

Investigation of the problem of multifunctional work of a bulldozer-loader by reducing the mathematical model to pairwise nested numerical methods

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Abstract. The article attempts to substantiate the urgency of the developed bulldozer-loader with the facts of the consequences of emergencies associated with the objects of work, describes the technical, economic and overview characteristics of existing bulldozer-loaders. For a previously compiled mathematical model with a system in conveniently simplified quasilinear ordinary differential equations, the motions of the centers of mass of the working bodies with generalized coordinates and generalized velocities are described. The force fields of motion drives and equilibrium force fields are recommended and calculated. A numerical method of shooting was used, a general algorithm and a program for solving the problem were compiled, which were implemented. The results of the calculation were obtained, showing the tendencies of the correctness of the algorithms.

1 Introduction

Practice shows that with the phased development of modern technologies for the development and creation of new, more profitable road-building machines (RDM) and their implementation, the interests for the development of even improved versions of them are also growing. In the mountainous road conditions of the Kyrgyz Republic (KR) and other countries, the need for improved multifunctional RDM is only growing. From time to time, almost periodically occurred incidents of adverse situations in the mountainous and foothill conditions of the Kyrgyz Republic, convinces the relevance of the development and creation of a bulldozer-loader with transforming multi-purpose working bodies. Here are some of the many cases of the latest media data: Catastrophic landslide in the Kara-Keche gorge on 09/15/2020 (Fig. 1).

During the day, road builders eliminate the risks of snow mass descent. Bishkek, February 20.02.23. Sputnik. On the Bishkek-Naryn-Torugart highway in the Naryn region, restrictions were imposed on the passage of cars, the presidential plenipotentiary office in the region reported. Now the road is open, tomorrow it will be blocked from 9:00 to 18:00.

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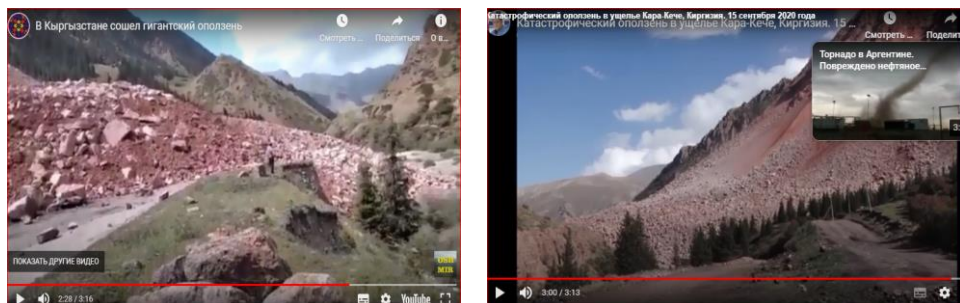


Fig. 1. Landslide in the Kara-Keche gorge. On the road section of the gorge.

The authorities of the Naryn region reported that shelling was carried out in dangerous areas on February 18-20. Forcibly lowered 1.2 million cubic meters of snow mass. More than 25 thousand cubic meters blocked the road, but this area was quickly cleared (Fig. 2).

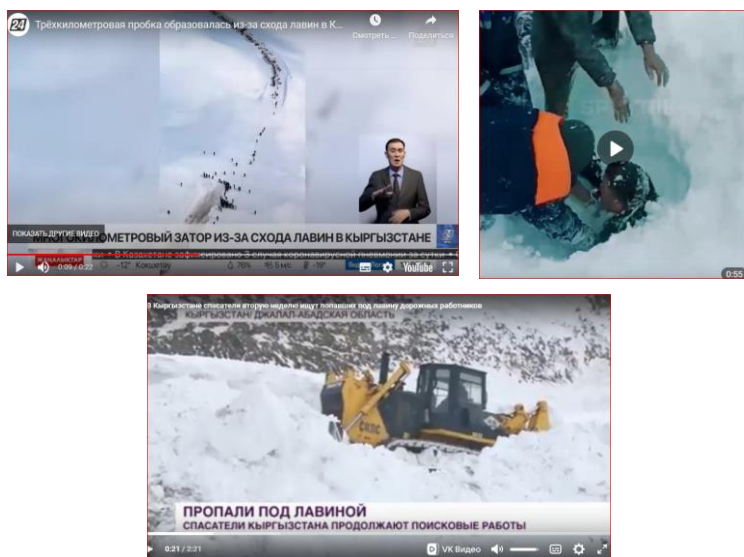


Fig. 2. On the section of the Bishkek-Torugart highway. Timely excavation of the victims.

In the Jalal-Abad region, an avalanche descended on the hostel of the gold mining company "Full Gold Mining". According to the Ministry of Emergency Situations, under the snow mass were four people - employees of the company. The snow mass covered four people, they were rescued (Fig. 2). All the above examples show that for cleaning from avalanches, snow cover or snow drift, in addition to filling the roadway with reagents, at this stage of existing structures and means, several machines will be required, since these phenomena occur at the pass sections at any time year, and with great frequency. At the same time, failure to perform one of the types of work can lead to a halt in several other types of work and the formation of traffic jams and congestion on mountain roads, and to unforeseen consequences. One of the main goals of further research is to increase the possibilities and advantages of the bulldozer-loader being developed with transforming multi-purpose working bodies, its place and role in the environment of other technical means for performing work both in parallel and in complementary roles in one whole joint work. We are talking about the task of determining the most profitable option for selecting, accessing, manufacturing or acquiring components and improving operating conditions in order to save

fuel and lubricants, the cost of driver services and other criteria. Familiarization with review materials, with an Internet source shows that the transition to a more rigorous formulation of the optimization problem is, for the time being, the implementation of preparatory work related to the stage of studying the details of this problem. In the conditions of the Kyrgyz Republic and other countries, the reasons for the non-final approach to the study are: the multifactorial choice of criteria and parameters of the target optimization function due to the relatively little study of various types of possible accidents on road sections; insufficient availability of machine and tractor parks in nearby settlements; unpredictability of the nature and scope of work; uncertainty about how to organize and conditions for access to assistance, and so on. Therefore, a detailed study of those factors that can be relatively significant is still required, which could become a prerequisite for the further stage of a clear formulation of the optimization problem, about the benefits in the operation of the RDM [1, 5].

