Excitation field and parameters of traction superconducting synchronous motor of pipeline transport

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Abstract. Movement of passenger crews in a rarefied air tube at speeds over 600 km/h involves the effect of magnetic levitation and a traction linear superconducting synchronous motor with a stator winding distributed along the length of the tube and a superconducting excitation system on the crew. Conditions of magnetic levitation stipulate that the position of the crew in the tube is characterized by six degrees of freedom, so a traction linear superconducting synchronous motor must be considered as a multi-stage dynamic system. However, in the "normal" operational mode, the problem is reduced to consideration of the system with one degree of freedom with the coordinate that determines the position of an inductor with superconducting winding and without ferromagnetic cores relative to the running magnetic field of the stator winding. The specifics of magnetic levitation of the moving crew, which consists in the possibility of its spatial displacement, cause the appearance of geometric parameters characterizing progressive and rotational perturbations of the coordinates of the center of inertia of the crew in expressions for mutual inductances between the superconducting excitation system, electromagnetic shield, and the stator phase windings.

1 Introduction

The movement of passenger crews in a rarefied air tube at speeds exceeding 600 km/h involves the effect of magnetic levitation and a traction linear superconducting synchronous motor (TLSSD) with a stator winding distributed along the length of the tube and a superconducting excitation system (SSV) on the crew [1].

The conditions of magnetic levitation stipulate that the position of the crew in the tube is characterized by six degrees of freedom, so it is necessary to consider the TLSSD as a multistage dynamic system. The need for rigid spatial restraint of fast-moving passenger crews in the limited inner space of the tube, which is related, first of all, to the safety of operation of the transport system under consideration, determines the need to take this specificity and the features of superconductor motor design into account when calculating the magnetic field and electromagnetic parameters of the TLSSD.

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2 Magnetic field

In the first approximation, the traction linear superconducting synchronous motor (TLSSM) can be represented by the scheme, the cross-section of which is shown in Fig. 1.

In the figure: $S$ is the symmetrical three-phase stator winding, $f$ is the SSV. The winding $S$ is rigidly fixed on the tube wall and the SSV on the crew, the length of the crew is assumed to be equal to $l_0$. The crew does not carry magnetic cores.

In contrast to the well-known Hyper Loop transport system [2-12], the design under consideration involves a departure from the spindle shape of the crew for reasons of stability concerning rotational perturbations (relative to the longitudinal axis of the crew).

The TLSSD is a system with six degrees of freedom, but in operation, the position of the crew is quite deterministic.

In the "normal" operating mode of the TLSSD, the speed of the running magnetic field of the stator winding and the speed of the crew is the same. Also, the gap ($\varepsilon$) between the parallel planes, in which the windings $S$ and $f$ are located, is unchanged. Therefore, the problem of investigation of TLSSD in "normal" mode is reduced to consideration of a single-stage system with inductor coordinate (SSV) relative to the running magnetic field of stator winding ($\theta$).

The position of the SSV will be determined in the reference frame $X_sY_sZ_s$ rigidly connected with the winding $S$ (Fig. 2).
The stator winding \( S \) lies in the coordinate plane \( Z_s = 0 \), and the axis \( X_s \) coincides with the longitudinal axis of the stator winding. The vector \( \mathbf{e}_{X_s} v_0^l + \mathbf{e}_{Z_s} q_0^l + \epsilon \) sets the position of the center of inertia of the inductor (point \( C \)), here \( v_0 \) is the velocity of the motion of the center of inertia at the “normal” mode. \( \mathbf{e}_{X_s}, \mathbf{e}_{Z_s} \) are the orts. In the future, the presence of the lower index "0" will indicate that it belongs to the "normal" mode. As the generalized coordinates of the translational motion of the inductor, we can take the projections of the vector \( \mathbf{e} \) on the
coordinate axes of the system $X_SY_SZ_S$ ($\mathbf{E}_X, \mathbf{E}_Y, \mathbf{E}_Z$). In "normal" mode we have $\varepsilon_x = \frac{\pi}{\tau} 0, \varepsilon_y = 0, \varepsilon_z = \varepsilon_0 = \text{const}$. 

To consider the possibility of rotational motion of the vehicle, we introduce two reference systems with origins at point C: the inertial system $XYZ$, which moves forward, and the system $xyz$, which is rigidly connected with the inductor. As generalized coordinates, we take the angles $\Phi_x, \Phi_y, \Phi_z$ rotation of the inductor around the $X, Y$ and $Z$ axes, respectively. The vectors of angular velocities coincide with the corresponding axes of the $XYZ$ system at given positive directions of angles $\Phi_x, \Phi_y, \Phi_z$.

