Braking force measurement errors on a stand with horizontal support discs

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Abstract. The authors have developed and are investigating the original design of a stand for testing the braking properties and suspension of wheeled vehicles, which provides the shape of the tyre contact spot with the support surface closest to real road conditions. The originality of the stand was confirmed by the decision to grant a patent for an invention of the Russian Federation. The advantages of implementing the proposed stand design, as well as disadvantages and ways to overcome them, are considered.

1 Introduction

Power-type roller stands are widely used to evaluate braking properties of wheeled vehicles (WV) and transport production machines (TPM) during technical inspections, under operating conditions, at service stations and motor transportation companies. However, at all known advantages of roller stands (convenience and simplicity of application, compactness, universality in relation to the types of tested vehicles, possibility to ensure promptness of tests and high level of automation of measurements, etc.), the results of WV tests on them can significantly differ from the results of road tests. One of the main reasons for this is the discrepancy between the contact patch, directions, and values of the acting forces and reactions of the wheel on the support rollers and the contact patch, and the acting forces and reactions of the wheel of the vehicle travelling on the road.

2 Methods and Materials

In [1], a power braking bench of original design is proposed, in which the shape of the tyre contact patch with the support surface, which is closest to real road conditions, is provided during the testing of the braking properties of the car.

The stand design consists of two modules with original support devices - in the form of horizontal support discs driven in rotation by a traditional power drive, the schemes are shown in Figures (Fig. 1-2).

Each module of the stand consists of a frame (1) with bores (2), guides (3) for vertical movement of the frame, fixed stationary. On each frame is mounted a support disc (4) with

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an axis (5) resting on a tapered bearing (6). The drive of rotation of the support disc is made in the form of an electric motor (motor-reducer) (7) placed outside the contour of the frame and mounted balanced, flexible gear (belt) transmission (8), the leading sprocket (9) of which is placed on the shaft (10) of the electric motor, the driven (11) - on the axis (5) of the drive of rotation of the support disc. A lever (12) is fixed on the electric motor body, which acts on the force sensor (13) when a reactive torque occurs. A sensor (14) is mounted on a special bracket for continuous determination of the position of the wheel rotation plane of the diagnosed axis of the vehicle relative to the axis of rotation of the support disc. The frame rests on a vibration platform (15) with a weight (collision) sensor (16) mounted thereon, additional weight sensors (16') are also placed between the frame and the support bearing (6) of the disc (4). The vibration pad (15) rests on the connecting rod of the crank mechanism (17) to make vertical oscillations.

3 Results

Application of the stand of the proposed design is carried out in the way described below and allows to obtain the following results, unattainable with the use of previously known designs of stands. In the mode of testing the braking qualities of the car, electric motors with a given low speed rotate the support discs, driving the wheels of the diagnosed car. When the driver acts on the brake pedal, a reactive torque is generated in the support disc drives, with the sensor 13 measuring the amount of reactive force, and the computer calculates the amount of reactive torque.

Further, based on the value of the arm (distance from the plane of rotation of the wheel of the diagnosed axis of the car to the axis of rotation of the support disc), which is determined by processing the signal from the laser sensor (14), the computer software calculates the value of the braking force in the contact patch of the wheel with the support disc, and taking into account the weight on the wheel, the specific braking force is calculated. In this mode, the proposed stand can measure braking forces as a conventional [3, 4] roller brake stand, but the shape of the wheel contact patch with the supporting surface will be close to real road conditions.

When switching on the oscillation drive of the vibrating platform (15), the frame with the support disc starts to make forced oscillations in the vertical plane, i.e. imitates a "rough" road, which can allow testing the working brake system in conditions as close as possible to
the real road. By adjusting the amplitude and frequency of vibration pad vibrations, as well as the speed of rotation of the support disc, it is possible to simulate a wide range of road conditions and, at the same time, to test the effectiveness of the working brake system and modern electronic driver assistance systems. In addition, using the vibration oscillation mode, it is possible to assess the technical condition of the vehicle suspension.

The originality of the stand is confirmed by the decision to grant a patent for invention of the Russian Federation [1].