2 Material and methods

When developing and creating new working equipment, the following issues and factors that directly affect the vital indicators of ensuring the safety and throughput of mountain roads were considered:







- features of construction, repair and maintenance of roads, taking into account the terrain;
- the height of the location of roads above sea level;
- climatic factors affecting the condition of roads: the frequency of manifestation of mudflows,
- landslides, thickness of snow cover, avalanches and others (Fig. 1; 2);
- analysis of methods of construction, repair and maintenance of roads and their effectiveness;
- analysis of the technological process of work in high-altitude conditions;
- analysis of existing structures and equipment and their functionality.

Construction equipment consists of an internal combustion engine, transmission, running gear or running system, working equipment. Their parameters include: mass; dimensions; working and transport speeds; technical characteristics of working bodies. The operating mode of the RDM is characterized by higher total resistances, low speeds, and traction qualities are most fully used in it.

An increase in the production of single-bucket loaders with various types of interchangeable equipment allows the industry to release many excavators, cranes and bulldozers employed in these works. The main parameter of single-bucket loaders is the nominal load capacity, which must be ensured when the machine is moving. The main parameter of single-bucket loaders is the nominal load capacity, which must be ensured when the machine is moving. According to this parameter, they are divided into light loaders (up to 0.5 t), light (0.6 ... 2.0 t), medium (2.1 ... 4.0 t), heavy (4 ,1 ... 10 t) and heavy-duty (over 10 t). Modern loaders have a nominal load capacity of up to 15 tons.

Some prerequisites of influencing factors can be identified. Some types of RDM can be cited as an example, which can be found on their websites. The Kyrgyz Republic has more opportunities to purchase (import) RDM from other developed countries of the world. Despite the fact that in the Kyrgyz Republic there are no relevant RDM manufacturing plants, there is a certain circle of specialists who are actively working on theoretical research on the development and creation of RDM, designed for proposals and recommendations for submitting an order for the creation of a prototype for their joint creation in the countries of the Commonwealth of Independent states (CIS) [1, 7].

Table 1. Bulldozers-loaders of different countries with photos and technical data

Names of backhoe loader	Type of front loader	Weight (T.)	Bucket capacity	Cylinder displacement	price	Maximum travel speed
Backhoe loader Hidromek HMK 102S		9.5	2.6	4.4 litre	5 mln, rub	40 km/h
front loader XGMA XG 955		17	3	6 litre	4.6 mln, rub	38 km/h
backhoe loader Катерпильер 444		8.81	3.997	4.4 litre	5.5 mln, rub	40 km/h
front loader Hitachi ZW310		22.32	4.1	12.8 litre	8.4 mln, rub	34.5 km/h
front loader New Holland W170		14	1.26	6.8 litre	3.9 mln, rub	38 km/h
backhoe loader XCMG WZ30-25		9.5 T	1 m ³	8.8 litre	4.8 mln, rub	33 km/h

Imported products are consistently in high demand, despite the fact that their cost is disproportionately higher. The consumer has learned to count money and makes a choice in favor of an initially more expensive, but economical machine during operation. In the Kyrgyz Republic, there are still feasible opportunities to purchase bulldozers-loaders, where in terms of access to them it is observed. The Kyrgyz Republic has the opportunity to trade with many countries. Based on reliable advertising sources, some of the data in table 1 of the purchase of bulldozers-loaders were advertised.

Not only new, but also used cars are imported from abroad. The market of secondary imported equipment has significantly revived in recent years. Currently, about a dozen large and medium-sized players and about the same number of small intermediaries are actively involved in the resale of used equipment after major repairs carried out by larger repair and service enterprises. Reasons: with the help of used equipment, you can start your business faster; it is much cheaper. And very often firms, especially in developing and economically relatively weak states, use used cars.

In [2], the idea of creating a new direction for the development of road construction engineering, patented by Kyrgyzpatent, the Eurasian Patent Office as a transforming working body of a bulldozer-loader, is presented. The recommended design of the working body based on a single-bucket front loader can operate, depending on the technological process, in the bulldozer mode and in the loader mode without mechanical replacement of one working body with another. Its advantage is the ability to turn from one type of working body to another by manipulating from the driver's cab with the help of control levers. At the same time, after the transition from one type of function to another, it reliably performs the function of another body, since the working body becomes different without changing its position on different planes due to the transformation of nodes and parts. Approximate types of common working equipment of a bulldozer-loader of road construction, loading and unloading and municipal machines are shown in fig. 3.

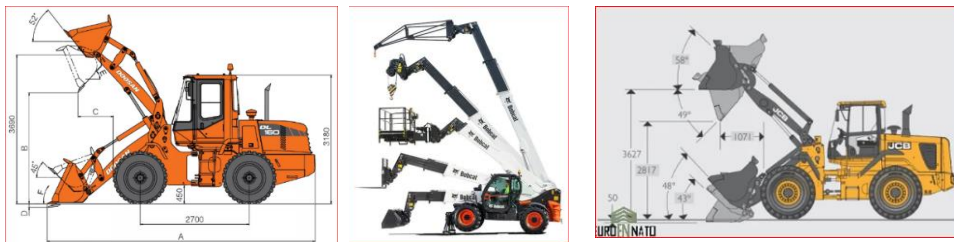


Fig. 3. Bulldozers-loaders with transforming working equipment.

To develop and create multi-purpose working equipment of a bulldozer-loader with a transforming working body, it is necessary to conduct a theoretical study to increase its functionality in the following modes: a) traditional bulldozer; b) single-bucket loader; c) advancement forward by telescopic pushing bars; d) turning the retractable parts of the telescopic pushing bars; e) regulation of the blade angle of the bucket loader (Fig. 4, 5).

Since the process of performing work is non-stationary, that is, it varies over time, the sequence of work of mechanisms can be organized in different modes.

An attempt to compile a mathematical model with a system of second-order ordinary differential equations in a more convenient, simplified, but nonlinear form, without much neglect of some of their nonlinear terms, can be achieved by choosing polar coordinate systems, where the main unknown variables - parameters consist of changes in angular values. Accordingly, we will introduce three angular generalized coordinates: $\varphi(t)$; $\theta(t)$; $\beta(t)$, one generalized linear $r(t)$.

The first stage is movement during the entire cycle of work of the DSM in the mode of a bulldozer or freewheel, usually in a horizontal direction (Fig. 4 and 5), where the choice of the Cartesian coordinate system Oxy (or Axy) at the origin O (or A) is convenient the fact that all weight forces and gravity forces \vec{G}_3 , \vec{G}_2 , \vec{G}_1 are directed along the Oy axis.

