The stress state analysis of lattice tower support for 330 kV overhead line using the «USL» software package

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Abstract. In conditions of market economy, overhead power transmission line (OPTL) support structures must be low-cost and of guaranteed quality, the manufacture of which will require a minimum amount of steel. Therefore, it is necessary to improve overhead line supports, which is possible by clarifying the forces in the support elements and when designing using numerical methods. In this article, a mode of deformation of a corner dead-end support of OPTL is analysed, which is considered a spatially multiple indeterminate open system with flexible joints. Attention is given to the basic problems that can occur in setting initial parameters and constructing design models for such structures. There are a number of problems connected with a refined specification of internal longitudinal stresses in the components of an OPTL structure. In designing, the joint operation of the lattice components of the support spatial pattern model is analysed, and the inclusion of stiffening diaphragms and diagonal element members in the operation on all four faces is taken into account. On the basis of the design and the extension of the results, the internal forces obtained and the ones specified in the components of the pattern OPTL support on the similar values of loads were thoroughly compared. When calculating the spatial model of an anchor-corner support as a result of the joint work of lattice elements, a decrease in internal forces in the elements of the spatial model is observed on average by 6.8%, compared to the forces determined in a typical overhead line support from the same load values. It has been established that it is quite acceptable to calculate overhead line lattice structures in the USL software package since the error in determining the forces in the rods is within 2% and the computer time consumption is less than in other software packages.

1 Introduction

Currently, about 40 thousand km of overhead power transmission lines on metal supports are operated in the power systems of Russia, the total weight of which reaches 600...
thousand metric tons. The massive nature of the construction of overhead line supports raises the issue of increasing the efficiency, durability, and reliability of energy construction; therefore, finding additional ways to save steel during the construction of high-voltage lines and reviewing existing standard projects is an important task [1-4].

In the market economy, overhead line support structures should be low-cost and of guaranteed quality, for the manufacture of which a minimum amount of steel will be consumed. Therefore, it is necessary to improve the overhead line supports, which is possible by clarifying the efforts in the elements of the supports and when designing using numerical methods [5-7].

An overhead power transmission line is a complex engineering structure in which flexible elements (wires and cables) work together with rigid ones (supports), and at the same time, the entire network is prestressed [8-11].

The elements of the supports are considered spatial systems loaded with forces that are also located in space. These elements, in most cases, have a prismatic or pyramidal shape with small angles of inclination of the belts to the longitudinal axis. There is an opinion [12, 13] that in these cases, it is sufficient to calculate spatial elements by breaking down loads into components in the planes of the faces and reducing them to the calculation of flat trusses under the action of a system of forces lying in the plane of the truss.

This article [14-16] outlines the basic principles of calculating complex engineering structures implemented in the SCAD PC and similar computing packages based on the finite element method.

2 Materials and methods

Consider the application of the finite element method (FEM) for spatial hinge-rod systems, which are overhead line structures, in the USL software package. When using this method, it is possible to clearly formulate three groups of equations, as is done in the theory of elasticity: 1) static, describing the equilibrium state of the system; 2) geometric, establishing a connection between deformations and displacements; 3) physical, which links forces and deformations [17, 18].

For example, consider the lower section of the anchor-angular support U330-2+9, shown in Figure 1. In such a spatial truss, there are 44 nodes and 104 rods (excluding horizontal diaphragms). Some nodes, due to design features and accepted assumptions, can only move in a certain plane [19-21]. In the overhead line support, such nodes are located on the side faces at the intersection of the struts. If the coordinate axis system is arranged in such a way that one of them is normal to the face of the support, then the static equation corresponding to this axis cannot be set up. Therefore, in this truss, the number of static equations is 100: for 20 nodes on the edges, 3 equations are set up for each; for 20 nodes on the side faces, 2 equations are set up for each. The supporting four nodes are fixed with ball bearings. The degree of static indeterminacy is 4. It is impossible to solve the problem of determining the forces in the rods of such a spatial truss using only the equations of equilibrium. Such systems are statically indeterminate [22, 23].

