of wheel rolling without slipping, with slipping in the absence of friction force and the possibility of rolling wheels with simultaneous sliding on high-speed sections of the track profile

We give a mathematical proof of the impossibility of rolling wheels with slip in the absence of friction (with an ideal connection) and the possibility of rolling wheels with slip on high-speed sections of the track profile, as non-ideal connections, and slipping of wheel sets on sections of brake positions.

Then, comparing formula (24) with formula (15), we note that they contain a multiplier in parentheses that is identical in appearance and in physical meaning. Otherwise, we obtained a formula for determining the speed of movement when rolling the wheelsets of a car without slipping on a non-ideal (with friction) inclined plane.

Assumptions accepted. Let us assume that the gravity car ***G*** along the rail thread (as an inclined plane with a non-ideal surface) makes a plane-parallel motion. We assume that during the translational motion of the car, its wheelsets roll (and/or roll) without slipping under the action of the projection of gravity ***G*** on the axis $C_вx$ (i.e. **Gx** = G**cosψ**) along the rail threads so that the speed of its center v_C is not equal to zero , i.e. $v_C \neq 0$ (Fig. 1).

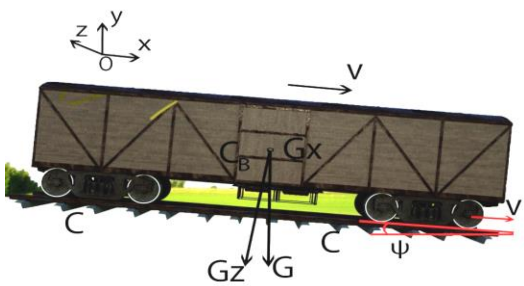

Fig. 1. Drawing for the movement of the car along the rail threads

On fig. 1 shows: ***G*** - gravity force of the wagon with cargo; ***Ox'y'z'*** - fixed coordinate systems, the origin of which is located on the conditional top of the hill; ***A*** and ***B*** are fixed points between which the car can move; $C_вxyz$ – moving coordinate systems located in the center of mass of the car ***Cv***; **P** - point of contact and / or contact of the wheelset with the rail; ***C*** - center of mass of wheelsets; S_{kb} - the center of the wheelbase of the car; ***v*** - is the portable translational speed of the car; v_C - is the speed of the center of inertia ***C*** of the wheel sets, and v_C = v; **τ - τ** - common tangent to the trajectory of the wheel, as a circle, and the rail; ***ψ*** - is the angle of inclination of the track profile.

We accept that the center of inertia of two wheelsets ***C*** is located in the center of the wheelbase of the S_{kb} car. We assume that the origin of the moving coordinate axes C_vxz are located at the center of inertia, which coincides with the center of mass of the car C_v. We neglect the wobble and lateral offset of the car wheelsets around the ***Cz*** axis and in the ***Cvxy*** plane, respectively. Let us consider the movement of wheelsets with a radius r in the plane Cwxz, as well as the influence on the movement of the car of any kind of resistance [5, 6, 20] (basic ωo, air and wind $\omega_{st} = w_{st}$, from arrows ωstr, from curves ω_{cr}, from snow frost ω_{sn}) ***Fc***. We assume that the car moves along the slope of the hill under the influence of the projection of the force of gravity ***Gx*** on the axis ***Cvx***, although, if necessary, the projection of the force of the tail and / or head wind of small magnitude ***Fv.x*** on the axis ***Cvx*** is not excluded [5, 6, 20].

Task formulation. It is required to find the acceleration of the center of inertia ***C*** of the wheel sets a_C (and, consequently, of the car ***a*** during its translational motion, and $a = a_C$) and write down the condition under which it is possible for the wheelsets of the car to roll without sliding, taking into account the rolling friction $F_{tr.k}$ of the wheelsets relative to the rail threads, as well as to determine the conditions under which the wheelsets begin to simultaneously roll and slide along the track profile, as an imperfect connection.

Construction of a mathematical model of the plane-parallel movement of the wheel sets of the car, as solid bodies (problem solution). In this article, we will show the solution of an engineering problem by compiling differential equations for the plane-parallel movement of car wheelsets, as solid bodies, and bringing the forces of inertia to the main vector and the main moment;

VII.3.1. The solution of the problem. Applying the principle of release from geometric statics constraints, we construct a design scheme for the movement of a car along a non-ideal inclined plane, and then, discarding non-ideal bonds (rail threads), we replace their influence with normal N_1, N_2 and tangents $F_{\tau 1}$ and $F_{\tau 2}$ components of the reaction of bonds (rail threads) R_1 and R_2 (Fig. 2).

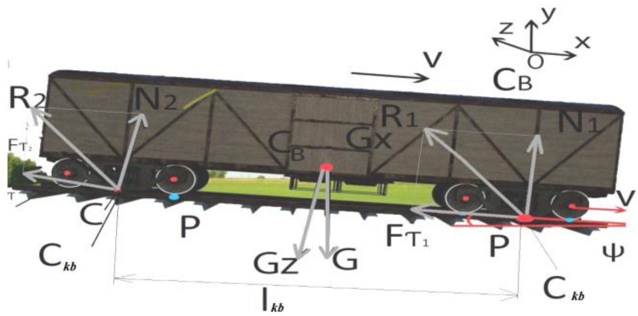

Fig. 2. Calculation scheme of the movement of the car along the track profile

On the pic. 2, the same designations are adopted as in Pic. 1, except for: N_1, N_2 and $F_{\tau 1}$, $F_{\tau 2}$ are the normal and tangential components of the reaction of the bonds R_1 and R_2, and $(N_1, F_{\text{tr}1}) \in R_1$ and $(N_2, F_{\text{tr}2}) \in R_2$ conditionally applied to the points C_b, coinciding with the centers of the wheelbase (KB) of the wagon l_{kb}.

