

Synthesis of control and observation laws power system based on generalized formula Ackermann for MIMO systems

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Abstract. For an electric power system (EPS), as a dynamic system with many inputs and many outputs (Multi Inputs Multi Outputs System MIMO), compact analytical formulas are obtained for calculating the coefficients of the controller matrix and the observer matrix in the state of the solution of the synthesis problem, providing a given placement of eigenvalues along full state vector. These formulas are generalizations to MIMO systems of the well-known Ackermann formula used to design the control of systems with one input and many outputs (Single Input Multi Outputs System SIMO). The approach is based on the transformations used in the original method (multilevel) decomposition method, as well as a nondegenerate similarity transformation in the form of the real controllability matrix. The obtained formulas are applicable to dynamic systems, for which the dimension of the state space is a multiple of the dimension of the inputs (controls). This limitation is removed by using the Yokoyama transform. These formulas differ in terms of parameterization of the set of equivalent laws. An example of the synthesis of a control law for a synchronous generator in a complex EPS is considered in order to preserve the existing modes of electromechanical oscillations and meet additional requirements (roughness with respect to disturbances and/or increased sensitivity to changes in controlled parameters in a given region or frequency band).

Introduction and problem statement

One of the most well-known explicit calculation formulas used for the synthesis of controllers and observers of linear stationary dynamic systems in the state space with one input and one output, including electric power systems (EPS), are the Ackermann and-Bass Gura formulas [04].

Let a fully controlled stationary linear dynamic SIMO-system (Single Input Multi Outputs System) be given

$$\dot{x} = Ax + Bu + Qv \quad (1)$$

where

$x \in \mathbb{R}^n$ is the state vector, u is the scalar input, $v \in \mathbb{R}^m$ is the set of real numbers, $A \in \mathbb{R}^{n \times n}$ for the case of continuous time and $A \in \mathbb{R}^{(n-1) \times (n-1)}$ for the case of discrete time.

The condition for complete controllability of the system (1) or complete controllability of the pair corresponds to the nonsingularity of the Kalman controllability matrix (\tilde{A}, B)

$$\chi(\tilde{A}, B) = \begin{pmatrix} B & A B & \dots & A^{n-1} B \end{pmatrix} \in \mathbb{R}^{n \times n} \quad (2)$$

Let also the characteristic polynomial of the matrix A equals

$$\Delta(\lambda) = \lambda^n + \alpha_{n-1} \lambda^{n-1} + \dots + \alpha_1 \lambda + \alpha_0 \quad (3)$$

where

U_k is the identity matrix of order n , $\lambda_j \in \mathbb{C}$ are the coefficients of the characteristic polynomial, \mathbb{C} is the set of complex numbers.

Using the feedback law on state variables

$$u = -F x + \dot{v} \quad (4)$$

required to provide a closed system

$$\dot{x} = (A - BF)x + B \dot{v}$$

the following characteristic polynomial:

$$\Delta(\lambda) = \lambda^n + \alpha_{n-1} \lambda^{n-1} + \dots + \alpha_1 \lambda + \alpha_0 \quad (5)$$

If we introduce the notation for the matrix polynomial

$$\tilde{\Delta}(\lambda) = \lambda^n + \alpha_{n-1} \lambda^{n-1} + \dots + \alpha_1 \lambda + \alpha_0 \quad (6)$$

then the calculation formula of Ackermann, which determines the vector of coefficients of the controller (dually, the observer of the state (4) looks like [0-4])

$$F = (0 \quad 0 \quad \dots \quad 0 \quad 1) \cdot \begin{pmatrix} \alpha_0 & \alpha_1 & \dots & \alpha_{n-1} \\ \alpha_1 & \alpha_2 & \dots & \alpha_n \\ \dots & \dots & \dots & \dots \\ \alpha_{n-1} & \alpha_n & \dots & \alpha_{2n-1} \end{pmatrix}^{-1} \cdot B \quad (7)$$

or taking into account (2), (6)

$$F = (0 \quad 0 \quad \dots \quad 0 \quad 1) \cdot \chi^{-1}(\tilde{A}, B) \cdot \tilde{\Delta}(\tilde{A}) \quad (7)$$

The main advantage of the Ackermann calculation formula (7), is its explicit form, which allows using the known parameters of the system, its characteristic polynomial (3) and a given arrangement of poles (polynomial roots), expressed in coefficients of the charac-

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teristic polynomial(5), calculate the feedback that provides the closed system with the required location of the poles (setting the roots). The same applies to the BassGura formula [4], which is not shown here.

In this paper, we consider a generalization of the Ackermann formula(7) for the case of a mathematical model of an EPS in the form of a dynamic system with many inputs(Multi Inputs Multi Outputs System MIMO). This task is relevant, since there are explicit calculation formulas for synthesizing the feedback law for MIMO systems similar to the Ackermann formula. Some exceptions are [5, 6]. In [5] to solve the problem, the Sylvester matrix equation and the associated inversion of the matrix polynomial(6), and in [6] is a special form of the control law, which significantly limits the area of its use.