Denote by $A(m,n)$ the matrix of coefficients for unknown quantities in the system of static equations, $\vec{F}$ - the vector of external nodal loads, - the vector of unknown quantities (forces in the rods). The system of static equations in matrix form has the form [24] (1):

$$A \cdot \Delta = \vec{F}$$

System of geometric equations (2):

$$\Delta = A^* \cdot \bar{U} \quad \text{or} \quad \bar{U} = A^* \cdot \Delta = 0,$$
where \( \overline{A} \) is a matrix, transposed with respect to a static matrix \( A \), \( \Delta = \{\Delta_1, \Delta_2, \ldots, \Delta_n\} \) - vector of deformations and \( \overline{U} = \{u_1, u_2, \ldots, u_n\} \) - vector of displacements.

System of physical equations (3):
\[
\overline{N} = \overline{K} \cdot \Delta
\]
where \( \overline{K} = (k_{ij}), (i = j = 1, 2, \ldots, n) \) – diagonal stiffness matrix (4):
\[
k_{ij} = \begin{cases} E \cdot S_i / l_i & i = j \\ 0 & i \neq j \end{cases}
\]
here \( E \) – modulus of elasticity, a constant value; \( S_i \) – the cross section of the \( i \)-th rod; \( l_i \) – the length of the \( i \)-th rod.

Thus, we have three types of equations.

The total number of equations in this system is \( m + 2n \). There are also \( m + 2n \) unknowns in this system: \( n \) forces in the rods, \( n \) deformations of the rods, \( m \) projections of node displacements on the coordinate axis. Such a system of equations has a unique solution. Consequently, for a hinge-rod elastic system, we obtain a single distribution of forces, deformations and displacements from the given loads. Such a system is a mathematical model for calculating a hinge-rod elastic system.

From the system of physical equations, we express the vector of forces through the vector of deformations and substitute it into the system of static equations (5):
\[
\overline{A} \cdot \overline{K} \cdot \Delta = \overline{F}
\]

In the resulting matrix equation, instead of the deformation vector, we substitute an expression from the geometric system (6):
\[
\overline{A} \cdot \overline{K} \cdot \overline{A}^{-1} \cdot \overline{U} = \overline{F}
\]

In the last matrix equation, the displacement vector is unknown, and the number of equations in the last system coincides with the number of unknowns. Solving this system, we have (7):
\[
\overline{U} = (\overline{A} \cdot \overline{K} \cdot \overline{A}^{-1})^{-1} \cdot \overline{F} = \overline{B}^{-1} \cdot \overline{F}
\]
where \( \overline{B} = (\overline{A} \cdot \overline{K} \cdot \overline{A}^{-1}) \), and \( \overline{B}^{-1} \) is its inverse matrix.

After finding the displacement vector, using a system of geometric equations, it is possible to find the deformation vector (8):
\[
\overline{U} = (\overline{A} \cdot \overline{K} \cdot \overline{A}^{-1})^{-1} \cdot \overline{F} = \overline{B}^{-1} \cdot \overline{F}
\]

And, finally, the vector of forces in the rods is found from the system of physical equations (9):
\[
\overline{N} = \overline{K} \cdot \Delta = \overline{K} \cdot \overline{A}^{-1} \cdot (\overline{A} \cdot \overline{K} \cdot \overline{A}^{-1}) \cdot \overline{F}
\]

Denote by \( \overline{C} \) the matrix (10):
\[
\overline{C} = \overline{K} \cdot \overline{A}^{-1} \cdot (\overline{A} \cdot \overline{K} \cdot \overline{A}^{-1})^{-1}, \text{ then } \overline{N} = \overline{C} \cdot \overline{F}
\]

If we calculate the matrix elements \( \overline{C} \), then by multiplying the matrix \( \overline{C} \) by the load vector, we can calculate the forces in the rods, depending on various external loads. The expression for the matrix \( \overline{C} \) is very cumbersome, since to obtain the matrix \( \overline{C} \), you need to multiply five matrices and calculate the inverse for one. If the elements of all matrices are...
stored in machine memory and matrix multiplication is performed directly, then it will take a lot of time.