Basic assumptions. Let us assume that the normal N and tangential $F\tau$ components of the constraint reaction R are conditionally applied in the center of the wheelbase of the S_{kb} car (see Pic. 2). In this case, the normal components of the N_1 and N_2 bond reactions are equal, respectively,

$$N_1 = N_2 = G_z/2 = G\cos\psi/2,$$

and the tangent components $F_{\tau 1}$ and $F_{\tau 2}$, according to the Coulomb law [35, 36, 38 – 41]:

$$F\tau 1 = Ftr1 \leq fN1 \text{ and } F\tau 2 = Ftr2 \leq fN2$$

taking into account the fact that

$f = 0.125$ – coefficient of sliding friction in motion between the rolling circles of wheelsets and the surfaces of rail threads $f = 0.25$, and according.

Otherwise, the normal component of the bond reaction is $N = 2N_1 = G\cos\psi$, and the tangential components are $F\tau = 2F_{tr} = 2fN = fG\cos\psi$.

Moreover, the friction force $F_\tau = F_{tr}$ for carriage wheels is directed opposite to the speed v_C of the center of inertia of the wheels C (moreover, $v_C \neq 0$) and the friction force F_{tr} is applied to the wheels at points P (see Fig. 1). It should be noted that when solving the problem of establishing wheel rolling without slipping, it is impossible to determine the friction force by the formula $F_{tr} = fG\cos\psi$, since this takes place in the case of slipping of the wheel contact point P along the rail threads. When the wheel is rolling without sliding, the friction force F_{tr} can be much less than $fG\cos\psi$, i.e. $F_{tr} \leq fG\cos\psi$.

We assume that the wheelsets of the car, when rolling without sliding, perform a plane-parallel motion along a non-ideal inclined plane (see Pic. 2).

Taking into account that the angular velocity ω = ω from the moving wheelset characterizes its relative motion, we can assume that around the axis Cy_1 (and/or around the point C) the car wheelset rotates with the angle of rotation φ (see Pic. 2 12 in [36 TrNTU №11 2018]).

It is well known [3, 2, 4] that the angular velocity ω of a wheelset in rotational motion:

$$\omega = \dot{\varphi}(t)$$

We will keep in mind that during the movement of the wheelset, with which the system of movable axles $Cx_1y_1z_1$ is rigidly connected, the distance between the axles of the wheelset and rail threads (see Pic. 2 12 in [36 TrNTU No. 11 2018]), with which, in turn, turn, the system of fixed axes $Ox'y'z'$ is rigidly connected, remains constant (i.e. r = const).

Solution methods. The movement of the wheelset of a car, according to the kinematics of a rigid body, can be decomposed into two movements (see pages 301 and 302 in, as the principle of independence of the translational movement of the body and its rotation in the case of plane-parallel movement:

firstly, to a portable translational movement together with translationally moving coordinate axes Cx_1, the beginning of which is located in the center of inertia C of the wheelset;

secondly, on the relative rotational movement around the axis Cy_1 passing through the center of inertia C.

In this regard, we write the differential equations of the plane-parallel motion of the wheel sets of the car, as solid bodies, in the form [6]:

$$\left.\begin{aligned} M\ddot{x}_C &= \sum_{k=1}^{n} F_{kx}^{e}\ ;\ _{\text{c}} \\ M\ddot{z}_C &= \sum_{k=1}^{n} F_{kz}^{e}; \\ J_C\ddot{\varphi} &= \sum_{k=1}^{n} m_C(F_k^{e}), \end{aligned}\right\} \tag{1}$$

where,

$\boldsymbol{M}$ – mass of wheel pairs of the wagon;

$\sum_{k=1}^{n} F_{kx}^{e}$ – the sum of the projection of external forces F_k^e on the moving axle Cx_1, in which k varies from 1 to n;

$\sum_{k=1}^{n} F_{kz}^{e}$ – the sum of the projection of external forces on the movable axis Cz_1;

J_C – the moment of inertia of the wheelset relative to the center of inertia C;

$\sum_{k=1}^{n} m_C(F_k^{e})$ – sum of moments of external forces F_k^e relative to the axis Cy_1, passing through the center of inertia C of the wheelset, in which k varies from *1* to n.

The first two equations in (1), according to the theorem on the movement of the center of inertia of a system of material points, written in projections on the coordinate axes Cx_1 and Cz_1 [2, 4, 7], describe the translational translational motion of wheel sets together with the translationally moving coordinate axes $Cx_1y_1z_1$, the beginning which is located in the center of inertia *C* of the wheelset.

The third equation of the system (44), as a mathematical notation of the theorem on the change in the main moment of the momentum of the system of material points in relative motion with respect to the center of inertia *C* in relation to the case of rotation of a rigid body around the movable axis Cy_1, which moves forward [12, 14, 15] , describes the relative rotational motion of the system of wheelsets with an angular velocity $\omega = \omega_{ot}$ around the movable axis Cy_1 passing through the center of inertia *C* of the wheelset.