The arbitrary position of the inductor can be considered as a combination of its three rotations, characterized by the following combinations of angles: $\phi_x \neq 0, \phi_x = \phi_z = 0$; $\phi_y \neq 0, \phi_y = \phi_z = 0$; $\phi_z \neq 0, \phi_x = \phi_y = 0$. Then the connection between the reference systems $XYZ$ and $X_fY_fZ_f$ is given by the following expressions:

$$X = x_f \cos \phi_y \cos \phi_z,$$
$$Y = y_f + q_0 \frac{\sin \phi_x}{\cos \phi_z} + x_f \cos \phi_y \cdot \sin \phi_z,$$
$$Z = z_f - q_0 \frac{\cos \phi_x}{\cos \phi_y} - x_f \sin \phi_y \cdot \cos \phi_z.$$

Communication between systems $X_SY_SZ_S$ and $XYZ$ is carried out by expressions: $X_S = X + v_0 t + \varepsilon_X, Y_S = Y + \varepsilon_Y, Z_S = Z + q_0 + \varepsilon_Z + \varepsilon_0$.

Let us calculate the magnetic field of the TLSSD. Due to the absence of ferromagnetic cores in the inductor, the system is linear, so the fields of windings $S$ and $f$ can be considered independently.

If we assume that the SSV is a periodic (along the axis $x_f$) The structure of electrically unconnected identical flat rectangular solenoids (with dimensions $2b\times2a$ on the middle coil) and arranged in the coordinate plane $Z_f = 0$, is the vector potential of the excitation field (neglecting the end effects) in the frame of reference $X_fY_fZ_f$ is defined by the ratios [3]:

$$\mathbf{A} = e_x A_x + e_y A_y,$$

where $e_x, e_y$ are the orts.

$$A_{x_f} = \sum_{n=1,3,...}^\infty \int a_n \sin \frac{\pi n}{\tau} x \sin ky \, dk,$$  

(1)
\[ A_{yf} = \sum_{n=1,3,...}^{\infty} \int_{0}^{\infty} a_y \cos \frac{\pi n}{\tau} x \cdot \cos ky \, dk, \]  

(2)

Here \( n \) is the harmonic number, \( k \) is the wave number, \( \tau \) is the pole division,

\[ a_x = -\frac{4\mu_0 f}{\pi^2 k n} \sin \frac{\pi n}{2} \alpha \beta e^{\pm k_n z}, \quad a_y = \frac{\pi n}{\tau k} a_x, \]

(3)

\( I_f \) is the excitation ampere turns, \( k_n = \sqrt{\left(\frac{\pi n}{\tau}\right)^2 + k^2} \), \( \alpha = \sin ka \), \( \beta = \sin \frac{\pi n}{\tau} b \).

Since the higher harmonics of the excitation field do not play a special role in the electromagnetic processes in the TLSSD because of the relatively high levitation height and due to the design measures known in the theory of electric machines, they can be further disregarded. In this case (1) and (2) take the form

\[ A_{xf} = \int_{0}^{\infty} a_x(1,k) e^{-k|z|} \sin \frac{\pi}{\tau} x_0 \cdot \sin ky \, dk, \]

\[ A_{yf} = \int_{0}^{\infty} a_y(1,k) e^{-k|z|} \cos \frac{\pi}{\tau} x_0 \cdot \cos ky \, dk, \]

where \( a_x(1,k) \), \( a_y(1,k) \), \( k_1 \), calculated using the expression (3) at \( n = 1 \).

The excitation field in the frame of reference \( X_S Y_S Z_S \) is defined by the following formulas:

\[ A_{Xs} = \int_{0}^{\infty} a_x(1,k) e^{k_1 \left[ Z_S - q_0 + \cos \varphi_y \cos \varphi_z + \operatorname{tg} \varphi_y \left( X_S - \varepsilon_X \right) \right]} \cdot \sin \frac{\pi}{\tau} \cdot \frac{X_S - \varepsilon_X - \nu_0 t}{\cos \varphi_y \cdot \cos \varphi_z} \cdot \sin k \left[ Y_S - \nu_0 t - q_0 \frac{\sin \varphi_y}{\cos \varphi_z} - \operatorname{tg} \varphi_y \left( X_S - \varepsilon_X \right) \right] \, dk, \]

