Improved parametrization of the vertical turbulent exchange coefficient, taking into account the shallow water bodies specifics

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Abstract. The article is devoted to the problem of improving the parameterization of the coefficient of turbulent exchange in shallow water bodies with significant depth differences and intensive vertical mixing. The article presents the basic equations of the model of taking into account the wave effect on the inhomogeneous vertical coefficient of turbulent exchange. This model is integrated into the system of Navier-Stokes equations taking into account the variable density, temperature and salinity, which describes the movement of a liquid under the influence of various forces, including pressure and viscosity, as well as forces caused by the action of waves. The article presents the discretization of the Navier-Stokes equations and the transfer equations by the finite difference method in three measurement directions. The additional term describing the wave effect on the vertical velocity in a shallow-water reservoir depends on the characteristics of the wave field: amplitude, angular frequency, wave number and coordinates. Using a model that takes into account the wave effect in three measuring directions and inhomogeneous turbulent exchange in the vertical direction, detailed information about the dynamics of waves in the simulated section of a shallow reservoir was obtained. The forecast of changes in hydrodynamic wave processes of the coastal zone is constructed. Inhomogeneous vertical turbulent exchange can vary significantly in different parts of a shallow reservoir and affect wave activity. Taking into account these changes makes the model more accurate and relevant to real conditions.

1 Introduction

In the context of modern studies of hydrodynamics and ecosystems of shallow water bodies, the problem of improving the parameterization of the coefficient of vertical turbulent exchange appears as a key aspect for understanding and modeling complex physical processes. Coastal zones of seas, rivers and lakes have unique characteristics, including significant depth differences and high activity of vertical mixing.

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Shallow-water ecosystems, such as coastal zones, lakes and rivers, play an important role in maintaining biodiversity and providing water resources for humanity. These areas are sensitive to changes in water quality and chemical composition that are associated with vertical mixing. Shallow water areas can become a place of occurrence of natural disasters: tsunamis, floods and storms. Understanding turbulent mixing in such areas is critical for predicting and mitigating the effects of these events. Shallow-water ecosystems can be vulnerable to pollution, climate change and other anthropogenic impacts. Understanding vertical mixing processes helps predict and manage these threats.

The claim that shallow-water ecosystems may be vulnerable to pollution, climate change and other anthropogenic impacts is supported by data and research in the field of hydrodynamics. Modern hydrodynamic studies confirm the importance of understanding and modeling hydrodynamic processes in shallow ecosystems for effective management of their vulnerability to anthropogenic and climatic changes. This helps to develop strategies for the conservation and sustainable use of such ecosystems. Thus, the development of improved methods of parametrization of the turbulent exchange coefficient, taking into account the characteristics of shallow water volumes, is a critically important task that allows for a better understanding and management of these natural resources and to minimize the potential impact on them of the environment and humans.

This article attempts to develop an improved parametrization of the vertical turbulent exchange coefficient, which not only takes into account the features of shallow water bodies, but also focuses on the specifics of intensive vertical mixing. The influence of these factors on the hydrodynamic processes of shallow-water systems remains poorly studied, and the proposed model seeks to fill this scientific gap.

In the context of hydrodynamics, the term "vertical-horizontal anisotropy" reflects the phenomenon of uneven distribution of physical characteristics of the medium in space. Anisotropy implies that these characteristics change vertically and horizontally in different directions. Within the framework of the proposed article, vertical-horizontal anisotropy indicates that turbulent processes and transfer of matter in shallow water bodies change both in the vertical direction (from the surface to the bottom or back) and in the horizontal direction (at various points of the reservoir). This may be due to the geometric features of the reservoir, the presence of obstacles or the peculiarities of currents. The proposed parametrization of turbulent exchange is aimed at accounting for and describing this anisotropy for more accurate modeling of physical processes in shallow water conditions.

2 Materials and Methods

2.1 Models of parametrization of the turbulent exchange coefficient in shallow water bodies

There are various grounds for classifying models of parametrization of the turbulent exchange coefficient in shallow water conditions. *Constant turbulence models* assume that the coefficient of turbulent exchange is constant throughout the water

column. This approach is simple and convenient for practical use, but does not take into account changes in turbulence depending on depth and other factors. The constant coefficient can be determined by calibration based on experimental and laboratory data.