4 Discussion

To examine the feasibility and efficiency of such a test stand, leading experts in the field of bench testing of automotive safety systems were involved [7-11], who, having critically analysed the design, formulated several problematic issues related to the use of the method and design of the test stand under consideration. In the present paper, we will consider some of these problems with respect to the testing of braking properties and try to provide answers to the questions raised.

One of the important ones, according to experts, is the issue of positioning the vehicle on the test stand in the longitudinal plane. While the displacement in the transverse direction is monitored by the laser sensors of the test stand and taken into account in the algorithm for calculating the braking force, the displacement in the longitudinal direction is not monitored. Earlier [2] it was shown (see Fig. 2) that to minimise the errors of braking force measurement the axis of the test wheel and the axis of the support disc should intersect, i.e. the axis displacement relative to each other (longitudinal displacement of the car relative to the test stand) should be close to zero. But in practice such a position may be difficult to achieve, because the tangential force $T$ (see Fig. 3), developed by the drive of the support disc rotation, will tend to "shift" the car in the direction of the support disc rotation when the braking force $R_x$ increases in the process of braking and wheel locking. Therefore, it is necessary to obtain quantitative estimates of the influence of the wheel position in the direction of the longitudinal axis of the car on the errors of braking force measurement and subsequently take them into account in the practical implementation of the test stand.

![Fig. 2. Binding of the XOY coordinate system to the test stand](image)

Let us consider the positioning of the wheel on the support disc, when the axis of rotation of the wheel does not coincide with the axis of rotation of the support disc (see Fig. 3). We denote the longitudinal displacement of the axis of rotation of the wheel relative to the axis of rotation of the support disc for some moment of time $i$ by the variable $S_i$. The distance from the axis of rotation of the supporting disc to the central plane of the wheel is denoted...
by $l_i$, the radius of the centre of the wheel contact spot with the supporting surface of the disc is denoted by $r_i$. The forces of adhesion of the wheel with the supporting disc are not taken into account yet.

Let us assume that the parameters $S_i$ and $l_i$ (parameters of the wheel position on the support disc) are continuously measured by the sensors of the test stand. Then

$$S_i^2 + l_i^2 = r_i^2 \quad (1)$$

$$\varphi = \arcsin \left( \frac{S_i}{r_i} \right) \quad \text{or} \quad \varphi = \arccos \left( \frac{l_i}{r_i} \right) \quad (2)$$

Let at the moment of reaching the maximum braking force $R_x$ on the test wheel, the wheel is blocked and the test stand is switched off, i.e. tractive effort torque $M_{tract}$, developed by the disc rotation drive becomes equal to the braking torque $M_{break}$. The system is in equilibrium, i.e. the system is in equilibrium

$$M_{tract} = M_{break} \quad \text{or} \quad M_{tract} = M_{break} \quad (3)$$

$$M_{tract} = T \cdot r_i \quad (4)$$

where $T$ is the tangential force from the test stand drive torque at the current value $r_i$;

$$M_{break} = R_x \cdot l_i + R_y \cdot S_i \quad (5)$$

where $R_y$ is wheel reaction to the lateral component $T_y$ tangential force tending to deform the tyre in the transverse plane, $R_y = -T_y$, i.e. equal in modulus, but differently directed (see Fig. 4).

$R_x$ is the actual (to be measured) braking force acting in the centre plane of rotation of the wheel is directed against the movement of the backing plate.

$$T \cdot r_i = R_x \cdot l_i + R_y \cdot S_i \quad (6)$$

**Fig. 3.** Positioning the wheel on the support disc so that the axis of rotation of the wheel does not coincide with the axis of rotation of the support disc
Expressing the parameters $l_i$ and $S_i$ by $r_i$ and $\varphi$ and substituting into the balance equation (6), we obtain

$$T \cdot r_i = R_x \cdot r_i \cdot \cos \varphi + R_y \cdot r_i \cdot \sin \varphi$$  \hspace{1cm} (7)$$