![Diagram](image)

**Fig. 1.** Spatial design scheme for the lower part of support U330-2+9: a – design model; b - numbering of belts and edges.

Let's approach the solution to this problem from the point of view of saving time, i.e. we will reduce the number of mathematical operations.

Considering the matrices \( \overline{A} \) and \( \overline{B} \) for our example with support (Fig. 1), we can see that most of the coefficients are zero, and therefore it is rational to work only with non-zero elements. So, in the matrix \( \overline{A}(m, n) \) of 10400 elements, only 508 elements are non-zero; in the matrix \( \overline{B}(m, n) \) of 10,000, 1484 are non-zero, and taking into account symmetry, only 742.

After the non-zero elements of the matrix \( \overline{B} \) are placed in memory, we can proceed to solving the matrix equation (11):

\[
\overline{B} \cdot \overline{U} = \overline{F}
\]  

(11).

The result of its solution is a displacement vector \( \overline{U} \). After finding the displacement vector from the geometric equations, the deformation vector is found, and then, using physical relations, the force vector.

To solve the above matrix equation, it is convenient to use the Khaletsky method [5]. This method consists of the fact that the matrix \( \overline{B} \) is represented as a product of two triangular matrices, one of which has zeros above the main diagonal and the other below the main diagonal, and along the main diagonal - ones. At the same time, since the matrix
is symmetric, the elements of the upper triangular matrix are expressed in terms of the elements of the lower triangular matrix.

Note that one lower triangular matrix can be stored in memory. There will not be very many non-zero elements in this matrix, although there are already significantly more than in the matrix \( B \). The number of non-zero elements can be counted without calculating the matrix directly. The fact is that the lower triangular matrix in the Khaletsky scheme is formed by columns, and in some row non-zero elements appear no earlier than in the row under the same number in the matrix \( B \). In the calculation of the two-chain anchor-angular support U330-2+9, the number of non-zero elements is approximately 1300.

### 3 Results and discussions

#### 3.1 Analysis of anchor-corner support U330-2+9

The purpose of this calculation is to analyse the stress-strain state of the anchor-angular support of the overhead line in software systems and compare the results of calculating the internal forces with a typical calculation.

The actual design scheme of a metal lattice tower-type support is a spatial rod repeatedly statically indeterminate system with hinged nodes.

The calculation of the overhead line support, taking into account all its properties, exact geometric dimensions, and strict interaction of elements in nodes, is not implemented at the present stage due to its complexity. Therefore, in a typical design, the spatial structure of the overhead line support, which perceives and transmits all loads and impacts to the foundations, is replaced by design schemes and divided into elements—flat trusses (Fig. 2, a). The support is schematized, and secondary factors that do not affect the reliability and required accuracy of the calculation are discarded. Studies of the actual operation of tower-type lattice supports have shown that such an approximation leads to very small errors in the magnitude of the normal forces acting on the rods of the entire system.

The spatial model of the overhead line support is shown in Figure 2, b.

To create a spatial model of the support, the following assumptions were made.

In the design scheme of the rod structure, the rods were replaced by their longitudinal axes.

The real support devices were replaced by ideal support links, and the collected loads from the surface of the rods were transferred to the axes.

The calculation of the anchor-angular support of the overhead line U330-2+9 is performed for loads in the 3rd wind region for normal and emergency modes, i.e., for 2 loading schemes [3] (Table 1).

The sequence of input of initial data for creating models is: construction of a calculation scheme, description of the conditions for fixing the support structure in space, assignment of the rigidity of structural elements, creation of loading schemes for the support structure, compilation of calculated combinations of loads, calculation, analysis of calculation results, and comparison of the internal forces obtained in the design schemes.