In what follows, all transformations, taking into account duality, turn out to be valid for the problem of synthesis of the state observer.

Let a fully controlled MIMO system be given, which is understood as a mathematical model of EPS

$$\dot{x} = Ax + Bu \tag{8}$$

with state and control matrices of multiple dimensions $m \in \mathbb{R}^{p \times n}$, $n \in \mathbb{R}^{n \times 1}$, $G \in \mathbb{R}^{n \times m}$, where the matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix} \tag{9}$$

made up of the first block columns of the Kalan controllability matrix is reversible. It is required to determine the matrices of regulators (observers) such that the state matrix "object regulator" ("object - observer") $mF n$ has given eigenvalues λ_i . At the same time, in the future we will compare the sets of solutions to this problem obtained by various analytical methods: using multilevel decomposition[7] and on the basis of a nondegenerate similarity transformation.

1. Solution using multilevel decomposition

Let us perform the following similarity transformation [8]:

$$\begin{aligned} \dot{x} &= x^{-1} A x = d \begin{bmatrix} U_{ax}(\lambda) & 0 \\ 0 & U_{ax}(\lambda) \end{bmatrix} h^{-1} x^{-1} m^{-1} n h \\ \dot{x} &= x^{-1} A x = d \begin{bmatrix} U_a & 0 \\ 0 & U_a \end{bmatrix} h \end{aligned} \tag{10}$$

For the resulting pair of matrices, we synthesize the state controller with the matrix K

In this case, when the condition(9), the number of decomposition levels of the pair of matrices (A, B) (including the zero level) is $G[7]$. The levels do not generate (the rank of the control matrices at each of the levels is l). Decomposition is performed according to the formulas [7]

$$\begin{aligned} A_0 &= A, \quad A_1 = A, \\ A_{i-1} &= A_i - \lambda_i A_i^{-1} A_i, \quad A_{i-1} = A_i - \lambda_i A_i^{-1} A_i \\ i &= 0, 1, \dots, GF2, \end{aligned}$$

using semiorthogonal left zero divisors

$$A^D = [U_{ax}(\lambda) \quad 0; \quad 0 \quad U_{ax}(\lambda)], \quad E = 0, 1, \dots, G \in \mathbb{R}$$

The state matrices by decomposition levels take the form

$$\begin{aligned} A_i &= H \begin{bmatrix} U_{ax}(\lambda) & 0 \\ 0 & U_{ax}(\lambda) \end{bmatrix} I = \\ &= N \begin{bmatrix} U_{ax}(\lambda) & 0 \\ 0 & U_{ax}(\lambda) \end{bmatrix} O \\ i &= 0, 1, \dots, GF2, \\ A_{i-1} &= A_{i-1} \end{aligned}$$

The control matrices by decomposition levels have the form

$$\begin{aligned} B_i &= d \begin{bmatrix} U_a \\ 0 \end{bmatrix} h = d \begin{bmatrix} U_{ax}(\lambda) \\ 0 \end{bmatrix} h, \quad E = 0, 1, \dots, G \in \mathbb{R}, \\ B_{i-1} &= B_i \end{aligned}$$

Here and below, the designation corresponds to the block located at the intersection of i -th block line and j -th a block column of a matrix M divided into blocks of the same size $l \times l$.

Assign matrices with given eigenvalues by decomposition levels

$$\begin{aligned} \lambda_1 \lambda_2 \dots \lambda_{p-1} \lambda_p \in \mathbb{R}^{l \times l}, \\ \text{such that} \\ + A E^{-1} Q = \lambda_i \end{aligned}$$

The controller matrix at the uppermost (1) -th level has the form

$$K_{p-1} = A_{p-1}^{-1} K_{p-1} F \lambda_{p-1}^{-1} = A_{p-1}^{-1} K_{p-1} F \lambda_{p-1}^{-1}$$