Prandtl-Kolmogorov models are based on the theory of turbulence and take into account the profile of changes in turbulence in the vertical direction. One of the most widespread variants is the *k-ε* (*k-epsilon*) model, which takes into account two parameters - turbulence intensity (*k*) and frequency (*ε*). However, such models require a large amount of data and computing resources and may not be completely accurate in shallow water conditions [1-3].

Models with a variable coefficient of turbulent exchange assume that the coefficient of turbulent exchange varies depending on the conditions of the water body. In shallow areas with strong vertical gradients of velocity and density, the turbulence coefficient may be higher than in deeper waters. Such models are more accurate in describing shallow water conditions, a model of this type will be used for inclusion in a three-dimensional model of hydrodynamics within the framework of this study [4-5].

Shallow water bodies can undergo stratification, which means vertical separation of water into layers of different densities and temperatures. *Models taking into account stratification* take into account the effect of this phenomenon on turbulence. The Boussinesq and Kraus-Turner models take into account vertical gradients of density and temperature to determine turbulent exchange [6].

Direct observations and experimental studies are an important source of data for parametrization of the turbulent exchange coefficient in shallow water conditions. They may include measurements of velocity and turbulence at different points and depths of a water body. The choice of a specific model or method depends on the goals and conditions of the study. It is important to take into account that shallow-water conditions may differ significantly from deep-water ones, therefore parameterization should be adapted to the specific characteristics of the research site.

2.2 The model of accounting for the wave effect on the inhomogeneous vertical coefficient of turbulent exchange

The model of wave action on the inhomogeneous vertical coefficient of turbulent exchange allows us to study the effect of waves on vertical mixing and turbulence in shallow waters. The model takes into account how waves affect the vertical movement and mixing of water masses, which may be important for understanding the dynamics of coastal zones. Waves can be caused by wind, tides, hydro-dynamic processes and other factors. The main parameters that the model takes into account are wave height (*H*), wave period (*T*) and wave propagation direction.

The waves transfer their energy to the water, causing fluctuations in the vertical direction. As a result, there is a vertical movement of water masses. This movement can be periodic and depend on the characteristics of the waves [7-10].

Suppose that the vertical component of the wave velocity w_w in the case of homogeneous waves is described as:

$$w_w(z,t) = acos(kz - \omega t),$$

where a is the amplitude of the vertical component of the wave velocity, k is the wave number, ω – is the angular frequency of the wave [11].

To describe the vertical component of the wave velocity for inhomogeneous waves, the following expression can be used:

$$w_w(z,t) = a\cos(k_x - \omega t)\cos(k_y - \omega t)\cos(k_z - \omega t),$$

where k_x, k_y, k_z are the wave numbers [12].

Vertical mixing under the influence of waves can be described as follows:

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial P}{\partial z} + \nu\nabla^2 u - \frac{1}{\rho}\nabla\cdot\tau_w,$$

where $\frac{\partial u}{\partial t}$ is the change in horizontal velocity over time, ρ is the density of water, $\frac{\partial P}{\partial z}$ is the vertical pressure gradient, ν is the kinematic viscosity, $\nabla^2 u$ is the Laplace operator of horizontal velocity, $\nabla\cdot\tau_w$, is the divergence of stresses caused by the action of waves on the water surface.

This equation is part of the Navier-Stokes system of equations, taking into account variable density, temperature and salinity, which describes the movement of a liquid under the action of various forces, including pressure and viscosity, as well as forces caused by the action of waves.

