Theoretical preconditions for the development of mathematical models of the technology of desert plant drying

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Abstract. The formulation of stationary and non-stationary problems for drying dispersed materials is substantiated in the article. A mathematical model and its analytical solutions are developed to determine the patterns of the interdependence of temperature and moisture content. A mathematical model of the process of drying a dispersed material is based on the energy conservation law under heat and mass transfer in a multiphase medium.

1 Introduction

The basis of the drying theory is the dependence of the transfer of heat and water in wet materials when they interact with heated gases (a drying agent) and with hot surfaces, and in the process of irradiation with thermal and electromagnetic waves in the presence of phase transformations. The patterns of energy and mass transfer in wet materials during their dehydration are quite complex and insufficiently studied.

In developing a mathematical model of the drying process, the following assumptions are assumed: the wet material to be dried is a multi-phase (dry matter + water + air) and a multi-component (dry air + water vapor) medium.

The system of drying by penetrating with a drying agent is a process accompanied by heat and mass transfer between various phases and components of the system. Due to the multi-phase nature of the medium, the equations for its description must contain phase contact areas since the kinematic, dynamic, and thermodynamic interaction between the phases is conducted through them.

In general, the physical model of the process can be represented as:

1. Drying of the wet material takes place in a layer characterized by thickness \( h \) and surface area \( S \).

The layer consists of unit elements, characterized by volume \( V_1 \), surface area \( S_1 \), density and moisture content \( \omega \) (hereinafter, the following indices are taken: 1 - refers to dry matter (a skeleton); 2 - to moisture content; and 3 - to air).

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2. The space between unit objects inside the layer is filled with air with temperature $t_3^*$, water vapor density $\rho_{\text{п}}^*$, and flow rate $U_3^*$ (the air parameters outside the layer are denoted without the «*» sign).

3. Water evaporated from the material initially enters the air space inside the layer, and then, under the effect of convection (circulation), passes outside the layer and then moves to the surrounding medium.

Methods

Wet material is a colloidal capillary-porous body; the water in it is bound by various forms of bonds [1]. Water in the micro-pores and capillaries of the body has the least strong bond. When it is removed, the evaporation process can be considered as evaporation from the free surface, and the evaporation surface coincides with the surface of the body. As dehydration begins, the process of the deepening of the evaporation zone begins.

In mathematical modeling, the following concepts are used. We single out elementary volume $\Delta V$ inside the layer of wet material and denote by $\Delta V_i$ the volume of the $i$-th phase contained in volume $\Delta V$. Then,

$$\sum_{i=1}^{n} \Delta V_i = \Delta V$$

Let us introduce the concept of the relative volume of the $i$-th phase:

$$\alpha_i = \frac{\Delta V_i}{\Delta V}$$

$$\sum_{i=1}^{n} \alpha_i = 1.$$

The phases interact with each other through contact interfaces, which is the main characteristic of a multiphase medium. The area of the surface of the mutual contact of the $i$-th phase with the $j$-th phase in the selected volume $\Delta V$ is denoted by $\Delta S_{ij}$. The ratio of $\Delta S_{ij}$ to $\Delta V$ is the specific surface of mutual contact, denoted by:

$$\beta_{ij} = \frac{\Delta S_{ij}}{\Delta V}$$

$$\beta_{ij} = \beta_{ji}.$$

The total specific surface of the $i$-th phase $\beta_i$ is:

$$\beta_i = \sum_{j=1}^{n} \beta_{ij}$$

An important characteristic of a multiphase system is the characteristic linear size of phases $\delta_i$, proposed by Prof. G.G. Umarov [2]; it is an analog of its thickness or hydraulic radius. This size is expressed in terms of the relative volume of phase $\alpha_i$ and its total specific surface area:
\[ \delta_i = K_\phi \frac{\alpha_i}{\beta_i} \]

Let us introduce the phase density \( \rho_i \) into consideration, then the density of the “wet material + water + air” system is:

\[ \rho = \sum_{i=1}^{i=n} \rho_i \alpha_i \]

The volume of the dried material \( \Delta V_1 \) consists of the volume of dry matter (a skeleton) \( \Delta V_{11} \), the volume of water \( \Delta V_{12} \), and the volume of moist air \( \Delta V_{13} \), i.e.