Next, we sequentially perform the calculations of the controller matrices at the levels Using pseudoinverses and auxiliary matrices

$$\begin{aligned} i = GF2, GF3, \dots, 1, 0 \\ A_i^{-1} = A_i^{-1}, \quad A_i^{-1} = A_i^{-1} + A_{i-1}^{-1} A_i^{-1} A_i^{-1} = [U_a \quad 0] \\ \text{on } i\text{-th decomposition level define the controller matrix} \end{aligned}$$

$$\begin{aligned} K_i &= A_i^{-1} K_i F \lambda_i^{-1} \\ &= K_i^{-1} A_i^{-1} K_i F \lambda_i^{-1} \dots K_i^{-1} A_i^{-1} K_i F \lambda_i^{-1} \end{aligned}$$

the blocks of which are written in terms of blocks of the controller matrix at the $(i+1)$ level as follows:

$$\begin{aligned} K_{i-1} &= K_i^{-1} A_i^{-1} K_i F \lambda_i^{-1} \\ K_{i-1} &= K_i^{-1} A_i^{-1} K_i F \lambda_i^{-1} \\ &\dots \\ K_{i-1} &= K_i^{-1} A_i^{-1} K_i F \lambda_i^{-1} \\ K_{i-1} &= K_i^{-1} A_i^{-1} K_i F \lambda_i^{-1} + \lambda_i^{-1} K_i^{-1} A_i^{-1} K_i F \lambda_i^{-1} \\ &\quad \lambda_i^{-1} K_i^{-1} A_i^{-1} K_i F \lambda_i^{-1} \end{aligned}$$

Let's rewrite the expressions for the blocks of the controller matrix

$$\begin{aligned} K_{i-1} (i = 1, 2, \dots, GF) (i = 0, 1, \dots, GF1) \text{ at the } i\text{-th level of decomposition} \\ \text{using the values of the known blocks of the same level, located to the left of the current block} \end{aligned}$$

$$\begin{aligned} K_{i-1} &= K_i^{-1} A_i^{-1} K_i F \lambda_i^{-1} + \\ &+ A_i^{-1} K_i^{-1} A_i^{-1} K_i F \lambda_i^{-1} \end{aligned} \tag{11}$$

difference between this solution and (13) is that the matrix polynomial (16) does not have to be decomposed into matrix factors, as is done (14). The main thing is that the roots of the characteristic equation (16) coincided with the corresponding eigenvalues of the matrices Γ^0 .

3. Example

Consider an example of practical synthesis, when the task is to modify the previously obtained control law for synchronous generators so that the modes of electromechanical oscillations provided by the original control laws are preserved, while the modified laws would satisfy some additional requirements. Such requirements may include "enhanced" roughness with respect to disturbances (robustness), or, conversely, increased sensitivity to changes in controlled parameters in a given region or frequency band, etc. These requirements can be met by varying the given matrices Γ^0 , that the desired solution of the modal control problem under consideration, which belongs to the set of possible solutions (13), is not contained in the set (13), obtained using multilevel decomposition. In other words, consider the case when the matrix polynomial (16) does not decompose into matrix factors by analogy with the controller matrix (14).

Let the model be given EES including synchronous generators and represented by the following linearized equations (17):

$$\frac{\dot{x}}{x} = Q \frac{\dot{x}}{x} = F \frac{\dot{x}}{x} + G \frac{\dot{u}}{u}, \quad E = 1, 2, \dots, G = 0/2. \quad (19)$$

Here δ_j is the damping coefficient, J_j is the moment of inertia, ΔP_j is the change in active power, determined using the equations of the electrical network

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} Y \\ Z \end{bmatrix} \begin{bmatrix} \Delta U \\ \Delta \delta \end{bmatrix}$$

Where ΔP is the active power increment vector in all system nodes, ΔQ is the reactive power increment vector, ΔU is the voltage amplitude increment vector, $\Delta \delta$ is the voltage phase shift angle increment vector, $\Delta \delta$ is the generator slip increment vector, is the control vector (signals of PID-controllers). The elements of the Jacobian matrix are the corresponding partial derivatives $\frac{\partial P}{\partial U}, \frac{\partial P}{\partial \delta}, \frac{\partial Q}{\partial U}, \frac{\partial Q}{\partial \delta}$.

Under the assumptions made, the matrix and n in the vector equation of the mode EES take the form

$$m = \begin{bmatrix} \dot{u} & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \dot{u} & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ \dot{u} & - & - & 0 & - & - & - & 0 \\ \dot{u} & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \dot{u} & F \frac{\Delta P}{I_s} & F \frac{\Delta P}{I_s} & 0 & F \frac{\Delta P}{I_s} & F \frac{\Delta P}{I_s} & 0 & 0 \\ \dot{u} & F \frac{\Delta P}{I_s} & F \frac{\Delta P}{I_s} & 0 & F \frac{\Delta P}{I_s} & 0 & F \frac{\Delta P}{I_s} & 0 \\ \dot{u} & - & - & 0 & - & - & - & 0 \\ \dot{u} & F \frac{\Delta P}{I_s} & F \frac{\Delta P}{I_s} & 0 & 0 & 0 & 0 & F \frac{\Delta P}{I_s} \end{bmatrix}$$

$$n = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ - & - & 0 & - \\ 0 & 0 & 0 & 0 \\ - & 0 & 0 & 0 \\ 0 & - & 0 & 0 \\ - & - & 0 & - \\ 0 & 0 & 0 & - \end{bmatrix}$$

Here, as you can see, the dimension of inputs is a multiple of the dimension of states, and phase angles and slips are used as outputs.