In this case, we will take into account the possible local deformation of the wheels of the wheel sets of the car and the rolling surface of the rail threads. In this case, the contact between the wheel and the rail occurs not at one point *P* (see Fig. 12), but along a disproportionately small arc *PA*.

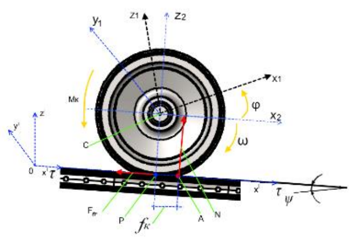

Fig. 3. Scheme of rolling wheelsets with sliding relative to rail threads, taking into account the moment of rolling friction

Designation in pic. 3 is the same as in Pic. 2 12 in [36 TrNTU No. 11 2018], except that in it: *A* and *P* are the points of application of the normal *N* and tangent $F\tau$ component of the reaction of bonds (rail threads) *R*; M_k – rolling friction moment; f_k is the coefficient of rolling resistance and/or rolling friction of wheelsets against rails (usually for steel wheels and steel rails f_k = 0.005·10-3 m.

In this case, the tangential component $F\tau$ of the reaction of the coupling (of a disproportionately small arc *PA*) *R* is the friction force F_{tr} and it is applied at point *P* of the MCV of the wheel, and the normal component *N* will be applied at point *A*, displaced relative to the center of inertia of the wheel *C* of the wheelsets at the length of the arc *PA* in side of the movement.

Therefore, it can be seen that the part of the gravity force of the car with the load $G\cos\psi$, applied to the center of mass *C* of the wheel, and the normal component *N* of the coupling reaction *R*, forms a pair of forces, called the moment of the couple of forces and / or the rolling friction moment M_k (see § 2.4 .2 in [7]). The shoulder of this moment is called the rolling resistance coefficient and/or the rolling friction coefficient f_k, which has the dimension of length (m) [4, 5].

Therefore, when compiling the differential equation for the plane motion of wheel sets, we will take into account the moment of rolling friction M_k in the number of external force effects on the wheel.

Proceeding from this, we rewrite the differential equations of the plane motion of wheelsets (i.e., translational motion with simultaneous rotational motion of the wheels around its own axis) of the car in the form (see pp. 264 – 266 in [5]):

$$\left.\begin{aligned} M\ddot{x}_C &= G\sin\psi - F_{\text{tr}}; \\ M\ddot{z}_C &= -G\cos\psi + N; \\ n_{\text{k}} J_C \ddot{\phi} &= -n_{\text{k}} M_{\text{k}} + F_{\text{tr}} r, \end{aligned}\right\} \tag{2}$$

where,
$F_{\text{tr}} \le fG\cos\psi$ – the friction force, which, when solving the problem of establishing the rolling of a wheel without slipping, remains an unknown value;
f – conditional coefficient of sliding friction of the contacting surfaces of wheelsets and rail threads, taking into account rolling friction in bearings:

$$f_{\text{pr}} = \frac{G_{01}\cos(\psi_{01})}{(G_{01}+G_{\text{kp}})\cos(\psi_{01})}\frac{r_{\text{p}}}{r} f \tag{3}$$

where,
$N = G\cos\psi$ – the normal component of the reaction of bonds (rail threads);

$n_{\text{к}} = 8$ – the number of wheels in the wagon bogies;

$J_C = G{\cdot}i_C^2/\text{g}$ or $J_C = G{\cdot}r^2/2\text{g}$ – the moment of inertia of the wheels of one wheel pair relative to the center of inertia C, provided that the wheel is considered to be a solid homogeneous disk with a radius r [46, 47], taking into account the fact that it contains:

$i_C^2 = r^2/2$ – the square of the radius of inertia of the wheel relative to its geometric axis Cy_1 (see Pic. 2);

$M_{\text{k}} = F_{\text{tr}} f_{\text{k}} = fG\cos\psi f_{\text{k}}$ – the moment of the rolling friction pair of one wheel pair [7] relative to the point A of the application of the normal component N of the reaction of ties (rail threads) (see Fig. 1);

f_{k} – coefficient of rolling friction of the wheelset on the rails, m (usually for hardened steel on hardened steel $f_k = 0.001$ m (see p. 42 [54 Ivanov P.S.])), equivalent to the arm of the rolling friction pair (see Fig. 2).

In the third equation of system (45), the negative sign of the rolling friction moment of the wheels $M_{tr.k}$ corresponds to the opposite direction of the angle of rotation φ of the movable axis Cx_1 relative to the other movable axis Cx_2, parallel to the axis Ox' of the fixed coordinate system $Ox'y'z'$ and rigidly connected to the conditional top of the hill (UVG) (see Fig. 3) in accordance with the rules of rotation of the unit radius vector of mathematics (see p. 179 [50 Bronshtein]).

Taking into account the accepted notation, we give the system of differential equations (45) the following form :

$$\left.\begin{aligned} \frac{G}{\text{g}}\ddot{x}_C &= G\sin\psi - F_{\text{tr}} \\ \frac{G}{\text{g}}\ddot{z}_C &= -G\cos\psi + G\cos\psi = 0 \\ n_{\text{k}}\frac{G}{\text{g}} i_C^2\ddot{\phi} &= F_{\text{tr}}(r - n_{\text{k}} f_{\text{k}}) \end{aligned}\right\} \tag{4}$$

Let's say that in system (4), due to the fact that the objectives of the study are to determine the condition of wheel rolling without slipping, then the friction force F_{tr} is the sought value due to the fact that for a non-ideal connection it is $Ftr \neq 0$.

