# Mathematical models of MOS transistors with induced and ion-doped conditions in energy engineering

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**Abstract.** Mathematical models of integrated circuit (IC) elements are one of the foundations in energy engineering in setting and solving IC design problems. When developing an IC, its elements and the circuit itself are created simultaneously, so the IC developer's natural desire is to use models of elements relative to structural parameters, which makes it possible to analyze and optimize the characteristics of the IC before producing trial batches. To date, a number of Metal Oxide-Semiconductor (MOS) transistor models have been developed regarding structural parameters, but their use, as a rule, does not provide acceptable analysis accuracy and in some cases leads to unnecessary computer time consumption, which is explained by their complexity. The issue of creating mathematical models of MOS transistors and IL-channels, which have found wide application in the creation of integrated circuits, is especially acute, but simple and, at the same time, sufficiently accurate mathematical models have not been created to date. Therefore, the main goal of this chapter is to conduct a comparative analysis of existing models of MOS transistors with induced and ion-doped channels and develop models that largely eliminate the shortcomings of the existing ones. The issues of determining the electrical parameters of MIS structures are also considered and experimental results of studying the accuracy of the developed models are presented.

## 1 Introduction

Analysis of the characteristics of integrated circuits under various conditions (changes in ambient temperature, operating mode, test tests), statistical analysis and optimization of electrical characteristics impose significant requirements on the efficiency of the model, which, while maintaining acceptable accuracy of the model, can be assessed by two factors:

- the cost of computer time to solve the problem:
- the quantitative composition of the parameters determined by the model and the complexity of their measurement.

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The first costs are determined by the complexity of the algebraic expression of the mathematical model. Measuring the parameters of a transistor is a very labor-intensive process, so to reduce these costs one should strive to reduce their number.

The requirements for satisfactory accuracy and high efficiency of the model are contradictory. As a rule, more accurate models are described by more cumbersome expressions that require more computer time to solve problems. However, practice shows that it is possible to find simple analytical dependencies for mathematical models while maintaining high accuracy. It should be noted that the accuracy of the analysis of IS characteristics is currently mainly determined by the accuracy of the models, because The error of numerical methods can be reduced to fractions of a percent. Machine design methods replace prototyping, which leads to an error of up to 15-20%. Therefore, this value can be taken as the upper limit of the error of machine analysis methods.

The model must be universal. A universal model is understood as a model that describes an element in a wide range of temperatures, currents and voltages; designed, as a rule, to solve a wide range of problems. However, such models lead to a decrease in their efficiency. Currently, developers are moving towards creating a library of models. Each model is designed for a specific class of circuits. This approach is more appropriate. Therefore, when developing models, one must strive to satisfy the listed requirements for efficiency, accuracy and versatility.

## 2 Materials and methods

Mathematical models of an MIS transistor are obtained based on equations describing the processes of charge transfer in the inverse layer, taking into account the assumptions

$$\frac{dp}{dt} = \frac{1}{q}\frac{djp}{dy} \tag{1}$$

Transport equation

$$jp = qpMpEy - qDp\frac{dp}{dy} \tag{2}$$

where q is the electron charge; Mp – hole mobility; p – hole concentration; Ey – longitudinal field strength in the channel; Dp – coefficient.

Where the field effect can be described by the Poisson equation:

$$\frac{d^4p}{dx^2} = -\frac{q(p-n+N_D-N_A)}{E_nE_o} \tag{3}$$

where n is the electron concentration; $N_D$– donor concentration; $N_A$– concentration of acceptances.

Let's substitute equation (2) into (1) using Einstein's equation $D_p = \frac{kT}{q}M_p$, and $Ey = -\frac{d\varphi}{dy}$. Ignoring the resulting expression for the channel depth, denoting the specific charge of holes $Q_r = q\int_0^{xk} pdx$, superficial effective mobility $M_{reff} = \frac{q}{Qp}\int_0^{xk} pM_p dx$ and, replacing approximately $\int_0^{xk} M_p\frac{dp}{dy}dx \cong \frac{d}{dy}\int_0^{xk} M_p pdx = \frac{d}{dy}(Q_rM_p)$, we obtain the continuity equation for the specific charge

$$\frac{dQ_r}{dt} = \frac{d}{dy}\left(Q_rM_{reff}.\frac{d\varphi}{dy} + \varphi_t\frac{d}{dy}(M_{reff}Q_r)\right) \tag{4}$$

Assuming that the charge is mainly localized at the surface of the semiconductor (i.e., neglecting the change in potential in depth from 0 to Xk Fig. 1), we obtain, based on the

Gauss theorem, for an arbitrary cross section of the MTD structure (provided that the transverse field in the inverse layer significantly more than longitudinal) condition of electrical neutrality:

$$Q_z + Q_r + Q_{pk} + Q_{os} = 0 \tag{5}$$

where $Q_z, Q_{pk}, Q_{os}$ are the specific charges of the gate, surface states and depletion layer.

