Force analysis of roller squeezing mechanism

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Abstract. Force analysis of roller squeezing mechanisms was conducted on the basis of analytical dependencies obtained to determine the energy-power parameters of roller squeezing of semi-finished leather products after dyeing. These dependencies were determined taking into account the hydraulic pressure during squeezing. It was revealed that the thickness of the leather layer and the radius of the roll have the greatest influence on the energy-power parameters. With a decrease in the thickness of leather and the radius of the roll, the gripping force can be reduced due to a decrease in the extent of the contact zone. A decrease in the roll radius reduces the torque and power required to rotate the roll.

1 Introduction

Roller mechanisms are the main working parts of most equipment in many branches of industry, including transport. Among them, we can highlight squeezing mechanisms that are directly related to wastewater disposal, and therefore, to the environmental safety of enterprises.

When squeezing wet materials, the energy-power parameters are the forces created by the roll pressing devices, the torque, and the forces required to rotate the rolls. They are of practical importance in the design and operation of a squeezing machine. The quality and effect of roller squeezing largely depend on these parameters.

Energy-power parameters are determined based on force analysis of roller mechanisms. Force analysis is conducted using kinetic-static methods, that is, taking into account the inertia forces applied to the links of the mechanism. However, due to the stationary operating conditions of roller mechanisms and the sufficient balance of their links, static methods [1] or quasi-static methods [2] can be used in force analysis.

The phenomena of contact interaction and fluid filtration occur simultaneously in roller squeezing mechanisms. Consequently, when performing force analysis in roller squeezing mechanisms, it becomes necessary to take into account the results of solving the problem of contact interaction and the problem of filtration (roller squeezing).

References [3-6] are devoted to the determination of forces and moments in roller mechanisms. The calculation formulas obtained in these studies have insufficient accuracy and a certain complexity. When determining the analytical dependencies of forces and moments in a roller squeezing mechanism, the phenomena of fluid filtration and filtration

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properties of the squeezed material, are not taken into account. Consequently, the obtained dependencies do not allow us to determine reliable values of forces and moments of roller squeezing mechanisms.

2 Materials and methods

This study is devoted to the force analysis of the roller mechanisms of a machine for squeezing semi-finished leather products after dyeing and fat-liquoring operations of leather.

The possibility of performing force analysis of roller squeezing mechanisms is largely prepared by previous studies conducted in various fields of industries to solve the problem of contact interaction and fluid filtration [7-24].

Figure 1 shows a scheme of a roller squeezing mechanism with three links: a layer of leather of thickness $\delta_1$, drive rollers of the same radius $R$ coated with elastic materials (a technical cloth) of the same characteristics and thickness $H$. This is the most typical scheme of roller machines for squeezing leather after dyeing and fat-liquoring operations.

![Fig. 1. Scheme of a roller squeezing mechanism.](image)

The semi-finished leather product at the stage of squeezing after dyeing and fat-liquoring operations has a uniform and thin thickness, so, in the contact zone, it does not recover its deformation.

Based on this, we consider that the roll contact curve consists of two sections – curve-lined and straight-lined. In the curved section, the leather and cloth are compressed, so, they are in the front part of the contact zone.

During the squeezing process, due to the action of reactive forces, the cross-section of the maximum deformation of leather is shifted from the line of centers towards the entrance of the leather layer into the contact zone [23]. Therefore, the straight section is located in the middle and rear parts of the contact zone.

According to Figure 1

\[-\phi_1 + \phi_3 \leq \theta_1 + \theta_3 \leq 0, \quad 0 \leq \theta_2 + \phi_3 \leq \phi_2 + \phi_3,\]
where $\varphi_1, \varphi_2$ – are the nip and exit angles, $\varphi_3$ – is the angle separating sections 1 and 2.

We determine the mathematical model of the curved section using the hypothesis of the constancy of the ratio of deformation rates of contacting bodies in the contact zone $\frac{\varepsilon}{\varepsilon^*} = k$ [25] and considering expression $\varepsilon = \frac{R - \eta_1}{H}$ and $\varepsilon^* = \frac{2}{\delta_1} \left( R - \frac{\cos(\varphi_1 + \varphi_3)}{\cos(\theta_1 + \varphi_3)} \right)$ (Figure 1), where $\varepsilon, \varepsilon^*$ – are the relative deformations of cloth and leather, respectively.

Then, we obtain:

$$ r_1 = \frac{R}{1 + mk} \left( 1 + \frac{a}{\cos(\theta_1 + \varphi_3)} \right), \quad -\varphi_1 + \varphi_3 \leq \theta_1 + \varphi_3 \leq 0, \quad (1) $$

where $a = mk \cos(-\varphi_1 + \varphi_3)$, $m = \frac{2H}{\delta_1}$, $k$ – is the indicator that determines the ratio of deformation rates of contacting bodies.