Analyzing the system of equations (4), we note that, according to the first equation, in the absence of a sliding friction force in motion (i.e. $F_{tr} = 0$), which corresponds to the case of an ideal connection, i.e. when the wheels and rail threads are considered as absolute rigid bodies, it is possible for the wheel to slide along the rail, since at $t = 0$: $\dot{x}_C = v_{\text{н}} = v_0$.

During the time *t* seconds, sliding on an absolutely smooth plane (ideal connection), the center of inertia of the wheel C (see Fig. 3) would go the way:

$$x_C = v_{\mathrm{n}} t + \mathrm{g} \sin \psi t^2 \tag{5}$$

where,
t – current time.

Here is the acceleration of the center of inertia of the wheel ***C*** for a perfect connection:

$$a_C = \mathrm{g} \sin \psi = \mathrm{const}$$

In this case, i.e. at $F_{tr} = 0$, the rolling of the wheel along the rail is impossible, since from the third equation of system (5) at $t = 0$:

$$\varphi = 0 \text{ and } \dot{\varphi} = 0$$

i.e. the wheel does not rotate ($\omega = 0$).

Since the entire time of movement of the wheels $z_C = -r$ (as the distance between the movable axle Cx_2 and the fixed axle Ox') is constant, then, $\ddot{z}_C = 0$ therefore, from the second equation of system (47) we find: $N = G\cos\psi$.

Since in system (5) the friction force $F_{\mathrm{tr}} \neq 0$ and the wheelsets of the car roll without slipping and the speed of their center of mass $v_C = \omega r$ (see formula (41) in paragraph VII.2.1) is parallel to the Cx_2 axis (see Fig. 3), then we can write down the *condition of rolling wheels without slipping* (pure rolling of wheels) and in this form:

$$\dot{x}_C = \dot{\varphi} r \tag{6}$$

Taking the time derivative of $\dot{x}_C$, we have:

$$\ddot{x}_C = \ddot{\varphi} r \tag{7}$$

which corresponds to the formula for the tangent component $\boldsymbol{a_\tau}$ of linear acceleration as the point moves along the curve:

$$a_\tau = \varepsilon r \tag{8}$$

where,

$\boldsymbol{\varepsilon} = \ddot{\varphi}$ – angular acceleration [36].

From formula (7) we find the angular acceleration of the wheel:

$$\ddot{\phi} = \frac{\ddot{x}_C}{r} \tag{9}$$

Further, to determine the equation of motion of the center of mass *C* of wheelsets, it is necessary to integrate the first equation of system (1), where on the right side there is a sliding friction force in the movement F_{tr} of wheelsets on the rolling surface of rail threads.

As noted earlier, when solving the problem of establishing a wheel rolling without sliding, it is impossible to determine the friction force by the formula $F_{tr} = fG\cos\psi$, since this takes place in the case of sliding of the wheel contact point *P* along the rail threads. When the wheel is rolling without sliding, the friction force F_{tr} can be much less than $fGcos\psi$, i.e. $F_{tr} \leq fGcos\psi$.

Therefore, to exclude the friction force F_{tr} from it, we will keep in mind that such a force, according to the rules of the kinematics of the motion of a rigid body, is always applied to the wheels at the points P of their contact with the rail threads (see Pic. 2 12 in [36 TrNTU No. 11 2018]), coinciding with the MCC P_v, where the velocity $vP_v = 0$, and move along with them [3, 4]. Therefore, the speed of the center of mass C of the wheel, according to the formula: $v_C = r\omega$.

Taking into account formula (51a), we represent the third equation of system (4) as:

$$n_{\text{k}} \frac{G}{\text{g}} i_C^2 \frac{\ddot{x}_C}{r} = F_{\text{tr}}(r - n_{\text{k}} f_{\text{k}})$$

from here, after elementary transformations, we obtain a formula for determining the friction force when rolling without slipping,

$$F_{\text{tr}} = \frac{G}{\text{g}} \frac{n_{\text{k}} i_C^2}{(r - n_{\text{k}} f_{\text{k}}) r} \ddot{x}_C \tag{10}$$

Substituting equality (9) into the first equation of system (4), we will have:

$$\frac{G}{\text{g}} \ddot{x}_C = G \sin\psi - \frac{G}{\text{g}} \frac{n_{\text{k}} i_C^2}{(r - n_{\text{k}} f_{\text{k}}) r} \ddot{x}_C$$

or

$$\frac{G}{\text{g}} \left(1 + \frac{n_{\text{k}} i_C^2}{(r - n_{\text{k}} f_{\text{k}}) r} \right) \ddot{x}_C = G \sin\psi$$

From here, after elementary mathematical calculations, we obtain a generalized mathematical expression for determining the linear acceleration of the center of mass C of the wheels of the car and / or the center of mass of the car C_v (see Pic. 2) in the final form:

$$\ddot{x}_C = \frac{\text{g} \sin\psi}{1 + \dfrac{n_{\text{k}} i_C^2}{(r - n_k f_{\text{k}}) r}} \tag{11}$$

If we take into account that $i_C^2 = r^2/2$ is the square of the radius of gyration of the wheels, as a solid homogeneous disk with radius r [36, 46, 47], and $n_k = 8$ is the number of wheels in the car bogies, the last formula can be given the form:

$$\ddot{x}_C = \frac{\text{g} \sin\psi}{1 + \dfrac{4}{1 - 8\dfrac{f_{\text{k}}}{r}}} \tag{12}$$