(4)

\[ A_{Ys} = \int_{0}^{\infty} a_y(1,k) e^{k_1 \left[ Z_S - q_0 + \cos \varphi_y \cos \varphi_z + \operatorname{tg} \varphi_y \left( X_S - \varepsilon_X \right) \right]} \cdot \cos \frac{\pi}{\tau} \cdot \frac{X_S - \varepsilon_X - \nu_0 t}{\cos \varphi_y \cdot \cos \varphi_z} \cdot \cos k \left[ Y_S - \nu_0 t - q_0 \frac{\sin \varphi_y}{\cos \varphi_z} - \operatorname{tg} \varphi_y \left( X_S - \varepsilon_X \right) \right] \, dk, \]

(5)

which are obtained from (4) by substituting

\[ x_f = \frac{1}{\cos \varphi_y \cdot \cos \varphi_z} \left( X_S - \nu_0 t - \varepsilon_X \right), \]


\[ y_f = Y_S - \varepsilon_f - q_0 \frac{\sin \phi_x}{\cos \phi_z} - \left( X_S - \varepsilon_X \right) \tan \phi_z, \]

\[ z_f = Z_S - \varepsilon_Z + q_0 \left( \frac{\sin \phi_x}{\cos \phi_z} - 1 \right) + \left( X_S - \varepsilon_X \right) \tan \phi_y. \]

Let's assume that the three-phase winding \( S \) is designed as a two-layer winding and the contribution of inter-coil connections is insignificant, so that the stator phase magnetic fields in the reference frame \( X_S Y_S Z_S \) will have a structure similar to the excitation field. Using (1) - (3), we can obtain formulas for calculating the phase fields.

3 Electromagnetic parameters of the TLSSD

The integral electromagnetic interaction in the TLSSD is characterized by the stator winding inductance \((L)\), the SSV inductance \((L_f)\), and the mutual inductance \((M)\) between them. Only \( M \) is a function of the generalized coordinates. Other parameters are determined by the scheme and geometry of the corresponding windings. Let us define the parameter \( M \). The magnetic flux \((\Phi)\) of the excitation field penetrating one winding of the stator winding is defined as follows

\[ \Phi = \oint_A dA. \]

If the center of the coil is defined by the coordinates \( \chi, 0, 0 \), so

\[ \Phi(\chi) = \int_{-b+\chi}^{b+\chi} \left[ A_{x_f}(X_S,a,0) - A_{x_f}(X_S,-a,0) \right] dX_S + \]

\[ + \int_{a_c}^{-a_c} \left[ A_{y_f}(-b+\chi,Y_S,0) - A_{y_f}(b+\chi,Y_S,0) \right] dY_S. \]  

We obtain \( \Phi \) taking into account (4) and (5).

Rotation of inductor will lead to the difference of excitation field fluxes meshing with individual turns of stator winding; therefore, it is necessary to determine magnetic fluxes of all turns of the stator core, followed by their summation [12-14]. At the same time, it should be taken into account that the turns on the neighboring pole divisions are included in opposite directions. As a result, we obtain a flux linkage of the phases of the stator winding with the field of excitation:

\[ \Psi_{af} = w_0 \sum_{m=-(p_f-1)}^{p_f} (-1)^{m-1} \Phi \left[ \chi = \left( m - \frac{1}{2} \right) \tau \right], \]

\[ \Psi_{bf} = w_0 \sum_{m=-(p_f-1)}^{p_f} (-1)^{m-1} \Phi \left[ \chi = \left( m - \frac{1}{2} \right) \tau + \frac{2}{3} \tau \right], \]
\[ \Psi_{cf} = w_0 \sum_{m=-(p_f-1)}^{p_f} (-1)^{m-1} \Phi \left[ \chi = \left( m - \frac{1}{2} \right) \tau + \frac{4}{3} \tau \right]. \]

where \( \Psi_{af}, \Psi_{bf}, \Psi_{cf} \) are the flux linkages of the individual phases, \( p_f \) is the number of pole pairs of excitation, \( \Phi[\chi] \) is the magnetic flux of the coil, determined by (6) at the corresponding value of \( \chi \), \( w_0 \) is the number of coil turns per pole and phase (\( w_0 = q W_k \gamma \)), where \( q \) is the number of coils per pole and phase, \( W_k \) is the number of turns in the coil, \( \gamma \) is the distribution coefficient).