3D model of hydrodynamics with variable coefficient of turbulent exchange

Consider a system of Navier-Stokes equations taking into account variable density, temperature and salinity, which describes the movement of a liquid under the influence of various forces, including pressure and viscosity, as well as forces caused by the action of waves, it includes [13-14]:

– the continuity equation:

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\mathbf{u}) = 0,$$

– the equations of motion:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + F_x - 2\Omega v\sin(\phi) + \nu\nabla^2 u,$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial y} + F_y + 2\Omega u\sin(\phi) + \nu\nabla^2 v,$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial z} + F_z + \nu\nabla^2 w,$$

– the transport equation for temperature:

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = Q - \frac{1}{\rho c_p}\frac{\partial(P\alpha T)}{\partial t} + \kappa\nabla^2 T,$$

– the transfer equation for salinity:

$$\frac{\partial S}{\partial t}+u\frac{\partial S}{\partial x}+v\frac{\partial S}{\partial y}+w\frac{\partial S}{\partial z}=R-\frac{1}{\rho c_p}\frac{\partial(P\beta S)}{\partial t}+\kappa\nabla^2 S\,.$$

where ρ is the density of the liquid (variable density), P is the pressure, u, v, w are the velocity components in three directions x, y, z, F_x, F_y, F_z are the forces caused by the action of waves (for example, waves on the surface of water), Ω is the angular velocity of the Earth's rotation, ϕ is latitude, T is temperature, S is salinity, Q is the source or sink of heat, R is the source or drain of salt, α, β are the coefficients of temperature and salinity expansion, respectively, c_p is the specific heat capacity at constant pressure, κ is the coefficient of thermal conductivity.

We will carry out the discretization of the Navier-Stokes equations and the transfer equations by the method of finite differences in three measuring directions:

– the continuity equation (variable density):

$$\frac{\rho_{i,j,k}^{n+1}-\rho_{i,j,k}^{n}}{\Delta t}+\frac{u_{i+1,j,k}^{n}-u_{i-1,j,k}^{n}}{2\Delta x}+\frac{v_{i,j+1,k}^{n}-v_{i,j-1,k}^{n}}{2\Delta y}+\frac{w_{i,j,k+1}^{n}-w_{i,j,k-1}^{n}}{2\Delta z}=0\,,$$

– the equations of motion (in three directions):

$$\frac{u_{i,j,k}^{n+1}-u_{i,j,k}^{n}}{\Delta t}+u_{i,j,k}^{n}\frac{u_{i+1,j,k}^{n}-u_{i-1,j,k}^{n}}{2\Delta x}+v_{i,j,k}^{n}\frac{u_{i,j+1,k}^{n}-u_{i,j-1,k}^{n}}{2\Delta y}+w_{i,j,k}^{n}\frac{u_{i,j,k+1}^{n}-u_{i,j,k-1}^{n}}{2\Delta z}=$$

$$=-\frac{1}{\rho_{i,j,k}^{n}}\frac{P_{i+1,j,k}^{n}-P_{i-1,j,k}^{n}}{2\Delta x}+F_{x,i,j,k}^{n}+$$

$$+\nu\left(\frac{u_{i+1,j,k}^{n}-2u_{i,j,k}^{n}+u_{i-1,j,k}^{n}}{\Delta x^2}+\frac{u_{i,j+1,k}^{n}-2u_{i,j,k}^{n}+u_{i,j-1,k}^{n}}{\Delta y^2}+\frac{u_{i,j,k+1}^{n}-2u_{i,j,k}^{n}+u_{i,j,k-1}^{n}}{\Delta z^2}\right),$$

$$\frac{v_{i,j,k}^{n+1}-v_{i,j,k}^{n}}{\Delta t}+u_{i,j,k}^{n}\frac{v_{i+1,j,k}^{n}-v_{i-1,j,k}^{n}}{2\Delta x}+v_{i,j,k}^{n}\frac{v_{i,j+1,k}^{n}-v_{i,j-1,k}^{n}}{2\Delta y}+w_{i,j,k}^{n}\frac{v_{i,j,k+1}^{n}-v_{i,j,k-1}^{n}}{2\Delta z}=$$

$$=-\frac{1}{\rho_{i,j,k}^{n}}\frac{P_{i,j+1,k}^{n}-P_{i,j-1,k}^{n}}{2\Delta y}+F_{y,i,j,k}^{n}+$$

$$+\nu\left(\frac{v_{i+1,j,k}^{n}-2v_{i,j,k}^{n}+v_{i-1,j,k}^{n}}{\Delta x^2}+\frac{v_{i,j+1,k}^{n}-2v_{i,j,k}^{n}+v_{i,j-1,k}^{n}}{\Delta y^2}+\frac{v_{i,j,k+1}^{n}-2v_{i,j,k}^{n}+v_{i,j,k-1}^{n}}{\Delta z^2}\right),$$