\[ \Delta V_1 = \Delta V_{11} + \Delta V_{12} + \Delta V_{13} \]

The relative volume of the \( j \)-th medium in the volume of the product:

\[ \psi_{1j} = \frac{\Delta V_{1j}}{\Delta V_1} \]

\[ \sum \psi_{1j} = 1 \]

The dimensionless parameter depending on the geometric shape of the phase:

\[ \delta_i = K_\phi \frac{\alpha_i}{\beta_i} \]

\[ \rho_i \]

\[ \Delta V_{1j} \]

\[ \Delta V_1 \]

\[ \psi_{1j} \]

\[ \eta_{ij} = \frac{\Delta V_{1j}}{\Delta V} \]

\[ \eta_{ij} = \psi_{1j} \]

\[ \eta_{ij} = \frac{\psi}{\alpha} \]

\[ \Delta V_{11} \]

\[ \Delta V_{12} \]

\[ \Delta V_{13} \]

\[ \Delta V_1 \]

\[ \psi_{1j} \]

\[ \eta_{ij} \]
The equation describing the law of conservation of a scalar variable is the basis, using it, we derive equations for the balance of other physical characteristics necessary to describe a multiphase and multicomponent medium. To obtain this equation, we consider an elementary volume with sides $\Delta x$, $\Delta y$, $\Delta z$ (Fig. 1). Flow $J$ of some scalar value $\Phi$ passes through this elementary volume. The flux flowing through one face $\Delta z \Delta y$, is denoted by $J_{yz}$. The flux inflowing through the opposite face is $J_{yz} + \left( \frac{\partial J_{yz}}{\partial x} \right) \Delta x$.

The balance of conservation of the scalar variable $\Phi$ along the $x$-axis has the following form:

$$J_{yx} \Delta z \Delta y - \left( J_{yx} + \frac{\partial J_{yx}}{\partial x} \Delta x \Delta y \Delta z \right) = - \frac{\partial J_{yx}}{\partial x} \Delta x$$

Similar expressions for the balance of the scalar quantity $\Phi$ take place along the $y$ and $z$-axes, i.e.:

$$J_{zy} \Delta x \Delta z - \left( J_{zy} + \frac{\partial J_{zy}}{\partial y} \Delta x \Delta y \Delta z \right) = - \frac{\partial J_{zy}}{\partial y} \Delta y$$

$$J_{zx} \Delta x \Delta y - \left( J_{zx} + \frac{\partial J_{zx}}{\partial z} \Delta x \Delta y \Delta z \right) = - \frac{\partial J_{zx}}{\partial z} \Delta z$$

The total balance of value $\Phi$ in volume $\Delta V = \Delta x \Delta y \Delta z$ due to its flow through the elementary volume is:

$$- \left( \frac{\partial J_{yx}}{\partial x} + \frac{\partial J_{zy}}{\partial y} + \frac{\partial J_{zx}}{\partial z} \right) \Delta V = - \text{div} \vec{J}_\Phi \Delta V$$

$$\Phi \vec{U}$$
\(- D_\phi \nabla \phi \)
where $\rho_i$ is the density of the $i$-th phase; $\mathbf{U}_i$ is the total velocity of the $i$-th phase; $\sum_j \epsilon_{ij}$ is the volume velocity of transition of the $j$-th phase to the $i$-th phase.

In the case of water evaporation into the vapor-air medium, the expression takes the following form:

$$
\rho_i \mathbf{U}_i + \text{div} \left[ \alpha_3 (\rho_n \mathbf{U}_n - D^*_n \text{grad} \rho_n) \right] = \epsilon_{n3}
$$

where $\rho_n$, $\mathbf{U}_n$ is the convective velocity of the air flow; $\rho_n \mathbf{U}_n \mathbf{U}_n \rho_n$ is the convective velocity of the air flow; $\epsilon_{n3}$ is the volume velocity of vapor-air transition.

For a Newtonian fluid (the fluid has no initial shear stress), the differential equation expressing the conservation of momentum in a given direction can be written likewise. However, one should take into account Stokes' viscous friction law and its difference from Fick's and Fourier's laws, which are used in describing the heat balance.