In a particular case, when the rolling friction moment of the wheels $M_{tr} = 0$ is not taken into account, i.e. at $f_\kappa = 0$ (see Pic. 3), the last mathematical expression will take the form:

$$\ddot{x}_C = \frac{g \sin\psi}{1 + \frac{n_k i_C^2}{r^2}},$$

or, considering that $i_C^2 = r^2/2$,

$$\ddot{x}_C = \frac{g \sin\psi}{1 + \frac{n_k}{2}} \tag{13}$$

Keeping in mind that $\ddot{x}_C = a_C$ is the acceleration of the center of mass ***C*** of the wheels, provided that the movement of this center is given in the form: $x_C = at^2/2$, and $n_k = 8$ is the number of wheels in the car bogies, we will give the last expression the form of a formula:

$$a_C = 0.25 g \sin\psi \tag{14}$$

or, considering that for small angles $\sin\psi \approx \psi = i$:

$$a_C = 0.25 g i \tag{15}$$

From here it becomes obvious that the acceleration of the center of mass *C* of the wheelsets when the wheels roll without slipping relative to the rails (*non-ideal connection*) is 4 times less than the projections of linear acceleration on an *ideal* inclined plane (see formulas (6) and (7) in paragraphs. I in.

In another particular case, when instead of wheels rolling without slipping, there is a *pure sliding* of wheel sets relative to the rail threads (if they are taken as ideal connections), when the moment of inertia of the rotating parts J_C is not taken into account (i.e. $J_C = 0$, and, therefore, $iC^2 = 0$), the mathematical expression (13) will take the form:

$$\ddot{x}_C = a_C = g \sin\psi \tag{16}$$

or, in the conventional notation and customary understanding,

$$a_C = g i \tag{17}$$

taking into account the fact that in it i is the slope of the track profile, ‰.

In this case, formulas (8) and (9) coincide with the projection of linear acceleration onto an ideal inclined plane (see formulas (6) and (7) in paragraph I in [36 BTI No. 9 2018]).

Formulas (15) and (17), obtained for particular cases, *confirm the correctness of the derivation of the generalized mathematical expression* for the acceleration of the center of mass *C* of the car wheelsets (53) (i.e. $\ddot{x}_C = a_C$).

In turn, this confirms the undeniably gross fallacy and / or inadmissibility of using the formulas derived for an ideal (*where the bonds are non-ideal* (*non-smooth and/or with friction*)) formulas derived *for an ideal* (smooth and/or friction-free) bond (see formulas (6) and (7) in item I in [36 BTI No. 9 2018]).

Otherwise, this confirms the incompatibility of formula (8) and (12) and/or (1).

The generalized mathematical expression for the acceleration of the center of mass *C* of the wheel sets of the car (53) allows, on the basis of formula (9), to determine the friction force in the movement F_{tr} of the wheel sets on the rolling surface of the rail threads when the wheel rolls without slipping in the form:

$$F_{\text{tr}} = G\sin\psi\left(1-\frac{1}{1+\frac{n_{\text{k}}i_C^2}{(r-n_{\text{k}}f_{\text{k}})r}}\right) \tag{18}$$

Such a friction force must act on the wheel of the wheel pairs of the car in order for it to roll without sliding along the rail threads (see p. 330 in [46 Targ]). It was previously indicated that the friction force $F_{tr} \le fN = fG\cos\psi$ (see non-strict inequality (34)). From this it is clear that by substituting expression (9) into the last inequality, we can find the condition for determining the friction coefficient in the form:

$$f \ge \frac{F_{\text{tr}}}{G\cos\psi} \tag{19}$$

If the latter condition is met, pure rolling will occur.

If condition (19) is not met, then the friction force Ftr cannot take the value determined by formula (18). In this case, the wheel will roll with slippage relative to the rail threads.

It is well known (see p. 330 in [46 Targ]) that in this case v_C and ω are not connected $v_C = \omega r$, since in this case the point of contact P of the wheel with the rail threads is not the instantaneous center of velocities (MCV). At the same time, the value of F_{tr} has a limiting value, i.e. $F_{tr} = fN = fG\cos\psi$. Therefore, the first and third equations of system (4) after simple transformations will take the form (see formula (20)):

$$a_C = \text{g}(\sin\psi - f\cos\psi) \tag{20}$$

$$\ddot{\phi} = \frac{1}{n_k i_C^2}\text{g}f\cos\psi(r\text{-}n_{\text{k}}f_{\text{k}}) \tag{21}$$

The center of mass C of the wheel in this case will move with acceleration a_C, and the wheel itself will rotate with angular acceleration. In this case, equality (7) and/or (8) will also be satisfied in the form $\ddot{x}_C = \ddot{\varphi}r$ or $a_C = \varepsilon r$.

The condition of wheel rolling without sliding (see formulas (43) and (49)) is satisfied only when, according to the Coulomb law [16, 17, 20], the friction force $F_{\text{tr}} \le fN = fG\cos\psi$ (see non-strict inequality (33)), appearing at the point of contact P of its contact with the rail (see Pic. 2), coinciding with the MCS P_v, where the speed $vP_v = 0$, moves with it [16, 18,19,20]:

$$G\sin\psi\left(1-\frac{1}{1+\frac{n_{\text{k}}i_C^2}{(r-n_{\text{k}}f_{\text{k}})r}}\right) \le fG\cos\psi$$

or, after simple transformations,

$$\mathrm{tg}\psi \le f\frac{1}{\left(1-\dfrac{1}{1+\dfrac{n_k i_C^2}{(r-n_k f_k)r}}\right)} \tag{22}$$

Note that condition (22) roughly characterizes the material and the physical state of the rolling surfaces of wheels and rails.