$$\frac{w_{i,j,k}^{n+1}-w_{i,j,k}^{n}}{\Delta t}+u_{i,j,k}^{n}\frac{w_{i+1,j,k}^{n}-w_{i-1,j,k}^{n}}{2\Delta x}+v_{i,j,k}^{n}\frac{w_{i,j+1,k}^{n}-w_{i,j-1,k}^{n}}{2\Delta y}+w_{i,j,k}^{n}\frac{w_{i,j,k+1}^{n}-w_{i,j,k-1}^{n}}{2\Delta z}=$$

$$=-\frac{1}{\rho_{i,j,k}^{n}}\frac{P_{i,j,k+1}^{n}-P_{i,j,k-1}^{n}}{2\Delta z}+F_{z,i,j,k}^{n}+$$

$$+\nu\left(\frac{w_{i+1,j,k}^{n}-2w_{i,j,k}^{n}+w_{i-1,j,k}^{n}}{\Delta x^2}+\frac{w_{i,j+1,k}^{n}-2w_{i,j,k}^{n}+w_{i,j-1,k}^{n}}{\Delta y^2}+\frac{w_{i,j,k+1}^{n}-2w_{i,j,k}^{n}+w_{i,j,k-1}^{n}}{\Delta z^2}\right),$$

– the transfer equation for temperature:

$$\frac{T_{i,j,k}^{n+1}-T_{i,j,k}^{n}}{\Delta t}+u_{i,j,k}^{n}\frac{T_{i+1,j,k}^{n}-T_{i-1,j,k}^{n}}{2\Delta x}+v_{i,j,k}^{n}\frac{T_{i,j+1,k}^{n}-T_{i,j-1,k}^{n}}{2\Delta y}+w_{i,j,k}^{n}\frac{T_{i,j,k+1}^{n}-T_{i,j,k-1}^{n}}{2\Delta z}=$$

$$=Q_{i,j,k}^{n}-\frac{1}{\rho_{i,j,k}^{n}c_p}\frac{P_{i+1,j,k}^{n}-P_{i-1,j,k}^{n}}{2\Delta x}+\frac{\kappa}{\Delta x^2}(T_{i+1,j,k}^{n}-2T_{i,j,k}^{n}+T_{i-1,j,k}^{n})+$$

$$+\frac{\kappa}{\Delta y^2}(T_{i,j+1,k}^{n}-2T_{i,j,k}^{n}+T_{i,j-1,k}^{n})+\frac{\kappa}{\Delta z^2}(T_{i,j,k+1}^{n}-2T_{i,j,k}^{n}+T_{i,j,k-1}^{n}),$$

– the transfer equation for salinity:

$$\frac{S_{i,j,k}^{n+1}-S_{i,j,k}^{n}}{\Delta t}+u_{i,j,k}^{n}\frac{S_{i+1,j,k}^{n}-S_{i-1,j,k}^{n}}{2\Delta x}+v_{i,j,k}^{n}\frac{S_{i,j+1,k}^{n}-S_{i,j-1,k}^{n}}{2\Delta y}+w_{i,j,k}^{n}\frac{S_{i,j,k+1}^{n}-S_{i,j,k-1}^{n}}{2\Delta z}=$$

$$=R_{i,j,k}^{n}-\frac{1}{\rho_{i,j,k}^{n}c_p}\frac{P_{i+1,j,k}^{n}-P_{i-1,j,k}^{n}}{2\Delta x}+\frac{\kappa}{\Delta x^2}(S_{i+1,j,k}^{n}-2S_{i,j,k}^{n}+S_{i-1,j,k}^{n})+$$

$$+\frac{\kappa}{\Delta y^2}(S_{i,j+1,k}^{n}-2S_{i,j,k}^{n}+S_{i,j-1,k}^{n})+\frac{\kappa}{\Delta z^2}(S_{i,j,k+1}^{n}-2S_{i,j,k}^{n}+S_{i,j,k-1}^{n}),$$