If during the initial period of time t_A-t_0 the base machine passes the distance x_A-x_0 in the direction of the Ox axis, collecting soil into the bucket, then at $\varphi_0 + \beta_0 = 0$, $x = x(t)$ can be chosen for the first generalized coordinate; for the generalized velocity $v_x(t) = \dot{x}(t)$.

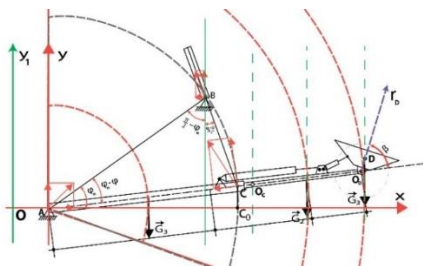


Fig. 4. Scheme for the first two stages of work

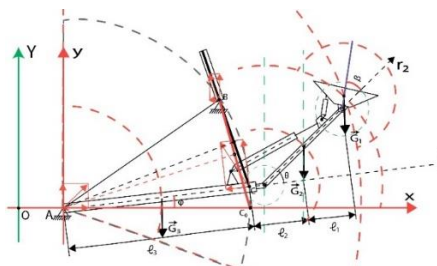


Fig. 5. Scheme of the first - third stages of work.

At the first stage, the portable movement of all links is performed for the period t_A-t_0 . For the time being, the study of the first stage of the work will be limited by reference to previously known results, if necessary, while reducing the number of degrees of freedom by one. We will also assume that the influence of the strokes of the working movement of the RDM on the work of the movement of the working bodies will be considered auxiliary and compensating when filling the bucket. At the initial stage of the study, for simplicity, the process of filling the bucket, taking into account the resistance of a heap of object cargo, is

commensurate with the weight of the final filling. Therefore, when compiling systems of equations for the movement of working bodies, we cannot take into account the portable movement of the free-wheeling RDM of the entire working cycle. In most cases, a change in the angle $\varphi(t)$ caused by the action of the lifting and lowering hydraulic cylinders fixed at point B, can occur after the (**first stage**) partial filling of the bucket, performed work in bulldozer mode.

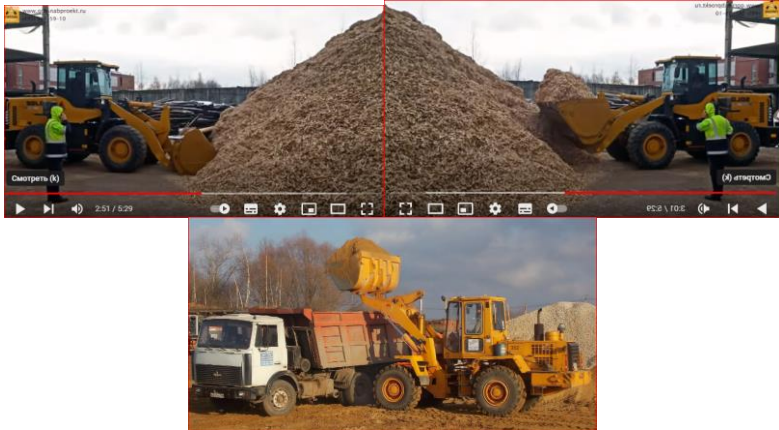


Fig. 6. Start and end of the bucket filling loader function.

At the **second stage**, t_A can be considered as the initial moment of time, and t_r as the final moment of time, where $t_A \leq t \leq t_r$. In this mode, the parameters r_4 and \dot{r}_4 can be expressed in terms of the lengths of the lifting and lowering hydraulic cylinder of length l_4 and $\varphi(t)$, $\dot{\varphi}(t)$, and the number of degrees of freedom is reduced by one [6]. In this case, the movements of the working bodies are described by the parameters: 1) change $\beta(t)$ and $\dot{\beta}(t)$ so that the load in the bucket does not crumble until the moment of loading (Fig. 4, 6);

2) changes $\varphi(t)$ and $\dot{\varphi}(t)$ simultaneously with $\beta(t)$ and $\dot{\beta}(t)$ from the initial value φ_A and $\dot{\varphi}_A$;

3) changes the position of the points of the mechanism $r(t)$ with $\dot{r}(t)$ under the action of telescopic pushing bars, simultaneously changing $\varphi(t)$ and $\dot{\varphi}(t)$, hence both $\beta(t)$ and $\dot{\beta}(t)$ (t).

At the **third stage of the movement**, when $t_r \leq t \leq t_\theta$, the value t_r can be considered as the initial moment of time, and t_θ as the final moment of time. Angle change $\varphi(t)$ corresponds to translational movement for angle changes $\theta(t)$ and $\beta(t)$. In turn, the change $\theta(t)$ characterizes the relative change in the polar angle at the origin of the coordinate point C.

An angle change $\theta(t)$ for lifting and lowering with a load will not be started until the extendable part of the telescopic pushing bars described by the variables $r(t)$ and $\dot{r}(t)$ completely leaves the area of the fixed part of the outer beam (Fig. 5).

To study each stage of the movement, certain mathematical models, methods for their solution, and corresponding calculation schemes will be required [6, 7].

At the initial stage of the study, the problem was studied and brief substantive materials were published in [5, 6]. However, in the models, the static states of the applied forces of hydraulic cylinders were not taken into account in detail in the structure of relative, portable and absolute movements separately of the movements of the RDM working cycle.

In this paper, movements are considered only in the second stage of the RDM operation. First, the calculations of the static state of comparison of forces at the initial moment of time are made. Preliminary estimates of the magnitude of the applied forces are recommended, which could impart movements to the corresponding links of the mechanisms.