As $R_y = -T_y = T \cdot \sin \varphi$, we get

$$T \cdot r_i = R_x \cdot r_i \cdot \cos \varphi + T \cdot r_i \cdot \sin^2 \varphi$$  \hspace{1cm} (8)$$

$$R_x = \frac{T \cdot r_i - T \cdot r_i \cdot \sin^2 \varphi}{r_i \cdot \cos \varphi} = \frac{T \cdot r_i (1 - \sin^2 \varphi)}{r_i \cdot \cos \varphi}$$  \hspace{1cm} (9)$$

Considering $\cos^2 \varphi + \sin^2 \varphi = 1$, we get

$$R_x = T \cdot \cos \varphi$$  \hspace{1cm} (10)$$

At $\varphi = 0$ \hspace{0.1cm} $R_x = 0, R_y = r_i, S_i = 0$;
At $\varphi = 90$ \hspace{0.1cm} $R_x = 0, R_y = T, S_i = r_i$;
At $0 < \varphi \leq 90$ lateral reaction $R_y \neq 0$ (is not equal to zero) and creates a braking torque equal to $R_y \cdot S_i$, which is taken into account by the measuring system of the test stand and introduces an error in the measurement of the actual braking force in the plane of wheel rotation $R_x$. For example, if (according to Fig. 3) $\varphi = 90$, then the braking system of the test vehicle may not be activated, but the test stand will still show the presence of braking force due to the braking torque generated by the lateral reaction of the wheel. $M_{\text{brake}} = R_y \cdot S_i$.

If in equation (8) we substitute (10) instead of $R_x$, we obtain

$$T \cdot r_i = T \cdot r_i \cdot \cos^2 \varphi + T \cdot r_i \cdot \sin^2 \varphi$$  \hspace{1cm} (11)$$

In the right part of the equation, the first summand characterises the component of the braking torque created by the braking force $R_x$ acting in the central plane of the wheel, and the second summand characterises the component of the braking torque created by the reaction $R_u$ of the wheel to the lateral component $T_u$.

It is obvious that reliable measurement of the braking force generated by the braking mechanism of the wheel on the test stand under consideration is possible only at small values
of displacement $S_i$ of the wheel axis relative to the axis of the support disc, i.e. at small values of the angle of the wheel $\varphi$. It is necessary to determine at what values $S_i$, $\varphi$ and $r_i$ the braking force measurement error will not exceed a certain specified value $\delta$, e.g. 1 \%.

In order to assess the influence of wheel positioning relative to the transverse axis of the test stand and to identify the influence of the radius of the wheel contact patch centre location on the support disc on the measured braking force, we calculate the numerical values of the braking force measurement error for different values of wheel displacement and different radii of the support discs.

Let us calculate the 1st variant of error calculation through the acting forces. Let at $\varphi=0$ the acting braking force $R_x$ at the point of application of the equal braking force (in the centre of the contact patch) be equal to the tangential force $T$ developed by the drive of the support disc and is equal to 2 kN.

As an example, consider the following algorithm for calculating the error:
- set the offset of the wheel axis, for example $S_i = 50$ mm and radius $r_i = 350$ mm;
- from formula (2) we determine $\varphi = \arcsin \left( \frac{S_i}{r_i} \right) = \arcsin\left(\frac{50}{350}\right) = 8.21^\circ$
- we calculate the braking force acting in the longitudinal plane of the car (in the plane of rotation of the tested wheel), taking into account the longitudinal displacement of the wheel on the support disc (it will be less than $T$ and will be determined by the formula (10)):

$$R_x = T * \cos \varphi$$
(12)

$R_x = 2000 * 0.989 = 1979.48$ H

- we calculate the errors of braking force measurement:

Absolute error, H

$$\Delta = T - R_x$$
(13)

$\Delta = 2000 - 1979.487 = 20.513$

Relative error, %

$$\delta = \frac{100 \left( T - R_x \right)}{T}$$
(14)

$\delta = 100 \times \frac{2000 - 1979.487}{2000} = 1.025$

Next, we will plan the design experiment: we will set the range of variation of the parameter $S_i$ from 10 to 150 mm with a step of 10 mm. We will vary the radius of the wheel contact patch centre location on the support disc $r_i$ in the range from 350 to 1000 mm with a step of 50 mm. In accordance with the above algorithm (according to the formulas (9-12)) we will make calculations for the investigated range of parameter $S$ and parameter $g_u$, the results will be summarised in Table 1.
Table 1. Relative error of braking force measurement depending on the value of longitudinal displacement $S_i$ and radius $r_i$, %