Let us consider the case of specifying the parameters of the control law in the PID-controllers, which provide the solution of the problem formulated earlier.

Let and be given an arbitrary nonzero real number $l = 2, G = 2, J = G, l = 4 \hat{a}$

Let also the matrices from (14) have the form $\Gamma^0 = \{ \ddot{u}, \dots, \ddot{u} \}$

$$\Gamma_{65}^0 = \begin{bmatrix} F \ddot{u} & 0 \\ 0 & F \ddot{u} \end{bmatrix}, \quad \Gamma_{66}^0 = \begin{bmatrix} \ddot{u} & \hat{a} \\ 0 & \ddot{u} \end{bmatrix}$$

Characteristic equation
$$+ \ddot{u} + \hat{a} \ddot{u} + \ddot{u} = \begin{bmatrix} \hat{a} & \hat{a} \\ 0 & \hat{a} \end{bmatrix} F \ddot{u} = 0$$

has roots coinciding with given eigenvalues (given modes of electromechanical oscillations), but the system of equations

$$\begin{cases} \Gamma_{4+} \Gamma_{5+} = F \Gamma_{65} \\ \Gamma_{4+} \Gamma_{5+} = \Gamma_{66} \end{cases} \quad (18)$$

turns out to be unsolvable with respect to the matrices $\Gamma_{4+} \Gamma_{5+}$.

Let further and $\Gamma^0 = [\ddot{u}, \ddot{u}, \ddot{u}, \ddot{u}]$

$$\Gamma_{65}^0 = \begin{bmatrix} F \ddot{u} & \hat{a} \\ 0 & F \ddot{u} \end{bmatrix}, \quad \Gamma_{66}^0 = \begin{bmatrix} \ddot{u} & 0 \\ 0 & \ddot{u} \end{bmatrix}$$

Characteristic equation
$$+ \ddot{u} + \hat{a} \ddot{u} + \ddot{u} = \begin{bmatrix} \hat{a} & \hat{a} \\ 0 & \hat{a} \end{bmatrix} F \ddot{u} = 0$$

has roots coinciding with the given eigenvalues, and the system of equations

$$\begin{cases} \Gamma_{4+} \Gamma_{5+} = F \Gamma_{65} \\ \Gamma_{4+} \Gamma_{5+} = \Gamma_{66} \end{cases} \quad (19)$$

is solvable with respect to matrices. One of the two possible solutions to system (21) has the following simple form: $\Gamma_{4+} \Gamma_{5+}$

$$\Gamma_{4+} = e^{\ddot{u}} F \frac{\ddot{u}}{\ddot{u}}, \quad \Gamma_{5+} = e^{\ddot{u}} \frac{\ddot{u}}{\ddot{u}}, \quad (20)$$

$$A \Gamma_{4+} = [\ddot{u}, \ddot{u}], \quad A \Gamma_{5+} = [\ddot{u}, \ddot{u}]$$

where as can be seen, the eigenvalues are located on the main diagonal of the corresponding matrices. Triangular matrices (22) have the following features: firstly, to exclude division by zero in the expressions of offdiagonal elements in matrices (22), it

is necessary to prevent the coincidence of the given eigenvalues (multiplicity of oscillation modes); secondly, the off-diagonal elements of matrices (22) contain the previously mentioned "adjusting" parameter β , which makes it possible to provide the previously mentioned "enhanced" roughness with respect to disturbances, or increased sensitivity to changes in controlled parameters in a given region or frequency band (while maintaining the given oscillation modes).

4. Conclusion

For the EES represented by a linear stationary dynamic system with many inputs and many outputs (MIMO system), compact highly efficient analytical formulas for calculating the coefficients of the feedback law are obtained when solving the problem of modal control (given placement of eigenvalues) by the full state vector. The resulting analytical formulas are generalizations to MIMO-systems of the well-known Ackermann formula applied to systems with one input and many outputs (SIMO systems). The transformations are based on a nondegenerate similarity transformation and the algorithm of the multi-step decomposition method. The obtained formulas are applicable to dynamic systems, in which the dimension of the state space is a multiple of the dimension of the system inputs. This limitation is removed by using the Youla transform. The resulting formulas differ in terms of the parametrization of the set of equivalent laws. It is shown that there are cases when the set of equivalent control laws obtained using an analytical formula based on a nondegenerate similarity transformation is wider than the set of control laws formed on the basis of the multi-step decomposition method. The synthesis example demonstrated the possibility of changing the previously obtained control law for synchronous generators in order to preserve the previous oscillation modes and satisfy some additional requirements for robustness and sensitivity. These requirements are provided by varying the parameters of special matrices. The implementation of the obtained laws involves the use of synchronized vector measurements.

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