From Figure 4, the forces of the weights $\vec{G}_1, \vec{G}_2, \vec{G}_3$ pairs of forces with moments M_A, M_B, M_C, M_D , reactions of the support forces $\vec{P}_B, \vec{P}_A, \vec{P}_C$, are known. To determine the reaction \vec{P}_B of bonds, we will compose a system of algebraic equations of static equilibrium:

$$\sum_{i=1}^{in} M_{Ai} = 0: -[G_3l_3+(G_2+G_1)(l_3+r_0+r) + G_1(l_1+\sqrt{2} l_D \sin(\beta - \frac{\pi}{4})) + 2P_B l_3 \sin \frac{\pi - \varphi_M - \varphi}{2}] \cos \varphi = 0;$$

$$\sum_{J=1}^{JN} X_J = 0: X_A - (G_3 + G_2 + G_1) \sin \varphi - P_B \cos \frac{\pi - \varphi - \varphi_M}{2} = 0; \tag{1}$$

$$\sum_{J=1}^{JN} Y_J = 0: Y_A - (G_3 + G_2 + G_1) \cos \varphi + P_B \sin \frac{\pi - \varphi - \varphi_M}{2} = 0.$$

From this system, it is possible to determine the reactions of bonds at $t_A \leq t \leq t_r$:

$$P_B(\beta, \varphi, r) = \frac{1}{2l_3 \sin \frac{\pi - \varphi_M - \varphi}{2}} [G_3l_3 + G_2(l_3 + r_0 + r) + G_1(l_3 + r_0 + r + l_1 + \sqrt{2} l_D \sin(\beta - \frac{\pi}{4}))]; \tag{2}$$

$$X_A(\beta, \varphi, r) = (G_3 + G_2 + G_1) \sin \varphi + P_B \cos \frac{\pi - \varphi - \varphi_M}{2} \cdot \frac{\varphi + \varphi_M}{2}; \tag{3}$$

$$Y_A(\beta, \varphi, r) = (G_3 + G_2 + G_1) \cos \varphi - P_B \sin \frac{\pi - \varphi - \varphi_M}{2} \cdot \frac{\varphi + \varphi_M}{2}. \tag{4}$$

To determine the required total force of the drives, in the general case equal to (let us denote) P_{Bh} they were able to communicate the movement of the bucket with the load and other forces indicated in equation (2) during lifting, the condition $P_{Bh} > P_B$. But, for lowering down in the opposite direction of the center of mass of the bucket with a load and other mentioned counteracting forces, $P_{Bh} < P_B$.

One convenient way to replace P_{Bh} is to determine the maximum value P_{Bmax} of all values in a continuous force field P_B so that the differences with the opposite vector of the total gravity correspond to the direction of the lift. The process of unloading the bucket is

carried out in reverse order. In this case, it is necessary to determine P_{Bmin} from all values in the force field P_B so that the differences with the opposite vector of the total forces correspond to the direction of lowering. The process of determining the forces of forward and reverse movement of hydraulic cylinders, characterizing the rest of the generalized variables, can be defined in a similar way.

We use polar coordinate systems (r, φ) with the origin at point A. This movement includes: translational movement along the r axis with the origin of the relative coordinate C, also in relation to this translational movement, relative rotational movement with the polar coordinate system (r_D, β) with the center at the point O_D , where r_D is the radius vector. The distance $O_D D = l_D$ can be a variable value, since the process of filling or emptying the bucket occurs over a non-instantaneous, and sometimes up to the same order as the time intervals of other relatively long processes of the RDM work cycle.

In this case, we will consider the time intervals for filling or emptying the bucket as negligible compared to other long processes of the working cycle, and for simplicity, we will assume that $O_D D = l_D = \text{const}$ is considered constant. We will compose an equation for the equilibrium of the moments of a pair of forces acting in the system (r_D, β) ,

$$\sum_{j=1}^{jn} M_{O_D, j} = 0; \quad P_{D \text{ ID sin}(\beta)} - G_{1 \text{ ID sin}(\beta)} = 0, \tag{6}$$

whence $P_D = G_1 = m_1 g$, where $m_1 = m_{\text{ковш}} + m_{\text{тряз}}$. (7)

To determine the required total force of the drives, in the general case equal to P_{Dh} , they were able to communicate the movement of the ladle with the load when the center of mass D of the ladle with the load was raised, the condition $P_{Dh} > P_D$. But, for lowering down by changing the angle of rotation β in the opposite direction of the center of mass D of the bucket with a load, the condition $P_{Dh} < P_D$. We also note that if the difference in forces is insufficient - during the lowering of the center of mass D of the bucket with the load, additional force is required to overcome the negatively directed force ($-P_{Dh}$) by additional regulation of the load of the hydraulic cylinders. One of the best replacement P_{Dh} methods is to determine the maximum value P_{Dmax} of all values in the field of forces P_D so that the differences with the opposite vector of the total moments of gravity correspond to the direction of the lift. The process of unloading the bucket and lowering it or bringing the blade to the emptying position by turning the bucket with the load is carried out in the reverse order, and in this case, the minimum value P_{Dmin} of all values in the field of the same forces P_D should be determined.

Consider the movement along the r axis with the origin of coordinate C, which includes the relative rotational movement with the coordinate system (r_D, β) centered at the point O_D . Proceeding from the translational-rectilinear nature of the portable movements: the center of mass of the extended parts of the telescopically pushing bars and the weight of the bucket with the load applied through the beginning O_D . To determine the static balance of

forces, we will compose an equation for the equilibrium of the acting components of the force vectors along the r axis

$$\sum_{k=1}^{KM} R_k = 0; \quad P_R \cos \varphi - (G_2 + G_1 \cos(\beta)) \cos \varphi = 0. \tag{8}$$

whence

$$P_R = G_1 \cos(\beta) + G_2 = (m_1 + m_2 \cos(\beta))g, \quad \text{where} \quad m_1 = m_{\text{bucket}} + m_{\text{load}} \tag{9}$$

In order for the total forces of the drives (hydraulic cylinders) P_{R_h} to be able to equally communicate the movement of the retractable parts of the telescopically pushing bars and the bucket with the load during lifting extension along the r axis, the condition $P_{R_h} > P_R$. But, for their reverse movement, the condition $P_{R_h} < P_R$. Similarly, to the above, we will determine the maximum value $P_{R_{\max}}$ of all values in the field of forces P_R so that the differences with the opposite vector of the total gravity correspond to the direction of movement of the lift. The process of reverse movements of the parts of the telescopically pushing bars and the bucket with the load is carried out in the reverse order. In this case, you should determine the minimum value $P_{R_{\min}}$ of all values in the field of the same forces P_R .