<table>
<thead>
<tr>
<th>Relative error of the braking force measurement depending on the $S_i$ and $r_i$, %</th>
<th>Radius of wheel contact patch centre location on the support disc, $r_i$, mm</th>
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<tbody>
<tr>
<td>$350$</td>
<td>$400$</td>
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<tr>
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<td>8</td>
<td>7</td>
</tr>
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<td>0</td>
<td>0</td>
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</table>

The results given in Table 1, we also present graphically (Fig. 5.) in the form of a surface reflecting the dependence of the relative error value of the braking force measurement results as a function of the value of the longitudinal displacement of the axis of the tested wheel relative to the axis of the support disc and the value of the radius of the wheel contact patch centre location on the support disc.

From the analysis of the obtained dependence it can be seen that the displacement of the wheel in the longitudinal plane of the test stand and the radius of placement of the centre of the wheel contact spot on the support disc have a noticeable effect on the value of the error of braking force measurement. At the value of the radius of the centre of the contact spot of the support disc $500$ mm, to ensure the error of braking force measurement up to $1\%$, the permissible displacement of the wheel should be no more than $70$ mm. At the radius of the centre of the contact patch of the support disc $750$ mm, the longitudinal displacement can be up to $100$ mm, and at the radius of $1000$ mm, to ensure the error of braking force measurement up to $1\%$, the permissible wheel displacement can be up to $140$ mm. This shows a general pattern - the braking force measurement error decreases as the offset value decreases and the support disc diameter increases.
Fig. 5. Dependence of the relative error value of the braking force measurement results as a function of the value of the longitudinal displacement of the axis of the tested wheel relative to the axis of the support disc and the value of the radius of the wheel contact patch centre location on the support disc (calculation variant 1)

5 The 2nd variant of error calculation will be carried out through acting moments

From equation (11) of the total braking torque, we find the fraction of the torque created by the lateral reaction $R_y$ of the wheel to the lateral component of the traction force from the test stand drive $T_u$. This fraction is presented in the numerator of formulas (14 and 15), while in the denominator of these formulas the total braking torque is presented:

$$\delta = \frac{T \cdot r_i \cdot \sin^2 \varphi}{T \cdot r_i \cdot \cos^2 \varphi + T \cdot r_i \cdot \sin^2 \varphi} \cdot 100, \%$$ (14)

or

$$\delta = \frac{T \cdot r_i \cdot \sin^2 \varphi}{T \cdot r_i \cdot (\cos^2 \varphi + \sin^2 \varphi)} \cdot 100 \% = 100 \cdot \sin^2 \varphi, \%$$ (15)

As we can see, in this used model of calculation of relative error, it (error $\delta$) depends only on the angle value $\varphi$. The results of the relative error calculation are given in the table, and the nature of the dependence $\delta=f(\varphi)$ is shown in Figure 6.

Table 2. Results of calculation of relative error $\delta$ of braking force measurement as a function of angle $\varphi$.

<table>
<thead>
<tr>
<th>$\varphi, ^\circ$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
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<tbody>
<tr>
<td>$\delta, %$</td>
<td>0</td>
<td>0.03</td>
<td>0.12</td>
<td>0.27</td>
<td>0.496</td>
<td>0.768</td>
<td>1.09</td>
<td>1.48</td>
<td>1.93</td>
<td>2.44</td>
<td>3.01</td>
</tr>
</tbody>
</table>

Since we have previously defined $S_i$ (longitudinal displacement of the wheel axis relative to the axis of the support disc) and $r_i$ (radius of location of the centre of the wheel contact spot relative to the axis of rotation of the support disc) as initial parameters of wheel position...
control on the test stand, we calculate and plot the dependence of the measurement error $\delta$ of the braking force as a function of the above parameters for the previously considered range and step of parameter variation, and the results are summarised in Table 3.

Fig. 6. Dependence of relative error $\delta$ of braking force measurement on angle $\varphi$.