If condition (22) is satisfied, then this means that in the process of rolling the car down the slope of the hill, deformation of the contacting surfaces of the wheels and rail threads occurs, and for this reason, a pair of forces (G_{zC}, N) arises (see Pic. 1 in [36 TrNTU No. 11 2018]).) and/or the moment of this pair of forces M_k (see Pic. 3, which prevents the wheels from rolling without slipping.

In a particular case, when the rolling friction moment of the wheels M_{tr} = 0 is not taken into account, i.e. when f_k = 0, from the last formula, you can get the condition:

$$\mathrm{tg}\psi \le f\frac{1}{\left(1-\dfrac{1}{1+\dfrac{n_k}{2}}\right)} \tag{23}$$

With regard to the wheels of the bogies of a four-axle car (n_k = 8), we rewrite (23):

$$\mathrm{tg}\psi \le 1.25 f\,. \tag{24}$$

Subject to non-strict inequality (21) and/or (22) and/or (23), the wheel will roll along the rail without slipping, i.e. there will be a pure sliding of the wheel along the rail (see p. 534 in [36], p. 330 in [5]).

Subject to non-strict inequality (21) and/or (22) and/or (23), the wheel will roll along the rail without slipping, i.e. there will be a pure sliding of the wheel along the rail (see p. 534 in [36], p. 330 in [5 Targ]).

The condition of wheel rolling without sliding in the form (61) can also be represented in the following form (see p. 330 in [5 Targ]):

$$f \ge \mathrm{tg}\psi\left(1-\frac{1}{1+\dfrac{n_k i_C^2}{(r-n_k f_k)r}}\right) \tag{25}$$

If the path profile angle ψ does not satisfy the nonstrict inequality (24), *then the wheel will roll, sliding along the rail* (see p. 265 in [35], p. 409 in [38]). In this case, there is no geometric relationship between the acceleration $\ddot{x}_C = a_C$ of the center of inertia of the wheels C and its angular acceleration $\ddot{\varphi}$ = εC. These two accelerations are obtained by substituting the condition $F_{tr} \le fN = fG\cos\psi$ (see non-strict inequality (34)) into the first and *last equations of system* (4).

So, for example, from the first and second equations of system (2) we obtain the conditions under which rolling occurs with simultaneous sliding, *which follow from the first and third equations of system* (4):

$$\ddot{x}_C \geq a_C \tag{26}$$

where,
a_C – acceleration of the motion of a body along a non-ideal inclined plane (see the formula (20)):

$$a_C = \mathrm{g}(\sin\psi - f\cos\psi)$$

$$\ddot{\phi} \leq \frac{2(r - n_\mathrm{k} f_\mathrm{k})}{n_\mathrm{k} r^2}\mathrm{g}f\cos\psi \tag{27}$$

As can be seen, the center of inertia of the wheels C in this case will move with acceleration a_C, and the wheels of the wheelsets will rotate with angular accelerations $\ddot{\varphi}$ = ε, determined by relations (21) and (23).

Let us rewrite non-strict inequalities (64) and (65) as equalities:

$$\ddot{x}_C = \mathrm{g}(\sin\psi - f\cos\psi) \text{ and } \ddot{\phi} = \frac{2(r - n_\mathrm{k} f_\mathrm{k})}{n_\mathrm{k} r^2}\mathrm{g}f\cos\psi$$

or

$$\ddot{x}_C = a_C \text{ and } \ddot{\phi} = \varepsilon \tag{28}$$

where,
a_C – acceleration of the center of inertia C of the wheel, determined by the first formula (28);
$\ddot{\varphi} = \varepsilon$ – angular acceleration of the center of inertia C of the wheel:

$$\varepsilon = \frac{2(r - n_\mathrm{k} f_\mathrm{k})}{n_\mathrm{k} r^2}\mathrm{g}f\cos\psi \tag{29}$$

Integrating each of equations (28), we obtain:

$$\dot{x}_C = a_C \cdot t \text{ and } \dot{\phi} = \omega = \varepsilon \cdot t \tag{30}$$

The difference between $\dot{x}_C$ and $\dot{\varphi}r = \omega r$ (see formula (49)) is the sliding speed of the point of contact P of the wheel with the rail:

$$v_x = \dot{x}_C - \dot{\varphi}r = \dot{x}_C - v_{rC} \tag{31}$$

where,

$$v_{rC} = \dot{\phi}r = \omega r \tag{32}$$

Differentiating equation (30) taking into account formula (17), after elementary mathematical calculations, we obtain:

$$\dot{v}_x = \ddot{x}_C - \ddot{\varphi}r = \mathrm{g}\cos\psi((\mathrm{tg}\,\psi - \mathrm{tg}\,\psi_0) \tag{33}$$

where,

$$\mathrm{tg}\ \psi_0 = \left(1 + \frac{2(r - n_\mathrm{k} f_\mathrm{k})}{n_k r}\right) f \qquad (34)$$

Hence, it is clear that, under the condition $\psi > \psi_0$, the value in the brackets of formula (34) is positive and the sliding speed will increase as the wheel moves along the slope of the hill profile (see p. 264 in [43]).