Depletion layer charge $Q_{os}$

$$Q_{os} = C_g A\sqrt{-4} \tag{6}$$

where $A = \frac{1}{Cg}\sqrt{2gE_nE_oN_d}$ – coefficient taking into account the influence of the depleted layer of the substrate with the impurity concentration $N_D$; $C_g = \frac{E_gE_o}{t_g}$ – specific capacitance of dielectric with thickness *tg*; gate charge:

$$Q_3 = C_g(U_3 - \varphi_{MDP} - \varphi) \tag{7}$$

$\varphi_{MDP}$ – contact potential difference in the MIS structure.

Substituting formulas (7) and (6) into (5), we find the charge of holes:

$$Q_p = -C_g(U_3 - \varphi_{MDP} - \varphi) - Q_{пс} - C_g A\sqrt{-4} \tag{8}$$

Let's denote $U_o$ – threshold voltage of the MIS device at which surface inversion occurs:

$$U_o = U_{MDP} - \frac{Q_{pk}}{C_g} - \varphi_0 - U_v \tag{9}$$

where $U_в = A\sqrt{\varphi_0}$; $\varphi_0 = 2{,}4$ – surface inversion potential; $\varphi_f$ – Fermi potential.

Substituting formulas (9) and (8) into (4) and using the absolute values of quantities (voltages and charges), we obtain a differential equation relating $\varphi$ and $Q_p$ :

$$\frac{d\varphi}{d_y} = \frac{d}{d_y}(M_{ref}(U_3^{/} - \varphi - A\sqrt{4})\frac{d\varphi}{d_y} - \varphi_t\frac{d}{d_y}(M_{ref}(U_3^{/} - \varphi - A\sqrt{4}))) \tag{10}$$

$$U_3^{/} = U_3 - U_0 + \varphi_0 + U_в \tag{11}$$

$$Q_p = C_g(U_3^{/} - \varphi - A\sqrt{4}) \tag{12}$$

$$U_0 = \varphi_{MDP} + \frac{Q_{pk}}{C_g} + \varphi_0 + U_v \tag{13}$$

The boundary conditions for expression (10) are:

when $R_c = R_u = 0$, $\varphi(0,t) = U_u^{/} + \varphi_0$, $(l,t) = U_c^{/} + \varphi_0$

when $R_c \neq R_u \neq 0$, $\varphi(0,t) = U_u + I_uR_u + \varphi_0$, $(l,t) = U_c + I_cR_c + \varphi_0$

where $U_u^{/}$, $U_c^{/}$- direction of source and drain, respectively, for the “active” region of the transistor (Figure 1).

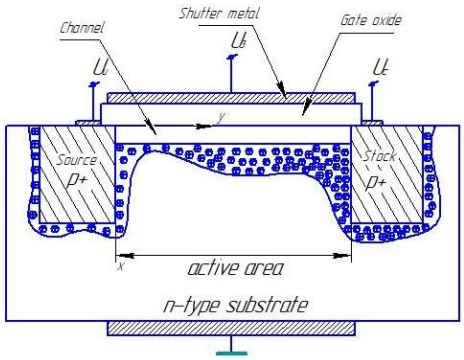


**Fig. 1.** Active region of the transistor

## 3 Research and results

Based on formula (10) at $\frac{dy}{dt} = 0$ when a statistical model was obtained. For a steep range of current-voltage characteristics, the diffusion component of expression (10) can be neglected. As a result, for the channel current we write :

$$I_c = -Q_p M_{ref} \frac{\partial y}{\partial t} \tag{14}$$

Averaging $M_{ref}$ over the length of the channels and introducing a new variable $\mathrm{U} = \varphi - \varphi_0$ substituting $Q_p$ into equation (14) and integrating it over the channel length from $U_u$ and $U_c$, taking into account the above boundary conditions, we obtain for the steep range of current-voltage characteristics:

$$I_c = \frac{1}{L}\int_{U_u}^{U_c} M_{ref}, Q_p dU = \frac{M_{ref} C_g W}{L}\Big\{(U_3 - U_c + U_b - U_u)(U_c - U_u) - \frac{(U_c - U_u)^2}{2} - \\ -\frac{2}{3}A\big((2\varphi_f + U_c)^{\frac{3}{2}} - (2\varphi_f + U_u)^{\frac{3}{2}}\big)\Big\} \tag{15}$$