Determining the resulting forces of drives in motion of the mechanical system RDM: $P_{B_{\max}}$; $P_{B_{\min}}$; $P_{D_{\max}}$; $P_{D_{\min}}$; $P_{R_{\max}}$; $P_{R_{\min}}$, we will write a system of quasi-linear ordinary differential equations of the second order, obtained using the Lagrange equations of the second kind [3, 6], taking into account these resulting drive forces,

$$\ddot{\beta} = \frac{\sqrt{2}}{l_D} \left[(l_3 + r_0 + r + l_1) \sqrt{2} l_D \sin\left(\beta - \frac{\pi}{4}\right) \cos\left(\beta - \frac{\pi}{4}\right) \cdot \dot{\varphi}^2 + \left(\bar{P}_{D_{\max}} - g \right) \frac{\sqrt{2}}{l_D} \cos\left(\beta - \frac{\pi}{4}\right) \sin \varphi \right]; \tag{10}$$

$$\ddot{\varphi} = - \frac{B}{Z} \cdot \dot{\varphi} - \frac{\bar{P}_{B_{\max}} - D}{Z} \cos \varphi; \tag{11}$$

$$\ddot{r} = \frac{m_1}{m_1 + m_2} \left[(l_3 + r_0 + r + l_1) \sqrt{2} l_D \sin\left(\beta - \frac{\pi}{4}\right) \right] \cdot \dot{\varphi}^2 + \left(\bar{P}_{R_{\max}} - g \right) \sin \varphi, \tag{12}$$

with initial and final conditions within the limits of change of generalized coordinates and generalized velocities at $t_A \leq t \leq t_r$:

$$\dot{\beta}_A \leq \dot{\beta} \leq \dot{\beta}_r, \quad \text{where} \quad \frac{d\beta}{dt} = \dot{\beta}_{(t)} = \dot{\beta}; \quad \dot{\beta}_{(tA)} = \dot{\beta}_A; \quad \dot{\beta}_{(tr)} = \dot{\beta}_r, \tag{10a}$$

$$\beta_A \leq \beta \leq \beta_r, \quad \text{where} \quad \beta_{(tA)} = \beta_A; \quad \beta_{(tr)} = \beta_r, \tag{10b}$$

$$\dot{\varphi}_A \leq \dot{\varphi} \leq \dot{\varphi}_r, \quad \text{where} \quad \frac{d\varphi}{dt} = \dot{\varphi}_{(t)} = \dot{\varphi}; \quad \dot{\varphi}_{(tA)} = \dot{\varphi}_A; \quad \dot{\varphi}_{(tr)} = \dot{\varphi}_r, \tag{11a}$$

$$\varphi_A \leq \varphi \leq \varphi_r, \quad \text{where } \varphi_r \leq \varphi_M; \quad \varphi_{(tA)} = \varphi_A; \quad \varphi_{(tr)} = \varphi_r, \quad (11b)$$

$$\dot{\varphi}_A \leq \dot{\varphi} \leq \dot{\varphi}_r, \quad \text{where } \frac{d\varphi}{dt} = \dot{\varphi}_{(t)} = \dot{\varphi}; \quad \dot{\varphi}_{(tA)} = \dot{\varphi}_A; \quad \dot{\varphi}_{(tr)} = \dot{\varphi}_r, \quad (12a)$$

$$r_A \leq r \leq r_l, \quad \text{where } r(tA) = r_A; \quad r(tr) = r_l, \quad (12b)$$

are selected based on their technical convenience and functional purpose. Here

$$B = 2[m_1(l_3 + r_0 + r + l_1 + \sqrt{2} \text{ID} \sin(\beta - \frac{\pi}{4}))(\dot{\varphi} + \sqrt{2} \text{ID} \cos(\beta - \frac{\pi}{4}) \dot{\beta}) + m_2(l_3 + r_0 + r) \dot{\varphi}]; \quad (11c)$$

$$D = g [m_3 l_3 + (m_2 + m_1)(l_3 + r_0 + r) + m_1(l_1 + \sqrt{2} \text{ID} \sin(\beta - \frac{\pi}{4}))] \cos \varphi; \quad (11d)$$

$$Z = m_1(l_3 + r_0 + r + l_1 + \sqrt{2} \text{ID} \sin(\beta - \frac{\pi}{4}))^2 + m_2(l_3 + r_0 + r)^2 + m_3 l_3^2. \quad (11e)$$

In this system, the left parts of equations (10), (11) have the dimension $1/t^2$, and (12) has the dimension $(l_3 + l_1)/t^2$, if $l_3 + l_1$ considered as the characteristic length of the parameter r .

To pass to dimensionless quantities, in particular, in time we will compare:

$$[\ddot{\beta}] \sim \left[\frac{1}{T_\beta^2} \right] \sim \left[\frac{g}{l_D} \right]; \quad \Rightarrow T_\beta^2 \sim \frac{l_D}{g}; \quad \Rightarrow T_\beta \sim \sqrt{\frac{l_D}{g}}; \quad (10^*)$$

$$[\ddot{\varphi}] \sim \left[\frac{1}{T_\varphi^2} \right] \sim \left[\frac{g}{2l_3} \right]; \quad \Rightarrow T_\varphi^2 \sim \frac{2l_3}{g}; \quad \Rightarrow T_\varphi \sim \sqrt{\frac{2l_3}{g}}; \quad (11^*)$$

$$[\ddot{r}] \sim \left[\frac{l_3 + l_1}{T_r^2} \right] \sim [g]; \quad \Rightarrow T_r^2 \sim \left[\frac{l_3 + l_1}{g} \right]; \quad \Rightarrow T_r \sim \sqrt{\frac{l_3 + l_1}{g}}; \quad (12^*)$$

since the ratios of the characteristic time scales T_β , T_φ , T_r are not the same. For characteristic values in time t and time step in terms of $\dot{\beta}$ and β ; $\dot{\varphi}$ and φ , \dot{r} and r in the equations of the system, based on the obtained conclusions (10*), (11*) and (12*), we assume

$$t_\beta = T_\beta \bar{t}; \quad t_\varphi = T_\varphi \bar{t}; \quad t_r = T_r \bar{t}, \quad (13)$$

$$\Delta t_\beta = T_\beta \Delta \bar{t}; \quad \Delta t_\varphi = T_\varphi \Delta \bar{t}; \quad \Delta t_r = T_r \Delta \bar{t}, \quad (13^*)$$

where \bar{t} - dimensionless time, t_β, t_φ, t_r - times commensurate with characteristic T_β, T_φ, T_r . When carrying out specific calculations, it is advisable to take them according to the ratios:

$$T_\beta = \sqrt{\frac{l_D}{g}}; \quad T_\varphi = \sqrt{\frac{2l_3}{g}}; \quad T_r = \sqrt{\frac{l_3 + l_1}{g}}. \tag{13**}$$