In a particular case, when the rolling friction moment of the wheels $M_{tr} = 0$ is not taken into account, i.e. at $f_k = 0$, from formula (43) either the following condition is obtained:

$$\ddot{\phi} \le \frac{2}{n_\mathrm{k} r} \mathrm{g} f \cos\psi \qquad (35)$$

or condition

$$\ddot{x}_C \le \frac{2}{n_\mathrm{k}} \mathrm{g} f \cos\psi \qquad (36)$$

As applied to the wheels of the bogies of a four-axle car ($n_k = 8$), we rewrite inequalities (35) and (36):

$$\ddot{\phi} \le \frac{0.25}{r} \mathrm{g} f \cos\psi \qquad (37)$$

$$\ddot{x}_C \le 0.25 \mathrm{g} f \cos\psi \qquad (38)$$

Calculation example VII.2. For example, let's study the movement of the car on the section of the third (C3) switch zone (SZ) of the downhill part of the hill. The initial data of the calculation example are the same as in example VII.1: $G = 908$ is the gravity force of a wagon with a load, kN; $f = 0.175$ is the sliding friction coefficient of metal on metal (according to p. 65 in [5]: $f = 0.15 ... 0.25$); $f_k = 0.001$ – coefficient of rolling friction of hardened steel on hardened steel (see p. 42 in [4]); $r = 0.475$ is the radius along the wheel rolling circle, m ; $n_k = 8$ - the number of wheels of the wheel pair of the car, pcs; $\psi_{6c3} = 0.002$ – slope angle of the NW slide, rad.; $l_{6c3} = 24.0$ is the length of the NW section of the slide, m, the time of movement of the car on the investigated section of the slide: $t_{6c3} = 7.934$ s.

Calculation results [53]. 1) Calculate the value of the square of the radius of inertia of the wheel relative to its geometric axis Cy_1 (see Fig. 2) $(i_C)^2$, m^2:

$$(i_C)^2 = r^2/2 = 0.475^2/2 = 0.113.$$

2) Let us calculate the linear acceleration $\ddot{x}_C = a_C$ of the center of mass C of the car wheels and/or the center of mass of the car C_v (see Fig. 2) using formula (53), m/s^2:

$$\ddot{x}_C = \frac{\mathrm{g}\sin\psi}{1 + \dfrac{n_\mathrm{k} i_C^2}{(r - n_\mathrm{k} f_\mathrm{k}) r}} = \frac{9.81 \cdot 0.002}{1 + \dfrac{8 \cdot 0.113}{(0.475 - 8 \cdot 0.001) \cdot 0.475}} = 3.871 \cdot 10^{-3}$$

or considering that $\ddot{x}_C = a_C$, $a_C \approx 0.004$ m/s^2.

In the particular case when $J_C = 0$ (and, therefore, $i_C^2 = 0$), the value $a_{Cnd.}$, calculated by formula (15) and/or (1), describing the movement of the car along an ideal inclined plane (coupling), m/s2:

$$a_{Sid.} = \mathrm{g}\sin\psi_{6c3} = 9.81 \cdot 0.002 \approx 0.02$$

which is 5 times higher than the value $a_C \approx 0.004$ m/s2 - for a non-ideal inclined plane (bond).

This means that if the car moves along an ideal inclined plane (coupling) (see Pic. 3), then there is a *pure sliding* of the wheelsets relative to the rails, which always corresponds to the condition (see Table 1 in [3]):

$$a_{sid} >> a_C \text{ and/or } \mathrm{g}\sin\psi >> a$$

Namely, this is what we especially draw the attention of the authors of the article [3], who stubbornly defend the correctness of the application of the formula for the velocity of free fall of bodies, taking into account the inertia of the rotating parts $v = \sqrt{2\mathrm{g}'h}$ (see formula (1) in [12]), derived for ideal constraints, for the case of non-ideal constraints , i.e. between the wheels of the car and rail threads.

3) Calculate the linear acceleration $a_{6c3} = a_C$ of the center of mass C of the wheel according to non-strict inequality (64) (see the first formula (67)), m/s^2:

$$a_C = \mathrm{g}(\sin\psi - f\cos\psi) = 9.81(0.002 - 0.175 \cdot 1) \approx -1.7 = |1.7|$$

Note that the negative sign of the acceleration of the center of mass C of the wheel means that the wheel, and, consequently, the car, moves uniformly, which is not typical for the switch zone when taking into account the effect of a tailwind of small magnitude ($F_{вx} \approx 3.2$ kN).

Comparing the results of calculating the acceleration of the center of mass C of the wheels, we make sure that the non-strict inequality (65) in the form of $\ddot{x}_C \geq a_C$ is observed, since $0.004 \geq |1.7|$.

Therefore, in accordance with condition (64), when the car moves along the slope of the marshalling yard, it is *as if the wheels are rolling with simultaneous sliding*.

4) Let's calculate the parameters characterizing the material and the physical state of the rolling surfaces of wheels and rails, according to the condition (61):

$$\mathrm{tg}\psi \leq f \frac{1}{\left(1 - \dfrac{1}{1 + \dfrac{n_\mathrm{k} i_C^2}{(r - n_\mathrm{k} f_\mathrm{k})r}}\right)} = 0.175 \frac{1}{\left(1 - \dfrac{1}{1 + \dfrac{8 \cdot 0.113}{(0.475 - 8 \cdot 0.001)0.475}}\right)} = 0.232$$

or, since $tg\psi_{6c3} = 0.002$, then $0.002 \leq 0.232$, i.e. the nonstrict inequality (61) is satisfied. In this case, the motion will be uniformly slowed down.