where $\rho_{i,j,k}^{n+1}$ is the density of the liquid at the time step *n+1* at the grid point (*i, j, k*), $u_{i,j,k}^{n+1}, v_{i,j,k}^{n+1}, w_{i,j,k}^{n+1}$ are the components of the velocity of the liquid at the time step *n+1* at the grid point (*i, j, k*) in the directions *x, y, z* respectively, $P_{i,j,k}^{n}$ is the pressure at the time step *n* at the grid point (*i, j, k*), Δt is the time step, $\Delta x, \Delta y, \Delta z$ are the spatial steps in the directions *x, y, z*, respectively, ν is the kinematic viscosity of the liquid, $F_{x,i,j,k}^{n}, F_{y,i,j,k}^{n}, F_{z,i,j,k}^{n}$ are the external force effects at the time step *n* at the grid point (*i, j, k*), in the directions *x, y, z*, respectively, $T_{i,j,k}^{n+1}$ is the temperature at the time step *n+1* t the grid point *(i, j, k),* $Q_{i,j,k}^{n}$ is the source/heat sink at the time step *n* at the grid point (*i, j, k*), κ is the thermal conductivity of the liquid, $S_{i,j,k}^{n+1}$ is the salinity (salt concentration) at the time step *n+1* at the nodal point of the grid *(i, j, k),* $R_{i,j,k}^{n}$ is the source/salt runoff at the time step *n* at the nodal point of the grid (*i, j, k*).

The vertical velocity of water induced by waves depends on the characteristics of the wave field and can be described in accordance with the equation for wave-type fluid motion. In this case, we will consider the vertical component of the velocity w in the system of Navier-Stokes equations, taking into account the wave action [15-16].

The equation of fluid motion for the vertical velocity component *w* can be written as follows:

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho}\frac{\partial P}{\partial z}+F_z+\nu\nabla^2 w+S_w,$$

where S_w is an additional term describing the wave effect on the vertical velocity.

An additional term describing the wave effect on the vertical velocity in a shallow reservoir can be expressed taking into account the wave theory and depends on the characteristics of the wave field in shallow water conditions.

The vertical component of the wave velocity w_w in the case of monochromatic waves is described as follows:

$$w_w(z,t) = acos(kz - \omega t),$$

in the case of inhomogeneous waves:

$$w_w(z,t) = a\cos(k_x - \omega t)\cos(k_y - \omega t)\cos(k_z - \omega t).$$

The additional term S_w can be expressed by:

$$S_w = \frac{\partial w_w}{\partial t} + u\frac{\partial w_w}{\partial x} + v\frac{\partial w_w}{\partial y} + w\frac{\partial w_w}{\partial z}.$$

Substituting the expression for $w_w(z,t)$ we get:

$$S_w = -a\omega sin(kz - \omega t) + u(-a\omega sin(kz - \omega t)\frac{\partial}{\partial x}) + v(-a\omega sin(kz - \omega t)\frac{\partial}{\partial y}) + w(a\omega kcos(kz - \omega t)),$$

or for three-dimensional inhomogeneous waves:

$$S_w = -\frac{\partial}{\partial t}\Big(a\cos(k_x x - \omega t)\cos(k_y y - \omega t)\cos(k_z z - \omega t)\Big) -$$

$$-u\frac{\partial}{\partial x}\Big(a\cos(k_x x - \omega t)\cos(k_y y - \omega t)\cos(k_z z - \omega t)\Big) -$$

$$-v\frac{\partial}{\partial y}\Big(a\cos(k_x x - \omega t)\cos(k_y y - \omega t)\cos(k_z z - \omega t)\Big) -$$

$$-w\frac{\partial}{\partial z}\Big(a\cos(k_x x - \omega t)\cos(k_y y - \omega t)\cos(k_z z - \omega t)\Big)$$

Thus, the additional term S_w makes a significant contribution to the vertical velocity and depends on the specific parameters of the wave field: amplitude, angular frequency, wave number or numbers and coordinates.

3 Results and Discussions

Using a model that takes into account wave action in three measuring directions and inhomogeneous turbulent exchange in the vertical direction, it is possible to obtain detailed information about the dynamics of waves in shallow waters. The model allows you to predict the height, length, direction, frequency and period of waves, the energy carried by waves at different points of a shallow reservoir. The dynamics of waves can affect the circulation and mixing of water in the reservoir.