The static component corresponding to the steady-state velocity pressure is taken according to SP 13330.2016 (4th edition) "Loads and Impacts" equal to \( q_0 = 0.5 \) kPa. The total wind pressure on the structure of the support \( P_{\text{calc}} = 9630 \) kg.
Table 1. Scheme of design loads to the support U330-2+9 compression method [8]

<table>
<thead>
<tr>
<th>Mode</th>
<th>Mode characteristics</th>
<th>Load scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>Normal mode</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td>The wires and cable are not broken and covered with ice.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>The wind is directed along the axes of the traverse.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$t = -5{}^\circ{}C$; $C = 20$ mm.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$q_n = 15$ kg/m²; $q^n = 20$ kg/m².</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4th ice region.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\alpha = 60^\circ$. Without a gravity difference.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Wire 2xAC 400, cable C-70.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>The scheme is calculated for the support trunk belts.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>The wire giving the highest bending moment to the support</td>
<td></td>
</tr>
<tr>
<td></td>
<td>is broken.</td>
<td></td>
</tr>
<tr>
<td></td>
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<td></td>
</tr>
<tr>
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<td>$t = -5{}^\circ{}C$; $C = 20$ mm; $q = 0$</td>
<td></td>
</tr>
<tr>
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<tr>
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<td>Wire 2xAC 400, cable C-70.</td>
<td></td>
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<tr>
<td></td>
<td>The scheme is calculated for the struts of the trunk of the</td>
<td></td>
</tr>
<tr>
<td></td>
<td>support and the belt of the traverse.</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 2. Design model of anchor-corner support U330-2+9

After the calculation, the internal forces in the overhead line support rods (longitudinal compression and stretching forces) were obtained for each load scheme. Based on the calculation results, the maximum forces arising in the support elements were determined, their analysis was performed, and the calculation results obtained in software complexes were compared with the forces determined when calculating a flat model of a typical support.

The internal forces obtained when calculating the lower part of the trunk of the anchor-angular support U330-2+9 are shown in Table 2, and the displacements of the entire trunk are shown in Table 3.
Table 1. Scheme of design loads to the support U330-2+9

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<td>The wire giving the highest bending moment to the support is broken. The cable is not broken. $t = -5^\circ C; C = 20 mm; q = 0$.</td>
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4th ice region. $\alpha = 60^\circ$. Without a gravity difference. Wire 2xAC 400, cable C-70. The scheme is calculated for the support trunk belts.

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**Table 2.** Results of determining the design forces in the lower part of the support shaft U330-2+9

<table>
<thead>
<tr>
<th>Design mode</th>
<th>Rod type</th>
<th>Location of the rod</th>
<th>Calculated forces in the rods of the lower part of the support, t</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>SP «USL»</td>
</tr>
<tr>
<td>Normal mode</td>
<td>Belt</td>
<td>Lower</td>
<td>163,9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Upper</td>
<td>153,3</td>
</tr>
<tr>
<td></td>
<td>Lattice</td>
<td>Lower</td>
<td>4,06</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Upper</td>
<td>8,01</td>
</tr>
</tbody>
</table>

![Design model of anchor-corner support U330-2+9](image)
4 Conclusions

1. When calculating the spatial model of the anchor-angular support of the overhead line U330-2+9, as a result of the joint work of the lattice elements (due to the inclusion of stiffening diaphragms and braces on all four faces), there is a decrease in internal forces in the elements of the spatial model by an average of 6.8%, compared with the forces determined in the typical support of the overhead line on the same load values.

2. The analysis of the results presented in table 2 confirms the position that when a load is applied in one of the planes of symmetry of the support, the face braces located in planes perpendicular to this load are included in the work, which in turn loads the belts of the lower section of the lattice structure of the support compared to the design of the support calculated by the method of breaking down into flat trusses (as is customary with this type of calculation).

3. A numerical example of the calculation of the support U330-2+9 shows that it is quite acceptable to calculate the lattice structures of overhead lines in the software package "USL," developed at the Moscow State University of Civil Engineering (National Research University), because the error in determining the forces in the rods is within 2%, and the cost of machine time is less than when calculating in the "SCAD" software package.

References

1. Ulitin V.V., GIORD, 96 (2007).


