Conditions for distortion-free voltage transfer in the transformer

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Abstract. The optimal design of low-power transformers, sonic and ultrasonic transformers, pulse transformers, differential transformers, etc. requires an in-depth study of their theory and design calculation. In this article, the influence of transformer parameters on the voltage transfer ratio is discussed, and recommendations are given for the selection of values of these parameters to reduce possible distortions. The calculation methods are illustrated by an example.

1 Introduction

Increasing demands on transformers with higher frequencies lead to the necessity of a detailed analysis of the transformer equivalent diagrams and the determination of the transmission ratio at low, medium, and high frequencies. It is necessary to find out how the parameters of the transformer circuit influence the resulting nonlinear distortions.

2 Methods

Influence of transformer circuit parameters on nonlinear distortions

Let us consider the peculiarities of the transformer operation in the most general case, namely in a sufficiently wide frequency range of the supplying sinusoidal voltage. In this case, the operation of the transformer at a fixed frequency can be considered as a special case of the general issue. To facilitate the solution, let us conditionally divide the entire operating frequency range into three frequency intervals: low, medium, and high. This approach follows from the possibility of simplifying the complete equivalent circuit of the transformer (Fig. 1), depending on the specific value of frequency, and, consequently, simplifying its mathematical description and calculation.

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Fig. 1. Complete equivalent diagram of transformer with source and load parameters.

At the low frequency, we can neglect the voltage drop on the leakage inductance and assume that the intrinsic capacitance $C'_0$ transformer does not shunt the load resistance $R'_L$. Then the equivalent circuit of the transformer is converted to the form shown in Fig. 2, a.

When the frequency is increased, the effect of the parameters $L_{1n}$, $R_H$, which take into account the processes in the magnetic core on the currents in the transformer, is reduced and can be neglected, which will allow a further simplification of the equivalent circuit (Fig. 2, b). Let's make some explanations. Indeed, if the growth of the inductive component of the resistance of the considered circuit, caused by $L_{1n}$ with increasing frequency is unquestionable, then the growth of the loss resistance $R_H$ is not so obvious. Total losses in the magnetic core $P_M$ can are calculated by the formula $P_M = Af^{3/2}B_m^2$ [1-2], where the amplitude of the magnetic induction $B_m$ at a sinusoidally varying field is determined by the well-known expression $B_m = U_i/(4\pi f S_M w_i)$. Or, since the selected transformer $S_M$ and $w_i$ do not change, we can write $B_m = A_i U_i/f$. Then, given that $R_H = U_i^2/P_M$, retrieve

$$R_H = \frac{U_i^2}{P_M} = \frac{U_i^2}{Af^{3/2}B_m^2} = \frac{U_i^2f^2}{Af^{3/2}A_iU_i^2} A_2 \sqrt{f},$$

($A_i$ and $A_2$ – coefficients of proportionality). The latter expression confirms the assumption made about the growth of increasing frequency for a given transformer design.

As the frequency increases further, it becomes impossible to neglect the effect of the voltage drop on the $L_S$, as well as the bypass effect $C'_0$, the equivalent circuit of the transformer takes the form shown in Fig. 2, c.

Fig. 2. Transformer substitution diagrams for different frequency ranges: a-low; b-medium; c-high

It is easy to see that the simplest from the position of analysis is the equivalent circuit of the transformer corresponding to the middle-frequency range.

It should be noted that any of the simplified equivalent diagrams in Fig. 2, which characterize the operating mode of a transformer, is ambiguously determined by a specific frequency value, but is a function of its parameters, which in turn depend on the geometry (design) of the transformer and the electromagnetic properties of the materials used.
Therefore, two transformers designed for the same power and voltage can behave differently in the same frequency range, depending on their particular design.

For further analysis, it is convenient to use the concept of transfer coefficient $K$, which is the ratio of the reduced voltage on the load to the transformer input voltage. Here are the expressions determining $K$ for each of the simplified equivalent circuits or, in other words, for conditional frequency ranges. For clarity, the circuit of Fig. 2, a is shown in the form of the scheme of Fig. 3.

![Fig. 3. Simplified transformer substitution diagram for the lower frequency range.](image)

Then,

$$K(HC) = \frac{I_{Z_{123}}}{Z_2} = \frac{Z_2Z_3Z_1}{Z_1Z_2Z_4 + Z_1Z_3Z_4 + Z_2Z_3Z_4 + Z_1Z_2Z_3} \quad (1)$$

where $Z_1 = R'_2 + R'_H$; $Z_2 = R_H$; $Z_3 = j\omega L_{1n}$; $Z_4 = R_1$ (includes and $R_i$ Fig. 1). Given the obvious correlations between transformer parameters, namely $R'_2 << R'_H << R_H$; $R_1 << R'_H << R_H$, expression (1) can be simplified:

$$K(HC) = \frac{Z_3}{Z_3 + Z_4} = \frac{j\omega L_{1n}}{R_1 + j\omega L_{1n}} = \frac{j\omega L_{1n}(R_1 - j\omega L_{1n})}{R_1^2 + \omega^2 L_{1n}^2} = \frac{\omega L_{1n}e^{j\phi}}{\sqrt{R_1^2 + \omega^2 L_{1n}^2},} \quad (2)$$

because $R_1 << \omega L_{1n}$.