The results of numerical experiments on modeling the propagation of wave hydrodynamic processes based on 3D model of hydrodynamics that takes into account the heterogeneity of turbulent mixing in the vertical direction at different time periods (10, 20, 30, 40 seconds) are presented (Fig. 1). Based on the devel-

oped programs, a forecast of changes in hydrodynamic wave processes of the coastal zone is constructed, the formation of turbulent structures is predicted.

Inhomogeneous vertical turbulent exchange varies significantly in different parts of a shallow reservoir and affects wave activity. Taking into account these changes makes the model more accurate and relevant to real conditions.

Thus, taking into account inhomogeneous vertical turbulent exchange makes modeling more realistic and useful for a wide range of scientific and engineering tasks related to shallow water bodies and their ecosystems.

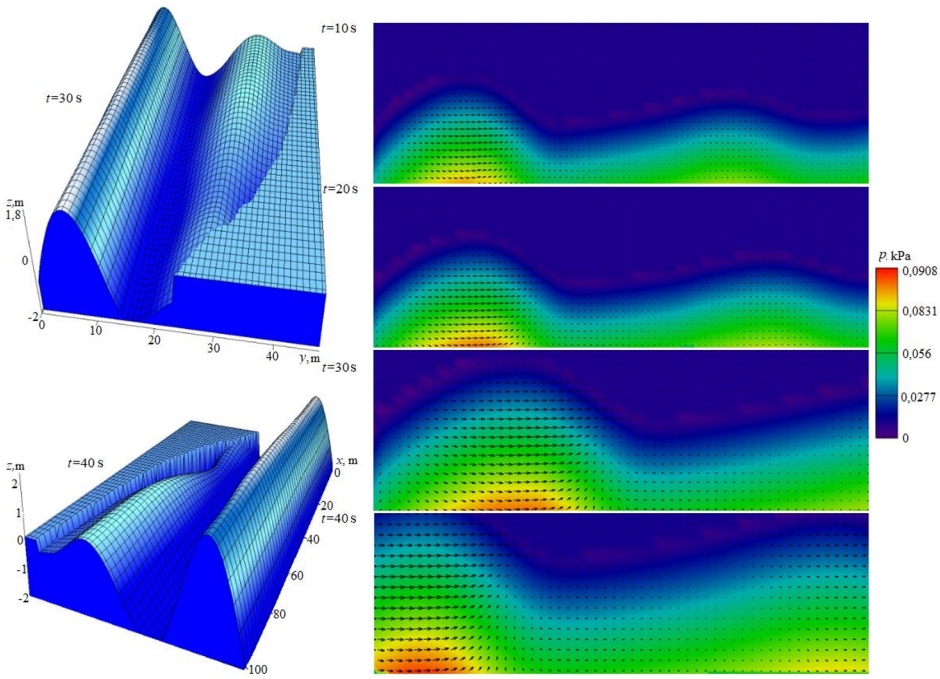

Fig. 5. Wave profiles and velocity vector fields at different time points.

4 Summary

The study and modeling of the parametrization of the turbulent exchange coefficient in shallow water volumes with significant depth differences and vertical mixing is an urgent and important task in the field of hydrodynamics and ecology. The paper describes the existing methods and models of parametrization of the turbulent exchange coefficient in shallow water conditions. A model of accounting for the wave effect on the inhomogeneous vertical coefficient of turbulent exchange is described. This model was used to improve the parametrization of the turbulent exchange coefficient included in the three-dimensional spatially inhomogeneous model of hydrodynamics.

A three-dimensional model of the hydrodynamics of shallow water bodies with a variable coefficient of turbulent exchange is presented, taking into account the wave effect on vertical mixing and turbulence in shallow water bodies. The model

includes a system of Navier-Stokes equations taking into account variable density, temperature and salinity, which describes the movement of a liquid under the influence of various forces, including pressure and viscosity, as well as forces due to the influence of gravitational waves.

The simulation results allow us to obtain detailed information about the dynamics of waves and currents in shallow waters, which is useful for understanding and forecasting environmental and climatic processes. An additional term describing the wave action takes into account the parameters of the wave field and makes the model more realistic.

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