Therefore, when the car moves along the slope of the marshalling yard, given the initial data of the calculation example according to condition (61), *the wheel will roll along the rail without slipping*, i.e. there will be a pure rolling of the wheel on the rail.

When a wheel rolls along a rail, according to formulas (43) and/or (49) and (51), the angular velocity ω_{6c3} and angular acceleration ε_{6c3} are respectively equal to:

$$\omega_{6c3} = v_{6c3}/r = 3.154/0.475 = 6.64 \text{ rad./s;}$$

$$\varepsilon_{6c3} = a_{6c3}/r = 0.032/0.475 = 0.068 \approx 0.07 \text{ rad/s}^2.$$

5) Check whether the non-strict inequality (65) is satisfied:

$$\ddot{\phi} \leq \frac{2(r - n_\text{k} f_\text{k})}{n_\text{k} r^2} gf \cos\psi = \frac{2(0,475 - 8 \cdot 0.001_\text{k})}{8 \cdot 0.475^2} 9.81 \cdot 0.175 \cdot 1 = 0.888$$

or considering that $\ddot{\varphi} = \varepsilon_6$, $\varepsilon_6 = 0.888$ rad/s^2.

Comparing the result $\ddot{\varphi} = \varepsilon_6 = 0.888 \approx 0.9$ rad/s^2 with data according to formula (51): $\varepsilon_{6c3} \approx 0.07$ rad/s^2, we make sure that the non-strict inequality (65) does not hold, since $\varepsilon_{6c3} < \varepsilon_6$, i.e., 0.07 < 0.9.

This, in turn, once again confirms the possibility of *rolling wheels without slipping* on the rolling surfaces of rail threads when the *car moves along the slope* of the marshalling yard.

6) Let's check the possibility of wheel rolling with simultaneous sliding according to formulas (66) - (70).

According to the first formula (67), the angular acceleration of the wheel, rad/s^2:

$$\varepsilon_{6c3} = \frac{2(r - n_\text{k} f_\text{k})}{n_\text{k} r^2} gf \cos\psi = \frac{2(0.475 - 8 \cdot 0.001)}{8 \cdot 0.475^2} 9.81 \cdot 0.175 \cdot 1 = 0.888$$

Let us calculate the speed of the wheel sliding along the rail according to the first formula (68), taking into account the fact that $a_{6c3} = |1.7|$, m/s:

$$v_{6c3} = a_{6c3} \cdot t_{6c3} = -1.7 \cdot 7.934 = -13.46$$

According to the second formula (68), the angular velocity of the wheel, rad/s:

$$\dot{\phi} = \omega_{6c3} = \varepsilon_{6c3} \cdot t = 0.888 \cdot 7.934 \approx 7.05$$

Let us calculate the speed of the center of mass *C* of the wheel using the formula (70), m/s:

$$v_{6c3} = \omega_{6c3} \cdot r = 7.05 \cdot 0.475 \approx 3.35$$

Let us produce the sliding speed $v_x = v_{xP}$ of the point of contact *P* of the wheel with the rail according to the formula (69), m/s:

$$v_x = \dot{x}_C - v_{rC} = -13.464 - 3.35 \approx -16.8$$

Analyzing the results of calculating the sliding speed $v_x = v_{xP}$ of the point of contact *P* of the wheel with the rail, we make sure that the sign of the speed turned out to be negative, which means that the wheel cannot roll with simultaneous sliding.

Therefore, in accordance with condition (19), when the car moves along the slope of the marshalling yard, it is as *if the wheels are rolling with simultaneous sliding*.

Summarizing the results of calculations, it can be argued that when the car moves along the downhill part of the hill, the conditions for *pure rolling of the wheels of the wheel pairs of the car on the rolling surfaces of rail threads* are observed, which is true and does not contradict the theory of rolling wheels of engineering mechanics.

Thus, the results of the studies performed on the basis of the provisions of geometric and analytical statics, supported by calculated data, thoroughly made it possible to prove that on the high-speed sections of the downhill part of the marshalling yard, the movement of wheelsets of the car occurs *with the rolling of the wheels on the rolling surfaces of the rail threads*.

4 Conclusion

1. Taking into account that the existing methods of hump calculations [4 - 9] are developed on the basis of a "deep" theoretical understanding of the results of extensive field experimental

studies to determine the specific resistance to movement, as non-ideal connections, and are empirical in nature.

2. A thorough critical analysis of the content of existing methods for calculating hump yards [2, 4 - 9] made it possible to identify the following undeniable assumptions and incorrectness, which, in our opinion, is unacceptable. So, for example, the theoretical provisions of the existing methodology for hump structural and technological calculations are based on the concepts of connections (ideal and non-ideal) known in theoretical and engineering mechanics [3–4]. However, these physically incompatible concepts are widely used, for example, in the same formula for determining the estimated height of the slides and the speed of rolling the car, both in high-speed sections and in the braking positions of the marshalling yard, which is fundamentally erroneous and unacceptable.

Otherwise, it is erroneous and / or unacceptable to use the formulas derived *for an ideal connection* when solving engineering problems of transport science (*where the connections are not ideal*) (see formulas (6) and (7) in paragraph I).

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