Thus,

$$K(HC) = 1e^{j\phi} \approx 1, \quad (3)$$

where $\phi = \arctg(R_i/(\omega L_{1n}))$ ≈ 0.

From (1, 2) we can see that the transfer coefficient is a complex value, whose modulus characterizes the change in output voltage depending on frequency, and the argument is the phase shift of the output voltage concerning the transformer input voltage as a function of frequency.

Thus, $HC$ range for the transformer is characterized by the virtual absence of phase ($\phi = 0$) and amplitude $|K(HC)| \approx 1$ distortion of the output voltage. The value of the output voltage is determined with sufficient accuracy by the transformation coefficient

$$n = \frac{W_1}{W_2}, \text{ i.e. } u_2 = \frac{u_1}{n}.$$

For the midrange ($CC$)

$$K(CC) = \frac{R'_H}{R_1 + R_2 + R_h} \approx 1; \quad (\phi \approx 0) \quad (4)$$
because usually. \( R'_H \gg R'_1 \) и \( R'_H \gg R'_2 \).

Thus, the midrange is similar in its transmission properties to the \( HC \).

Now let's determine the transfer coefficient for high frequencies. Let's represent the scheme of Fig. 2, c in the form of Fig. 4.

![Fig. 4. Simplified transformer substitution diagram for high-frequency ranges.](image)

\[
K(BH) = \frac{I_1 Z_{23}}{I_1 (Z_1 + Z_{23})} = \frac{Z_2 Z_3}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3} = \frac{Z_2 Z_3}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}
\]

where \( Z_1 = R'_1 + R'_2 + j \omega L_S \); \( Z_2 = -j(\omega C'_0) \); \( Z_3 = R'_H \).

Since at high frequencies \( \omega L_S \gg R'_1 + R'_2 \), so

\[
K(BH) = \frac{-jR'_H}{\omega C'_0 \left[ j \omega L_S (-j(\omega C'_0)) + j \omega L_H R'_H - jR'_H / \omega C'_0 \right]} = \frac{-jR'_H}{\omega L_S + jR'_H (\omega L_S C'_0 - 1)} = \frac{-jR'_H}{\frac{\omega^2}{\omega_0^2} - 1 + j \omega L_S}
\]

where \( \omega_0 = \frac{1}{\sqrt{L_S C'_0}} \) is the resonant frequency of the circuit formed by \( L_S \) and \( C'_0 \).

By \( \omega \ll \omega_0 \)

\[
K(BH) = \frac{-jR'_H}{-j(R'_H + j \omega L_S)} = \frac{R'_H}{\left\lvert (R'_H)^2 + (\omega L_S)^2 \right\rvert^{1/2} e^{j \theta}}
\]

The above formulas show that the amplitude of the output voltage over the entire frequency range is less than that calculated from the transformation ratio.

### 3 Results

From a principle point of view, a transformer can always be designed so that its operation is described by one of the simplest equivalent diagrams (Fig. 2, a or 2, b). However, in many cases, such a solution is not always the best, for example, in terms of its thermal behavior, dimensions of mass, and reliability, which is very important in the design.

At the same time, it is of practical interest to know in advance either from the measured parameters of the finished transformer or from the calculated data in what mode it will work. For this purpose, it is necessary to know the frequency values separating the specified frequency intervals.
In the low-frequency region (Fig. 2, a) the loss resistance $R_{II}$ must not shunt reactance due to magnetization inductance $L_{1n}$, and also this resistance must not shunt a circuit made up of $R'_2$ and $R'_{II}$ (or $R'_{II}$, considering that $R'_{II} \gg R'_2$). An inequality follows from these two conditions: $R'_{II} \ll \omega L_{1n} \ll R_{II}$.

Setting quite an acceptable accuracy for practical calculations (2% error), we obtain

$$\omega L_{1n} \leq 0.2 R_{II} \quad \text{and} \quad \omega L_{1n} \geq R'_{II}.$$ Considering that $R_{II} \approx U_1^2 / P_M$ and $R'_{II} \approx U_1^2 / P$, we will have

$$L_{1n} \leq 0.2 U_1^2 / (\omega P_M) = 3.2 \cdot 10^{-2} U_1^2 / (P_M f) \quad \text{and} \quad L_{1n} \geq 0.8 U_1^2 / (P f)$$

($P$—output power of the transformer). Then, the frequency limiting $HC$-range, will be determined by the inequality

$$3.2 \cdot 10^{-2} U_1^2 / (P_M L_{1n}) \geq f \geq 0.8 U_1^2 / (P L_{1n}) \quad (7)$$