To use one of the numerical methods for the system of equations (10), (11) and (12), we note that each of them directly without additional transformations can use the **shooting method**, which is a pair of nested methods: the outer one is for solving a nonlinear equation, inner - linear equation reduces the solution of the boundary value problem to the solution of two nested Cauchy problems. Indeed, we will use the twice repeated: explicit **Euler method**; and/or the *Runge-Kutta method*. To use one of the schemes of these numerical methods to the system of equations (10), (11) and (12), we will integrate twice, repeating the integration procedure for each of the following notation forms

$$\ddot{\beta} = \frac{d\dot{\beta}}{dt}; \quad \ddot{r} = \frac{d\dot{r}}{dt}; \quad \ddot{\varphi} = \frac{d\dot{\varphi}}{dt}, \tag{14}$$

presenting them in the form of the following equalities:

$$d\dot{\beta} = \ddot{\beta} dt; \quad d\dot{r} = \ddot{r} dt; \quad d\dot{\varphi} = \ddot{\varphi} dt, \tag{14*}$$

and in the transition to discrete nodal values, we write them in difference relations

$$\Delta\dot{\beta} = \ddot{\beta} \Delta t; \quad \Delta\dot{r} = \ddot{r} \Delta t; \quad \Delta\dot{\varphi} = \ddot{\varphi} \Delta t. \tag{15}$$

First, we will move from a continuous range of continuous values to a discrete range with variable values in the nodal values, taking into account the limits of their changes.

For non-stationary movement of the working bodies of the RDM capable of performing many functions of taking into account the parameters of spatial generalized coordinates and generalized velocities, the previously published materials provide for dividing the region of change according to one index from (β, φ, r) to $(\beta_i, \varphi_j, r_k)$, later to $(\varphi_i, \beta_j, r_k)$, and from

generalized velocities $(\dot{\varphi}, \dot{\beta}, \dot{r})$ to $(\dot{\varphi}_i, \dot{\beta}_j, \dot{r}_k)$. This approach is far from erroneous, and may not fully cover agile action management.

In this paper, all spatial indices i, j, k , are used, in addition, the change in time with a time step n should be simultaneously taken into account, making already three-dimensional arrays [7] four-dimensional. At the same time, for a static state, it is possible to designate and calculate by the formulas the distribution fields of the active forces of the drive at points D ; B ; R :

$$P_{D_{i,j,k}} = P_D(\varphi_{i,j,k}^1, \beta_{i,j,k}^1, r_{i,j,k}^1); \quad P_{B_{i,j,k}} = P_B(\varphi_{i,j,k}^1, \beta_{i,j,k}^1, r_{i,j,k}^1); \quad P_{R_{i,j,k}} = P_R(\varphi_{i,j,k}^1, \beta_{i,j,k}^1, r_{i,j,k}^1) \tag{16}$$

Here the upper indices are the time indices in the initial value for $n = 1$.

Based on the limits of change of continuous generalized coordinates $\beta(t), \varphi(t), r(t)$, according to (10b), (11b), (12b), we will introduce nodal values according to the formulas:

$$\beta_{i,j,k}^n = \beta_{i,j-1,k}^n + \Delta\beta, \quad j = 2, 3, \dots, JM, \quad i = 1, 2, \dots, IM, \quad k = 1, 2, \dots, KM, \quad (17a)$$

$$\varphi_{i,j,k}^n = \varphi_{i-1,j,k}^n + \Delta\varphi, \quad i = 2, 3, \dots, IM, \quad j = 1, 2, \dots, JM, \quad k = 1, 2, \dots, KM, \quad (17b)$$

$$r_{i,j,k}^n = r_{i,j,k-1}^n + \Delta r, \quad k = 2, 3, \dots, KM, \quad i = 1, 2, \dots, IM, \quad j = 1, 2, \dots, JM, \quad (17c)$$

where $n = 1, 2, \dots, N$, and steps along generalized coordinates are determined by the formulas:

$$\Delta\beta = (\beta_{i, JM, k}^n - \beta_{i, 1, k}^n) / (JM-1); \quad \Delta\varphi = (\varphi_{IM, j, k}^n - \varphi_{1, j, k}^n) / (IM-1); \quad \Delta r = (r_{i, j, KM}^n - r_{i, j, 1}^n) / (KM-1). \quad (18)$$

The Euler method is based on the idea of graphically constructing a solution to differential equations, but this method also provides a way to find the desired function in a numerical (tabular) form. Let's build, starting from the point $t_1 = t_A$, the nodal values $t_1 \leq t_n \leq t_N$, choosing the time step

$$\Delta t = (t_N - t_1) / (N-1), \quad t_n = t_1 + (n-1) \Delta t \quad (n = 1, 2, \dots, N), \quad (19)$$

and in the case of using the Runge-Kutta method, we will also use the fractional step [4]

$$\tilde{t}_n = t_{n-1} + \frac{\Delta t}{2} \quad (n = 1, 2, \dots, N). \quad (19a)$$

Further, based on the fact that all three equations are second-order ordinary differential equations, and each of them contains quasi-linear second derivatives, which can be written as systems of two first-order equations using formulas (14), (14*), (15), we first rewrite equations (10), (11) and (12) in the difference relations of systems of generalized velocities

$$\begin{aligned} \dot{\beta}_{i,j,k}^n = \dot{\beta}_{i,j,k}^{n-1} + \Delta t \{ & \mathfrak{RBE}_{i,j,k}^{n-1} \frac{\sqrt{2}}{1_D} \cos(\beta_{i,j,k}^{n-1} - \frac{\pi}{4}) \cdot \dot{\varphi}_{i,j,k}^{n-1}{}^2 + \\ & + [P_{D_{\max}} - g] \frac{\sqrt{2}}{1_D} \cos(\beta_{i,j,k}^{n-1} - \frac{\pi}{4}) \sin \varphi_{i,j,k}^{n-1} \}; \end{aligned} \quad (20)$$

$$\dot{\varphi}_{i,j,k}^n = \dot{\varphi}_{i,j,k}^{n-1} + \Delta t [BCH_{i,j,k}^{n-1} \dot{\varphi}_{i,j,k}^{n-1} + (DCH_{i,j,k}^{n-1} - P_{B_{\max}}) \cos \varphi_{i,j,k}^{n-1}] Z_{i,j,k}^{n-1} \quad (21)$$