Let $x_i$ be the velocity component of the $i$-th phase equal to $U_{xi}$, then the corresponding momentum equation (with the combined action of forced and natural convection) takes the following form:

$$
\frac{\partial \alpha_i \rho_i}{\partial \tau} + \text{div} \left[ \alpha_i \left( \rho_i \mathbf{U}_{xi} - \nu_i \text{grad} (\rho_i U_{xi}) \right) \right] =
$$

$$
= \alpha_i \rho_i \mathbf{g} - \alpha_i \frac{\partial P_i}{\partial x} + \sum_j \frac{\chi_T \mu_i^* \mu_j^*}{\mu_i^* \delta_j + \mu_j^* \delta_i} \beta_j \left( U_{sj} - U_{xi} \right)
$$

$$
\sum_j \frac{\chi_T \mu_i^* \mu_j^*}{\mu_i^* \delta_j + \mu_j^* \delta_i} \beta_j \left( U_{sj} - U_{xi} \right)
$$

The value of $\sum_j \epsilon_{ij}$ characterizes the interaction between the phases $\left(\frac{4}{4}\right)$.

The right-hand side of equation (29) is the source term $S_\Phi$, due to the influence of gravitational forces and the presence of distributed resistances to fluid flow, which appear due to the presence of the flow-off momentum in the streamlined elements of the porous structure. These resistances tend to vary in magnitude and direction.

The concept of the distribution of resistances not only simplifies the problem, it allows one to use correlations for volume coefficients of resistance and heat and mass transfer of porous structures in calculations of local characteristics of transfer. These correlations are often obtained from relatively simple experiments $\left(5\right)$, and their use allows for the calculation of complex fields of velocity, temperature, and concentration in the working space of the process equipment under consideration.
\[
U_i \frac{\partial \alpha_i \rho_i}{\partial \tau} + \alpha_i \rho_i \frac{\partial U_i}{\partial \tau} + \alpha_i \rho_i \vec{U}_i \text{grad} U_i + U_i \text{div} \alpha_i \rho_i \vec{U}_i -
\]
\[
-\nu_i \text{div} \text{grad} \alpha_i \rho_i U_i = \alpha_i \rho_i g_i - \alpha_i \frac{\partial P_i}{\partial x} + \sum \frac{x_i \mu_i^* \mu_j^*}{\mu_i \delta_j + \mu_j \delta_i} \beta_j (U_{ij} - U_i)
\]
\[
\alpha_i \rho_i \frac{\partial U_i}{\partial \tau} + \alpha_i \rho_i \vec{U}_i \text{grad} U_i + U_i \left( \frac{\partial \alpha_i \rho_i}{\partial \tau} + \text{div} \alpha_i \rho_i \vec{U}_i \right) =
\]
\[
= \alpha_i \mu_i^0 \text{div} \text{grad} U_i + \alpha_i \rho_i g_i - \alpha_i \frac{\partial P_i}{\partial x} + \sum \frac{x_i \mu_i^* \mu_j^*}{\mu_i \delta_j + \mu_j \delta_i} \beta_j (U_{ij} - U_i)
\]
\[
\frac{\partial \alpha_i \rho_i}{\partial \tau} + \text{div} \alpha_i \rho_i \vec{U}_i = 0
\]
\[
g_i = 0
\]
\[
\frac{\partial U_i}{\partial \tau} + U_i \frac{\partial U_i}{\partial x} + U_{yi} \frac{\partial U_i}{\partial y} + U_{zi} \frac{\partial U_i}{\partial z} = - \frac{1}{\rho_i} \frac{\partial P_i}{\partial x} + \nu_i \nabla U_i
\]
\[
\alpha_i = 1; \quad U_{xi} = U_i
\]
\[
\frac{\partial U_i}{\partial \tau} + U_i \frac{\partial U_i}{\partial x} + U_{yi} \frac{\partial U_i}{\partial y} + U_{zi} \frac{\partial U_i}{\partial z} = - \frac{1}{\rho_i} \frac{\partial P_i}{\partial x} + \nu_i \nabla U_i
\]
\[
\frac{\partial U_i}{\partial \tau} + U_i \frac{\partial U_i}{\partial x} + U_{yi} \frac{\partial U_i}{\partial y} + U_{zi} \frac{\partial U_i}{\partial z} = - \frac{1}{\rho_i} \frac{\partial P_i}{\partial x} + \nu_i \nabla U_i
\]
Thus, the validity of the recording of the momentum balance in the form of a generalized differential equation was proved. Now we consider the heat balance in a multiphase medium. The change in time of the heat content of the $i$-th phase, located in a unit volume of the medium is:

$$
\frac{\partial U_{zi}}{\partial \tau} + U_{xi} \frac{\partial U_{zi}}{\partial x} + U_{yi} \frac{\partial U_{zi}}{\partial y} + U_{zi} \frac{\partial U_{zi}}{\partial z} = -\frac{1}{\rho_i} \frac{\partial P_i}{\partial x} + v_i \nabla U_{xi} + \frac{1}{\rho_i \alpha_i} \sum \chi_{x \mu_i \mu_j} \frac{\mu_i}{\mu_j \delta_j - \mu_j \delta_i} \beta_{ij} (U_{zj} - U_{zi})
$$

where $\chi_{x \mu_i \mu_j}$ is the coefficient of effective thermal conductivity; $\mu_i, \mu_j$ are the fluxes of short-wave radiation; $\alpha_i$ is the flux of long-wave radiation.

Heat transfer through the contacting surfaces of phases is

$$
\sum \frac{\chi_{y \mu_i \mu_j}}{\mu_i \delta_j - \mu_j \delta_i} \beta_{ij} (t_j - t_i)
$$

where $\chi_T \approx 6$.

The intensity of the internal source associated with phase transformations is:

$$
\sum Q_{aij}
$$

We rewrite the resulting equation in the following form:

$$
\frac{\partial \alpha_i c_i \rho_i t_i}{\partial \tau} + \text{div} \left[ \alpha_i (c_i \rho_i t_i \vec{U}_i - \lambda^*_i \text{grad} t_i) + \vec{J}_{ki} + \vec{J}_{gi} \right] = \sum \chi_{x \lambda_i \lambda_j} \lambda^*_i \lambda^*_j \beta_{ij} (t_j - t_i) + \sum Q_{aij}
$$
\[ \frac{\partial \alpha_{i} c_{i} \rho_{i} t_{i}}{\partial \tau} + \text{div} \left[ \alpha_{i} \left( c_{i} \rho_{i} t_{i} \bar{U}_{i} - \lambda_{i} \text{grad}_{i} t_{i} \right) \right] = \sum_{j} \frac{\chi_{i} \lambda_{i}^{*} \lambda_{j}^{*}}{\lambda_{i}^{*} \delta_{j} + \lambda_{j}^{*} \delta_{i}} \beta_{ij} (t_{j} - t_{i}) + \sum Q_{ij} - \text{div} \left( \bar{J}_{ki} + \bar{J}_{gi} \right) \]

\[ \frac{\partial \alpha_{i} \rho_{i}}{\partial \tau} + \text{div} \alpha_{i} \rho_{i} \bar{U}_{i} = \sum_{j} \varepsilon_{ij} \]

\[ \alpha_{i} c_{i} \rho_{i} \left( \frac{\partial t_{i}}{\partial \tau} + \bar{U}_{i} \text{grad}_{i} t_{i} \right) = \text{div} \left( \alpha_{i} \lambda_{i}^{*} \text{grad}_{i} t_{i} + \bar{J}_{ki} + \bar{J}_{gi} \right) + \]

\[ + \sum \frac{\chi_{i} \lambda_{i}^{*} \lambda_{j}^{*}}{\lambda_{i}^{*} \delta_{j} + \lambda_{j}^{*} \delta_{i}} \beta_{ij} (t_{j} - t_{i}) - \sum (Q_{ij} - c_{i} t_{i} \varepsilon_{ij}) \]

\[ \alpha_{i} = 1; \ t_{j} = t_{i}; \ \varepsilon_{ij} = 0, \]

\[ c \rho \left( \frac{\partial t}{\partial \tau} + \bar{U} \text{grad} t \right) = \text{div} \left( \lambda^{*} \text{grad} t + \bar{J}_{k} + \bar{J}_{g} \right) \]

3 Conclusions

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