To find the upper boundary frequency $CC$-range (or the lower boundary of the $BC$-range) must be based on the condition $\omega L_S \ll R'_{II}$, i.e., neglect the influence of the scattering inductance on the transfer coefficient. If we limit ourselves (as before) to an error of 2% in determining $u_2$, it will result in a ratio of $5 \omega L_S \leq R'_{II}$, respectively for capacitance

$$1 / (\omega C'_0) \geq 5 R'_{II}.$$ Considering that $R'_{II} = U_1^2 / P$, retrieve

$$f \leq \left\{ \begin{array}{l} 3.2 \cdot 10^{-2} U_1^2 / (P L_S) \\ 3.2 \cdot 10^{-2} P / (U_1^2 C'_0) \end{array} \right\} \quad (8)$$

From the above it follows an important conclusion: to ensure the transmission of harmonic voltage with a transformer without frequency distortion of the signal in a given frequency range, it is necessary to impose the following restrictions on its parameters:

$$L_S \leq 3.2 \cdot 10^{-2} U_1^2 / (P f_B) ; \quad L_{1n} \geq 0.8 U_1^2 / (P f'_H) \quad (9)$$

where $f'_H$ and $f_B$—the lower and upper limit of the frequency range.

Example 1 Transformer made on PL 12,5*16-50 magnetic core from 50H alloy has the following parameters: $L_{1n} = 0.8 \, \text{Gh}$, $U_1 = 100 \, \text{V}$, $P = 200 \, \text{W}$, $L_S = 2 \cdot 10^{-4} \, \text{Gh}$. Determine the frequency range in which there will be no frequency distortion of the secondary voltage.

Lower boundary frequency: $f'_H \geq 0.8 U_1^2 / (P L_{1n}) = 50 \, \text{Gh}.$

Upper boundary frequency: $f_B \leq 3.2 \cdot 10^{-2} U_1^2 / (P L_S) = 8000 \, \text{Gh}.$

In converters, it is often necessary to transform a non-sinusoidal periodic voltage of a given shape into the load. Analyzing the expression for the transfer ratio of the transformer, it can be found that in the previously adopted conditional frequency ranges, called the low and medium frequency ranges, the value is practically constant. The boundaries of these frequency ranges are set by the following inequalities.
The upper boundary of the favorable frequency range is determined by the lower value of the frequency obtained from the right side of inequality (10). Since the used voltage waveforms decomposed into a Fourier series, have decreasing amplitudes of the higher harmonics with the increasing order number, it is advisable to design the transformer so that the frequency range (10) would be as wide as possible and it would contain the greatest number of harmonics with large amplitudes, which mainly determine the voltage waveform at the transformer output.

Example 2 Determine the parameters of a transformer transmitting a voltage to the load that has the shape shown in Fig. 5. Initial data: power $P = 200$ W; pulse repetition frequency $f = 400$ Gh; primary voltage amplitude $U_{1m} = 100$ V.

Decompose the primary voltage of a given shape in a Fourier series

$$u_1 = \frac{8U_m}{\pi^2} \sum_{1,3,5...}^{\infty} \frac{(-1)^{n-1}}{n^2} \sin n\omega t, \ n - \text{harmonic number}$$

Fig. 5. Periodic stress curve, for example 2.

Since the amplitudes of the harmonics decrease quite rapidly with increasing harmonic number, let us limit the frequency range in which $K = 1$, fifth harmonic, i.e. $f_v = 2 \cdot 10^3$ Gh. Find the necessary values by (10) $L_S$ and $C_0'$, considering that in order to consider the shape of the stress $U_{ef} = U_m/\sqrt{3}$, $R_h' = U_{ef}^2/P = U_{1m}^2/(3P)$

$$L_s \leq \frac{3.2 \cdot 10^{-2} R_h'}{f_v} = \frac{3.2 \cdot 10^{-2} U_{1m}^2}{f_v \cdot 3P} = 2.66 \cdot 10^{-4}$ Gh;

$$C_0' \leq \frac{3.2 \cdot 10^{-2} \cdot 3P}{f_v U_{1m}^2} = 0.96 \cdot 10^{-6} F.$$ 

4 Conclusion

Based on the use of the transfer coefficient, which is the ratio of the reduced load voltage to the input voltage of the transformer, the output voltage modulus and phase shift concerning the input voltage over a wide frequency range have been determined in this paper. With an accuracy acceptable for practical calculations, the relations between the parameters of the transformer's equivalent circuit are obtained, to provide harmonic voltage transfer without frequency distortion in a given frequency range. The results obtained can be recommended when using the filtering properties of power transformers [3-5] in the development of
secondary power sources [1, 7-10] and improving the efficiency of communication and information transmission [6, 11].

References

2. Yun Yang, Chengxiong Mao, Tianliu Wei, Dan Wang, Jie Tian, Multi-function combined operation and control strategy of electronic power transformer for power quality improvement (2017) doi.org/10.1002/tee.22474
8. A.A. Boburkhov, Sources of secondary power supply of onboard power amplifiers / Abstracts of the report at the 25th International Conference of Students and postgraduates "Radio Engineering, Electrical engineering and Power Engineering", Moscow, NRU MEI March 14-15, 31 (2019)
10. V.E. Patlakhov, Research and design of low-power high-frequency transformers optimal in mass. Dissertation., for the degree of Candidate of Technical Sciences, Samara State Technical University (Samara, 2006)