Table 3. Relative error of braking force measurement depending on the amount of longitudinal displacement $S_i$ and radii $r_i$, % (calculation option 2)

<table>
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<th>Wheel contact patch centre location radius on the support disc, $r_i$, mm</th>
<th>350</th>
<th>400</th>
<th>450</th>
<th>500</th>
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<td>0.51</td>
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</tbody>
</table>

The results given in Table 3 are presented graphically (Fig. 7) in the form of a surface reflecting the dependence of the relative error value of the braking force measurement results as a function of the value of the longitudinal displacement of the axis of the tested wheel relative to the axis of the support disc and the value of the radius of the wheel contact spot centre location on the support disc.
Comparing the calculation results for the two used variants (Table 1 and Table 3 and their corresponding graphs (Fig. 4 and Fig. 6) it can be stated that the values of relative errors calculated for variant 1 are smaller than those calculated for variant 2, which is determined by the difference of the used calculation models - in the first variant based on the analysis of braking forces, and in the second variant - based on the analysis of braking moments.

Summarising the results of calculations for two variants it is possible to determine the ranges of permissible parameters of wheel position control on the support disc - the value of longitudinal displacement $S_i$ and radius $r_i$, providing the error of measurement of the actual braking force within 1%:

- **calculation option 1**: with the radius of the centre of the contact spot 750 mm (the radius of the platform itself will be as follows $\approx 900 \div 1000$ mm) the permissible longitudinal displacement of the wheel axle (to one side) must be within the limits of $S_i=100$ mm, respectively the entire acceptable range (2$S_i$ - in both directions) will be 200 mm;
- **calculation option 2**: at a contact patch centre radius of 750 mm, the permissible longitudinal displacement of the wheel axle (to one side) must be within the limits of $S_i=70$ mm, respectively the entire acceptable range (2$S_i$ - in both directions) will be 140 mm.

Thus, on the basis of two variants of calculation it can be stated that for the platform with overall radius $\approx 900 \div 1000$ mm the permissible longitudinal displacement of the wheel axle (to one side) must be within the limits of $S_i=70 \div 100$ mm, respectively the whole acceptable range (2$S_i$ - both ways) will be $140 \div 200$ mm. These data should form the basis for substantiation of requirements to the design parameters of the test stand of the considered design and requirements to the parameters of control of positioning of the tested wheel on the disc support surface.

In order to analyse the comparability of the found value of the range of permissible displacement with the dimensions of the real contact patch of a passenger car wheel (see Fig. 8)
Fig. 8. Wheel contact patch on horizontal support surface: winter tyre 205/55 R16, wear rate 50%

We make a graphical method in scale (Fig. 9) superimposition of the obtained contact spot on the supporting discs of radius 500 mm and 1000 mm.

Fig. 9. Tyre contact patch positioning on support discs with radii of 500 and 1000 mm.

As can be seen from Fig. 9, with the radius of the centre of the contact spot of the wheel with the supporting disc 750 mm, the necessary condition of compliance of the longitudinal size of the contact spot with the value of the permissible range in 2S is provided, which allows to implement the stand of the proposed design for testing the braking properties of passenger cars.
Conclusions

The analysis of braking force measurement errors depending on the wheel position on the stand with support discs allowed us to establish the following regularities:
- at an increase of the longitudinal displacement $S_i$ of the axis of the controlled wheel relative to the axis of rotation of the support disc the error of measurement increases;
- when the radius $r_i$ increases, the centre of the wheel contact spot location with the surface of the support disc (and in fact, when the diameter of the support disc), the error of braking force measurement decreases;
- the ranges of values of longitudinal displacement of the wheel $S_i$, radius $r_i$, and angle $\phi$ are established, at which the error in the measurement of the braking force developed by the wheel tested on the stand of the proposed design will not exceed some established value $\delta$, %.

The obtained regularities should form the basis for the substantiation of requirements to the parameters of the brake bench of the considered design and requirements to the parameters of control of positioning control of the tested wheel on the disc support surface.

Further research should be directed to the study of the forces of adhesion of the tested wheel with the surface of the supporting disc, since the curvature of their contact path can lead to the appearance of significant angles of drift and, as a consequence, to the reduction of adhesion forces, slippage of the wheel relative to the supporting surface.

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