According to Figure 1, the mathematical model of a straight section is defined in the following form:

$$ r_2 = \frac{R \cos(\varphi_2 + \varphi_3)}{\cos(\theta_2 + \varphi_3)}, \quad (2) $$

With one known model of roll contact curve, we have a statically indeterminate problem. To solve it, it is necessary to introduce auxiliary conditions characterizing the interaction of the roller with the material being squeezed out and the fluid filtration for the specific case under consideration. We take the distribution of normal, shear, and hydraulic forces along the roll contact curve as an auxiliary condition. In polar coordinates with a pole in the center of the roll, the patterns of distribution of normal, shear, and hydraulic forces can be expressed by the following equations [14, 23]:

- for a curved section

$$ n_1 = n_{\text{max}} \left[ 1 - \frac{(\theta_1 + \varphi_3)^2}{(-\varphi_1 + \varphi_3)^2} \right], \quad (3) $$

$$ t_1 = \tan(\theta_1 + \varphi_3 - \mu_1) + C)n_1, \quad (4) $$

$$ p_1 = \frac{\alpha}{1 - \alpha} n_1, \quad (5) $$

where $C$ – is the dynamic coefficient of the roll, $\mu_1$ – is the angle between force $dN_1$ and radius $r_1$, $\alpha$ – is the coefficient determining the ratio of hydraulic pressure to the total pressure in the curved section, $n_{\text{max}}$ – is the maximum value of the normal stress;

- for a straight section

$$ n_2 = n_{\text{max}} \left[ 1 - \frac{(\theta_2 + \varphi_3)^2}{(-\varphi_2 + \varphi_3)^2} \right]^{\frac{1}{2}}, \quad (6) $$

$$ t_2 = \tan(\theta_1 + \varphi_3)n_2, \quad (7) $$

$$ p_2 = \frac{\beta}{1 - \beta} n_2, \quad (8) $$
where $\beta$ – is the coefficient that determines the ratio of hydraulic pressure to the total pressure in a straight-line section.

### 3 Results and discussion

Considering the roll in equilibrium under the action of applied forces, we obtain:

$$Q = N_y + T_y + P_y, \quad (9)$$

$$M_{rot} = M_{fr} - M_n - M_t - M_p \quad (10)$$

where $M_n, M_t, M_p$ – are the main moments of normal, shear, and hydraulic forces, $N_y, T_y, P_y$ – are the projections of the main normal, shear, and hydraulic forces onto the $Oy$ axis, equal to the sum of projections of the forces of the first and second sections onto the $Oy$ axis, that is:

$$N_y = N_{1y} + N_{2y}, \quad T_y = T_{1y} + T_{2y}, \quad P_y = P_{1y} + P_{2y}. \quad (11)$$

Assuming that $Q = Q_1 + Q_2$, from expression (9) we find

$$dQ_1 = dN_{1y} + dT_{1y} + dP_{1y}. \quad (12)$$

From Figure 2, it follows that

$$dQ_1 = (dN_1 + dP_1) \cos(\theta_1 + \varphi_3 - \mu) + dT_1 \sin(\theta_1 + \varphi_3 - \mu)$$

or taking into account expressions (4) and (5)

$$dQ_1 = dN_1 \left[ \alpha \cos(\theta_1 + \varphi_3 - \mu_1) + C \sin(\theta_1 + \varphi_3 - \mu_1) \right]. \quad (12)$$

![Fig. 2. Scheme of forces acting on the roller.](image)

The moduli of elementary normal forces are expressed as in [29]:

$$dN_1 = n_1 \sqrt{r_1^2 + r_1^2} d(\theta_1 + \varphi_3). \quad (13)$$
Transforming equalities (12) considering expressions (13), \( \cos \mu_i = \frac{r_i}{\sqrt{r_i^2 + r_i'^2}} \) and \( \sin \mu_i = \frac{r_i'}{\sqrt{r_i^2 + r_i'^2}} \), we obtain:

\[
dQ_1 = n_1 \left( \frac{r_i^2 + r_i'^2}{r_i \cos(\theta_1 + \varphi_3) + r_i' \sin(\theta_1 + \varphi_3)} + \frac{\alpha}{1 - \alpha} (r_i \cos(\theta_1 + \varphi_3) + r_i' \sin(\theta_1 + \varphi_3)) + C(r_i \sin(\theta_1 + \varphi_3) + r_i' \cos(\theta_1 + \varphi_3)) \right) d(\theta_1 + \varphi_3).
\]

From equality (1), we have:

\[
r_i' = \frac{R}{1 + mk} \cdot \frac{a \sin(\theta_1 + \varphi_3)}{\cos^2(\theta_1 + \varphi_3)}.
\]

Taking equalities (1) and (15) into account, we find

\[
r_i \cos(\theta_1 + \varphi_3) + r_i' \sin(\theta_1 + \varphi_3) = \frac{R}{1 + mk} \left( \cos(\theta_1 + \varphi_3) + \frac{a}{\cos^2(\theta_1 + \varphi_3)} \right),
\]

\[
r_i \sin(\theta_1 + \varphi_3) - r_i' \cos(\theta_1 + \varphi_3) = \frac{R}{1 + mk} \sin(\theta_1 + \varphi_3),
\]

\[
r_i^2 + r_i'^2 = \left( \frac{R}{1 + mk} \right)^2 \left( 1 + \frac{2a}{\cos(\theta_1 + \varphi_3)} + \frac{a^2}{\cos^4(\theta_1 + \varphi_3)} \right).
\]