$$\dot{r}_{i,j,k}^n = \dot{r}_{i,j,k}^{n-1} + \Delta t [(\mathfrak{R}2_{i,j,k}^{n-1} \frac{m_1}{m_1 + m_2} BE_{i,j,k}^{n-1} \cdot \dot{\varphi}_{i,j,k}^{n-1}{}^2 + (P_{R_{\max}} - g) \sin \varphi_{i,j,k}^{n-1}], \quad (22)$$

with appropriate initial conditions

$$\dot{\beta}_{i,j,k}^1 = \dot{\beta}_A, \tag{20a}$$

$$\dot{\varphi}_{i,j,k}^1 = \dot{\varphi}_A, \tag{21a}$$

$$\dot{r}_{i,j,k}^1 = \dot{r}_A. \tag{22a}$$

And then, similarly, repeating the process of writing according to the formulas

$$\dot{\beta} = \frac{d\beta}{dt}; \quad \dot{r} = \frac{dr}{dt}; \quad \dot{\varphi} = \frac{d\varphi}{dt}, \tag{23}$$

getting the following entries:

$$d\beta = \dot{\beta} dt; \quad dr = \dot{r} dt; \quad d\varphi = \dot{\varphi} dt, \tag{23^*}$$

and in difference relations representing

$$\Delta\beta = \dot{\beta} \Delta t; \quad \Delta r = \dot{r} \Delta t; \quad \Delta\varphi = \dot{\varphi} \Delta t, \tag{24}$$

and using (24) already for the obtained values from (20), (21) and (22), we can write

$$\beta_{i,j,k}^n = \beta_{i,j,k}^{n-1} + \Delta t \dot{\beta}_{i,j,k}^n, \tag{25}$$

$$r_{i,j,k}^n = r_{i,j,k}^{n-1} + \Delta t \dot{r}_{i,j,k}^n, \tag{26}$$

$$\varphi_{i,j,k}^n = \varphi_{i,j,k}^{n-1} + \Delta t \dot{\varphi}_{i,j,k}^n, \tag{27}$$

with appropriate initial conditions

$$\beta_{i,j,k}^1 = \beta_A, \tag{25a}$$

$$r_{i,j,k}^1 = r_A, \tag{26a}$$

$$\varphi_{i,j,k}^1 = \varphi_A. \tag{27a}$$

Here, for the sake of brevity, auxiliary notation for variables is used

$$\mathfrak{R}2_{i,j,k}^n = {}_{l3+r0+} r_{i,j,k}^n; \quad BE_{i,j,k}^n = {}_{l1+\sqrt{2}} \text{ID sin} \left(\beta_{i,j,k}^n - \frac{\pi}{4} \right); \quad \mathfrak{RBE}_{i,j,k}^n = \mathfrak{R}2_{i,j,k}^n + BE_{i,j,k}^n \tag{28}$$

$$PB_{i,j,k}^n = {}_{g(m3 \ l3 + m2} \mathfrak{R}2_{i,j,k}^n + {}_{+m1} \mathfrak{RBE}_{i,j,k}^n \left(2l_3 \text{sin} \frac{\pi - \varphi_M - \varphi_{i,j,k}^n}{2} \right); \tag{29}$$

considered $\beta_A = \pi/3$, and to the horizon $\pi/6$. Then the total initial value of the angles β and φ is

$$\beta_{A+} \varphi_A = \pi/3 - \pi/6 = \pi/6. \tag{30a}$$

In this case, the tangent line close to the lower edge of the “bud” of the bucket makes an angle $\beta_{HK} = \pi/6$ with the horizon, and the tangent line close to the upper edge of the “bud” of the bucket makes an angle $\beta_{\theta K} = \pi/2$ with the horizon. The values of the generalized velocities and coordinates at the initial of time $t_1 = 0$ of the relative motion of the RDM will be assumed

$$\dot{\beta}_{A=0}; \quad \dot{r}_{A=0}; \quad \dot{\varphi}_{A=0}, \tag{31}$$

$$\beta_{A=} \pi/3; \quad r_{A=} 13; \quad \varphi_{A=} \pi/6. \tag{32}$$

At the beginning of the maneuver plan, we assume that the angles β and φ will change in turn:

1) at constant $\varphi = \varphi_A$, the angle β will first change in the limit

$$\beta_{A-} \varphi_A \leq \beta \leq \beta_{A+} \varphi_A + \pi/6, \quad \Rightarrow \quad \pi/3 \leq \beta \leq \pi/2, \tag{32a}$$

and with respect to the horizontal line, the angle β varies within the limit

$$\pi/6 \leq \beta \leq \pi/3. \tag{32b}$$

In this case, only the first equations of the two systems of the problem under consideration are solved in turn: (20) with (20a) and (25) with (25a) without the remaining two pairs of equations with initial conditions (21) with (21a) and (26) with (26a); (22) with (22a) and (27) with (27a).

2) After that, at a constant $\beta = \beta_A - \varphi_A + \pi/6 = \pi/2 = \beta_C$ the angle φ changes in the limit

$$\varphi_A \leq \varphi \leq \varphi_A + \pi/6, \quad \Rightarrow \quad -\pi/6 \leq \varphi \leq 0, \tag{33}$$

and in this case, only the second equations of the two systems of the problem under consideration are solved in turn: (21) with (21a) and (26) with (26a) without the remaining two pairs of equations with initial conditions (20) with (20a) and (25) with (25a); (22) with (22a) and (27) with (27a).

At the end of this part of the work, the current values of the angles become $\varphi = \varphi_A + \pi/6 = \varphi_C$, $\varphi_C = 0$, and $\beta = \pi/2$, since when the angle increases φ by $\pi/6$, the translational motion also increases the angle β by $\pi/6$, thereby achieving the constraint

(30). However, up to the end value of the angle $\varphi_{IM} = \varphi_C + \pi/3$ there is still an unused angle value $\pi/3$.

3) Therefore, the angle φ continues to change in the limit

$$\varphi_C \leq \varphi \leq \varphi_C + \pi/3, \tag{33a}$$

and the angle β should change in the opposite direction

$$\beta_{JM} \geq \beta \geq \beta_{JM} - \pi/3. \tag{32e}$$

In this case, the first two pairs of equations of two systems of the problem under consideration are solved in turn: (20) with (20a) and (25) with (25a); (21) with (21a) and (26) with (26a) without the remaining pair of third equations with initial conditions (22) with (22a) and (27) with (27a).