For a flat region, we define the cutoff voltage as the voltage at which $Q_p = 0$. Then from formula (12) we find:

$$U_{OTK} = U_3 - U_0 - A\Big(\sqrt{\Big(\sqrt{(\varphi_0 + \frac{A}{2})^2} - U_3 - U_c} - \Big(\sqrt{\varphi_0} + \frac{A}{2}\Big)\Big) \tag{16}$$

$$U_0 = 2\varphi_f$$

For this mode, when $U_c > U_{OTK}$.

$$I_c = \frac{l_{OTK}}{1 - \frac{\Delta L}{L}} \tag{17}$$

where $l_{OTK}$– channel current, determined from formula (15) at $U_c = U_{OTK}$, and $\Delta L = B\sqrt{U_c - U_{OTK}}$is the width of the cutoff region by the amount by which the initial channel length $L$ decreases, where B is a constant.

This model does not take into account the dependence of $\mathrm{M}_{ref}$ on longitudinal and transverse fields, which is significant especially for small channel lengths (L≤5 μm) and the temperature dependence of electrical parameters.

In the model of Forman-Benchkovsky and Vadash [4], taking into account the dependence of $U_{ref}$ on $E_x$ is proposed in the following formula:

$$M_{ref} = M_0 \left[\frac{E_{kx}}{E_{xcp}}\right]^{C_1} \tag{18}$$

where $E_{kx} = 6 * 10^4 \frac{S}{cm}$ ; $C_1$ – Empirical constant = 0,15 for the ZP channel; $E_{xcp} = (U_3 - U_o + U_b + 0.5U_c) * \frac{\varepsilon_g}{\varepsilon_p X_g}$ – transverse field averaged over the channel length.

Substituting formulas (18) and (12) into (14) and integrating the last expression over the channel length, we obtain in the general case an implicit expression:

$$I_c = \frac{M_o C_g W}{L}\left[\frac{E_{kx}\varepsilon_p X_g}{E_g(U_3 - U_o + U_b + 0.5U_c)}\right]^{C_1} \int_{U_u + I_c R_u}^{U_c - I_c R_c} (U_3 - U_o + U_b - U - \sqrt{\varphi_o + U})du \tag{19}$$

where is $I_c$ found using the iterative method.
When $R_c = R_u = 0$ from (19) we get:

$$I_c = \frac{M_o C_g W}{L}\left[\frac{E_{kx}\varepsilon_p X_g}{E_g(U_3 - U_o + U_b + 0.5U_c)}\right]^{C_1}$$

$$\left\{(U_3 - U_o + U_b - U_u)(U_c - U_u) - \frac{1}{2}(U_c - U_u)^2 - \frac{2}{3}A\left[(\varphi_o + U_c)^{\frac{2}{3}} - (\varphi_o + U_u)^{\frac{2}{3}}\right]\right\} \tag{20}$$

In the prologue area: $I_c(U_c > U_{\text{отс}}) = \frac{I_{OTK}}{1 - \frac{\Delta L}{L}}$ (21)

$$\Delta L = \frac{U_c^{/} - U_{OTK}}{E_{xcp}} \tag{22}$$

$$\frac{1}{\Delta L} = \frac{1}{k(U_c^{/} - U_{OTK})^{\frac{1}{2}}} + \frac{\varepsilon_g}{\varepsilon_p X_g} * \frac{a(U_c^{/} - U_3^{/}) + b(U_3^{/} - U_{OTK}^{/})}{U_c^{/} - U_{OTK}^{/}} \tag{23}$$

The temperature dependence of the parameters is taken into account as follows:

$$M_o(T) = M_o(T_o)\frac{T_o}{T};\ \varphi_f = \varphi_T ln\frac{N_D}{n_i};\ n_i = \sqrt{1.5 * 10^{33} T^3 \exp(-1.21\, {}^{g}/_{kt})} \tag{24}$$

The disadvantages of the model are:

1. Not taking into account the dependence of $M_{ref}$ on $E_y$, which leads to large errors at L≤ 5 μm.

2. To calculate I_c, it is necessary to perform two iterative cycles:

- one – to calculate the boundary between steep and flat areas;

- the second – to calculate the current taking into account the parasitic resistances $R_u$ and $R_c$. This requires a lot of computer time.