Assuming \( q_0 \sim d_0, q_0 << l_0, q_0 >>> v_0 \), we have

\[ \Psi_{af} = I_f M \cos(\omega t + \frac{\pi}{\tau} \varepsilon_X), \quad \omega = \frac{\pi}{\tau} v_0, \]

\[ \Psi_{bf} = I_f M \cos(\omega t + \frac{2}{3} \pi + \frac{\pi}{\tau} \varepsilon_X), \]

\[ \Psi_{cf} = I_f M \cos(\omega t + \frac{2}{3} \pi + \frac{\pi}{\tau} \varepsilon_X), \]

where

\[ M = \frac{16\mu_0 w_0 B_B}{\pi \tau} \sum_{m=-(p_f-1)}^{p_f} (-1)^{m-1} \sin(m - \frac{1}{2}) \pi \int_0^\infty e^{-k_1 (\varepsilon_y + q_0 (1 - \cos \phi_y) + (m - \frac{1}{2}) \tau \tan \phi_y)} \frac{\alpha \alpha}{k_1}. \]

\[ \cdot \left\{ \left( \frac{\tau}{\pi} \right)^2 \cos k(\varepsilon_y + q_0 \sin \phi_y) + \frac{1}{k^2} \cos k[\varepsilon_y + q_0 \sin \phi_y + (m - \frac{1}{2}) \tau \tan \phi_y] \right\}. \]

In the general case, in addition to the values of \( L, L_f \) and \( M \) is necessary to take into account the influence of the electromagnetic shield located on the crew. The screen can be equivalent to two identically identical windings located along the same straight line parallel to the SSV solenoids [15]. Suppose that the magnetic axis of one of them (d-winding) coincides with the magnetic axis of the SSV, while the other winding (q-winding) is shifted \( \tau/2 \) on the motion of the crew. The flat screen is located at a distance \( \Delta \) from the SSV plane.

The self inductance \( (L_e) \) of any of the aforementioned windings is determined by the ratio

\[ L_e = \frac{32\mu_0 \tau}{\pi^3} \sum_{n=1,3,5,\ldots}^{\infty} \int_0^\infty k_n \left( \frac{\alpha_e \beta_e}{nk} \right)^2 dk, \]

\[ \alpha_e = \sin k a_e, \quad \beta_e = \sin \frac{\pi n}{\tau} b_e, \quad k_n = \sqrt{\left( \frac{\pi n}{\tau} \right)^2 + k^2}, \]
where $2a_e (2b_e)$ is the width (length) of the screen winding turns. There is no magnetic coupling between the SSV and the $q$-winding.

The mutual inductance between the field winding and the $d$-winding

$$M_{fd} = \frac{16\mu_0 w_0 \tau}{\pi^3} \sum_{n=1,3,\ldots}^{\infty} \int_0^{\infty} k_n \alpha \beta e^{-nk^2} dk$$

When the crew occupies an arbitrary spatial position, the parameters $m_{ds}$ and $m_{qs}$ (maximum values of mutual inductances between the track structure and the $d$, $q$-winding, respectively), which determine the mutual inductance between the track structure and the shield, strictly speaking, are unequal. However, let us consider them to be the same $m_{ds} = m_{qs} = m$. The value $m$ can be determined similarly to $M$. For the first harmonic of the field we have

$$m = \frac{16\mu_0 w_0 \beta}{\pi \eta} \sum_{m=-1}^{\infty} (-1)^{m-1} \sin \left( m - \frac{1}{2} \right) \int_0^{\infty} e^{-k_1 \left[ \varepsilon_e + \varepsilon_y + \eta \cos \phi_e - (m - \frac{1}{2}) \tau \cos \phi_y \right]} k^2 \cos k \left[ \varepsilon_y + (r_0 + \Delta) \sin \phi_x \right] + \frac{1}{k^2} \cos k \left[ \varepsilon_y + (r_0 + \Delta) \sin \phi_x \right] +$$

$$\left( m - \frac{1}{2} \right) \tau \cos \phi_x \right] \right) \int_0^{\infty} dk,$$

4 Conclusion

1. When studying the magnetic field of a traction superconducting linear synchronous motor, it is acceptable to consider only the first harmonic component, since the higher harmonics do not contribute appreciably to the electromagnetic interaction due to the relatively large gap between the crew excitation system and the stator winding distributed along the tube.

2. The specifics of the magnetic levitation of the moving TLSSD crew, which consists of the possibility of its spatial displacement, cause the appearance of geometric parameters in expressions for mutual inductances between the superconducting excitation system, electromagnetic shield, and stator phase windings, which characterize progressive and rotational perturbations of the coordinates of the crew center of inertia.

References