4) At constant β and φ , $l_3 \leq r \leq 2l_3 + l_1$ will change, and in this case, only the third equations of the two systems of the problem under consideration are solved in turn: (22) with (22a) and (27) with (27a) without the remaining two pairs of equations with initial conditions (20) with (20a) and (25) with (25a); (21) with (21a) and (26) with (26a).

Further, it is possible to bring the reverse sequence of actions corresponding to the unloading process, where they are accompanied in various combinations of pairwise solutions of the system equations similar to those given above, including the joint solution of all three pairs of equations with the corresponding initial conditions.

To check the performance and reliability of the compiled algorithm and program, the program was debugged with some approximate initial data. When performing the calculations, the process of determining the force of the drives of the DSM system was included:

$P_{B_{\max}}$; $P_{B_{\min}}$; $P_{D_{\max}}$; $P_{D_{\min}}$; $P_{R_{\max}}$; $P_{R_{\min}}$ so that the constant values of the forces are overcoming during their operation. Primary tabular results of calculations of generalized velocities and generalized coordinates for the nodal values of the argument - time show the tendency of the correctness of the algorithms for solving the problem.

3 Results

Let us briefly present the obtained output data of the result of debugging the program for some initial data.

For given: $g=9,81$; $m_1=1000$; $m_2=80$; $m_3=160$; $l_1=1,2$; $l_D=0,5$; $r_0=0,3$; $l_3=1,5$; and so other values have been obtained

$$P_{B_{\min}} = 10889.6; P_{B_{\max}} = 42353.04; P_{R_{\min}} = 0.0; P_{R_{\max}} = 9175.36; P_{D_{\min}} = 0.0; P_{D_{\max}} = 10594.8$$

Below the vertical table values, the time step changes t_n in 11 nodes, and horizontally the following values change in 6 nodes:

$$\hat{\beta}_{1,j,1}^n:$$

0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000

0.000	0.000	0.000	0.001	0.001	0.001
0.383	0.502	0.605	0.682	0.733	0.763
1.529	2.005	2.417	2.723	2.926	3.048
3.824	5.011	6.036	6.798	7.304	7.609
7.621	9.951	11.961	13.456	14.456	15.065
13.179	17.041	20.348	22.822	24.506	25.574
20.604	26.150	30.851	34.409	36.923	38.626
29.812	36.911	42.923	47.581	51.031	53.545
40.563	49.022	56.292	62.092	66.584	70.063

$\phi_{i,1,1}^n$:

0.000	0.000	0.000	0.000	0.000	0.000
0.029	0.028	0.026	0.023	0.019	0.014
0.058	-0.045	-0.082	-0.087	-0.119	-0.163
0.087	0.121	0.332	0.177	0.393	0.604
0.114	-0.116	-0.490	-0.255	-0.324	-0.391
0.133	0.038	0.036	0.204	0.115	0.085
0.142	0.029	-0.001	0.064	0.036	-0.030
0.135	0.022	0.006	0.031	0.011	-0.011
0.116	0.012	0.007	0.005	0.005	-0.003
0.097	0.012	0.007	0.003	0.002	0.000
0.089	0.009	0.007	0.003	0.002	0.000

$\beta_{1,j,1}^n$:

0.000	0.314	0.628	0.942	1.257	1.571
0.000	0.314	0.628	0.942	1.257	1.571
0.000	0.314	0.628	0.942	1.257	1.571
0.009	0.326	0.642	0.958	1.273	1.588
0.043	0.371	0.697	1.019	1.339	1.657
0.130	0.484	0.833	1.173	1.504	1.829
0.302	0.709	1.103	1.477	1.830	2.169
0.599	1.093	1.562	1.992	2.384	2.746
1.064	1.684	2.259	2.769	3.217	3.618
1.737	2.517	3.228	3.843	4.369	4.827
2.653	3.624	4.499	5.245	5.873	6.409

$\phi_{1,j,1}^n$:

0.000	0.209	0.419	0.628	0.838	1.047
0.000	0.209	0.419	0.628	0.838	1.047
0.002	0.211	0.420	0.630	0.839	1.048
0.005	0.209	0.416	0.625	0.832	1.039
0.010	0.215	0.434	0.635	0.854	1.072
0.016	0.209	0.407	0.620	0.836	1.051
0.023	0.211	0.409	0.632	0.842	1.055
0.031	0.212	0.409	0.635	0.844	1.054
0.039	0.214	0.409	0.637	0.845	1.053
0.045	0.214	0.410	0.637	0.845	1.053
0.050	0.215	0.410	0.637	0.845	1.053

4 Conclusion

It is noticeable that in the tabular output values there is a quite natural tendency of increasing or decreasing data. A full and detailed analysis of the obtained calculations is planned to be presented in subsequent scientific publications.

References

1. Development and creation of a bulldozer-loader with a transforming working body to improve the efficiency of construction, maintenance, repair of high-mountain roads and agricultural work. State scientific-research work report, Bishkek: KSUCTA, 2016. -88 p.
2. K.Isakov, Zh.Zh. Turgumbaev, A.Sh. Altybaev, A.A. Beishenaliev, Bulldozer-loader with a transforming working body. Eurasian patent No. 024772 (Moscow, 2016)
3. M.I. Bat, et al, Theoretical mechanics in examples and tasks (Moscow, Nauka, 1968)
4. N.N. Yanenko, Method of fractional steps for solving multidimensional problems of mathematical physics (Novosibirsk, Nauka, 1967)
5. K. Isakov, K.T. Osmonov, T. Toktakunov, Bulletin of KSUCTA **4(66)**, 586-593 (2019)
6. K. Isakov, K.T. Osmonov, T. Toktakunov, Bulletin of KSUCTA **4(66)**, 577-585 (2019)
7. K. Isakov, T. Toktakunov, K.T. Osmonov, A.B. Kanatova, Bulletin of KSUCTA **4(78)**, 1481-1491 (2022)
8. K. Isakov, N. Madanbekov, A. Altybaev, A. Chopoev, Ch. Omurbekov, E3S Web Conf. **402** (2023) <https://doi.org/10.1051/e3sconf/202340210012>
9. K. Isakov, N. Madanbekov, A. Altybaev, A. Chopoev, Ch. Omurbekov, Transportation Research Procedia **63**, 759–768 (2022) <https://doi.org/10.1016/j.trpro.2022.06.072>
10. K. Isakov, A. Altybaev, A. Beishenaliev, A. Chopoev, A. Matisakov, E3S Web Conf. **263**, 04021 (2021) <https://doi.org/10.1051/e3sconf/202126304021>