In [1], to take into account the influence of longitudinal and transverse fields on the mobility $M_{ref}$, a semi-empirical formula is used:

$$M_{ref} = \frac{M_0}{(1 + \frac{E_x}{E_{kx}})(1 + \frac{E_y}{E_{ky}})} \tag{25}$$

where $M_o$is the mobility of holes in weak fields; $E_{kx}, E_{ky}$ – critical values of field strength at which mobility will decrease by 2 times.

$$E_x = \frac{\varepsilon_g}{\varepsilon_п} * \frac{U_3^{/} - U}{tg};\ |E_y| = \left|\frac{d\varphi}{dy}\right| \tag{26}$$

Taking into account formulas (25) and (26), we can rewrite formula (10)

$$\frac{\partial\varphi}{\partial t}=\frac{M_0}{1+\frac{4}{2}\sqrt{4}}*\frac{\partial}{\partial y}\left\{\frac{U_3^{/}-\varphi-A\sqrt{4}}{\left[1+\frac{(U_3^{/}-\varphi)}{U_{kx}}\right]\left[1+\frac{L}{U_{ky}}\frac{\partial\varphi}{\partial y}\right]}-\frac{\partial}{\partial y}\left[\frac{U_3-\varphi-A\sqrt{4}}{\left[1+\frac{(U_3^{/}-\varphi)}{U_{kx}}\right]\left[1+\frac{L}{U_{ky}}\frac{\partial\varphi}{\partial y}\right]}\right]\right\} \tag{27}$$

$U_{kx}=\frac{\varepsilon_{\text{п}}}{\varepsilon_y}tgE_{kx};\ U_{ky}=LE_{ky}.$

Based on (27), a static model was obtained [2] with $\frac{\partial\varphi}{\partial t}=0$ and without taking into account the diffusion component in the steep region [3].

Substituting $\mathrm{M}_{ref}$ and $Q_p$ into formula (14) and integrating the resulting expressions along the channel length from 0 to L, we obtain:

$$I_c=\frac{M_0C_gU_{kx}U_{ky}W}{L[U_{ky}+U_c-U_u-I_c(R_c+R_u)]}\int_{U_u+I_cR_u}^{U_c-I_cR_c}\frac{U_v-U_o+U_b-U-A\sqrt{\varphi_0+U}}{U_{kx}+U_v-U_0+U_b-U}du \tag{28}$$

when $R_c=R_u=0$:

$$I_c=\frac{M_0C_gU_{kx}U_{ky}W}{L[U_{ky}+U_c-U_u)]}\int_{U_u^{/}}^{U_c^{/}}\frac{U_v-U_o+U_b-U-A\sqrt{\varphi_0+U}}{U_{kx}+U_v-U_0+U_b-U}du \tag{29}$$

Expression (28) is an implicit expression of the current-voltage characteristics of the MOS transistor. The current $I_c$ at given control voltages is determined by numerical methods.

The integral $\int$ consisting in formula (28) is expressed through elementary functions:

$$J_1=J_1-AJ_2;J_1=U_c^{/}-U_u^{/}-U_{kx}ln\frac{U_v-U_0+U_b-U_u^{/}-U_{ky}}{U_v-U_0+U_b-U_c^{/}-U_{kx}} \tag{30}$$

$$J_2=2(Z_1-Z_2)-aln\frac{(a+Z_2)(a-Z_1)}{(a-Z_2)(a+Z_1)};Z_1=\sqrt{\varphi_0+U_u^{/}} \tag{31}$$

$$Z_2=\sqrt{\varphi_0+U_c^{/}};a=\sqrt{U_3-U_o+U_b+\varphi_0+U_{kx}} \tag{32}$$

Equation (28-29) describes the current-voltage characteristics in the steep region. For a flat area with $U_c>U_{OTK}$, expressions (21-23) are used.

The temperature dependence of the parameters is taken into account by expressions (24).

The disadvantages of the model are:

Cumbersome expressions for $I_C$ and $\Delta L$, which leads to a lot of computer time;

Incorrect form of taking into account the influence of the longitudinal field, which does not provide reliable access to the cutoff region

## 4 Conclusion

An increase in accuracy in the design model was achieved by taking into account the dependence of mobility $M_{ref}$ on fields $E_x, E_y$ in differential form, which significantly complicated the mathematical expression of the model. This account was made on the assumption of large voltage drops along the length of the channel, which can be justified when designing discrete MOS transistors. In integrated MOS transistors, such drops do not exceed 10-15V, therefore, as practice has shown, the mobility can be averaged over the channel length.

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