Transforming equalities (14) considering expressions (3), (16), (17), and (18), we obtain:

\[
dQ_1 = \frac{R}{1 + mk} \left( \frac{a^2 + 2a \cos^3 \theta_1 + \cos \theta_1^4}{(a + 1 \cos^3(\theta_1 + \varphi_3)) \cos^2(\theta_1 + \varphi_3)} + \right.
\]

\[
\left. + \frac{\alpha}{1 - \alpha} \left( \cos(\theta_1 + \varphi_3) + \frac{a}{\cos^2(\theta_1 + \varphi_3)} \right) + C \sin \theta_1 \right) \left( 1 - \frac{(\theta_1 + \varphi_3)^2}{(-\varphi_1 + \varphi_3)^2} \right) d(\theta_1 + \varphi_3).
\]

The pressing force on the curvilinear section of the roll contact curve is determined by integrating \( dQ_1 \) over segment \([-\varphi_1 + \varphi_3; 0]\):

\[
Q_1 = \frac{R}{1 + mk} \int_{-\varphi_1 + \varphi_3}^{0} \left( \frac{a^2 + 2a \cos^3 \theta_1 + \cos \theta_1^4}{(a + 1 \cos^3(\theta_1 + \varphi_3)) \cos^2(\theta_1 + \varphi_3)} + \right.
\]

\[
\left. + \frac{\alpha}{1 - \alpha} \left( \cos(\theta_1 + \varphi_3) + \frac{a}{\cos^2(\theta_1 + \varphi_3)} \right) + C \sin \theta_1 \right) \left( 1 - \frac{(\theta_1 + \varphi_3)^2}{(-\varphi_1 + \varphi_3)^2} \right) d(\theta_1 + \varphi_3).
\]

After integrating and transforming expression (20), we find:

\[
Q_1 = \frac{Rn_{\text{max}} (\varphi_1 - \varphi_3)}{3(1 + mk)(1 + a)(1 - \alpha)} ((2(1 + a)^2 - C(1 + a)(\varphi_1 - \varphi_3)) -
\]

\[
- (2a + 1)(\varphi_1 - \varphi_3)^2)(1 - \alpha) + 2(1 + a)(1 + mk)\alpha).
\]
From Figure 2, it follows that
\[ dQ_2 = dN_2 + dP_2. \]  
\hspace{1cm} (22)

On the straight-line section of the roll contact curve, the leather layer is not deformed. Therefore, fluid filtration does not occur in this area. Consequently, \( \beta = 0 \) and \( p_2 = 0, \ dP_2 = 0. \) Then from equality (22), we obtain:
\[ dQ_2 = dN_2 = n_2 \sqrt{r_1^2 + r_2^2} \ d(\theta_2 + \varphi_2). \]  
\hspace{1cm} (23)

From equality (2), we find
\[ r_2' = \frac{R \cos(\varphi_2 + \varphi_3) \sin(\theta_2 + \varphi_3)}{\cos^2(\theta_2 + \varphi_3)}. \]  
\hspace{1cm} (24)

Taking equalities (2), (6) and (24) into account, from equality (23) we obtain:
\[ dQ_2 = \frac{Rn_{\text{max}} \cos(\varphi_2 + \varphi_3)}{\cos^2(\theta_2 + \varphi_3)} \left(1 - \frac{(\theta_2 + \varphi_3)^2}{(\varphi_2 + \varphi_3)^2}\right) \frac{1}{2} \ d(\theta_2 + \varphi_2). \]  
\hspace{1cm} (25)

After integration on segment \([0; \varphi_2 + \varphi_3]\) and transformation of expression (25), we obtain:
\[ Q_2 = \frac{Rn_{\text{max}}}{12} (10 - (\varphi_2 + \varphi_3)^2)(\varphi_2 + \varphi_3). \]  
\hspace{1cm} (26)

From dependencies (21) and (26), we find the forces produced by gripping devices of each roll in the following form:
\[ Q = \frac{Rn_{\text{max}}}{3} \left(\frac{(\varphi_1 - \varphi_3)}{(1 + mk)(1 + a)(1 - \alpha)}(2(1 + a)^2 - C(1 + a)(\varphi_1 - \varphi_3) - \\
- (2a + 1)(\varphi_1 - \varphi_3)^2)(1 - \alpha) + 2(1 + a)(1 + mk)\alpha + \frac{1}{4}(10 - (\varphi_2 + \varphi_3)^2)(\varphi_2 + \varphi_3)\right). \]  
\hspace{1cm} (27)

Pressure \( Q \) is a linear pressure (load intensity).

When designing roller squeezing machines, it is also necessary to know the specific pressure, defined as the ratio of linear pressure to the length of the roll contact curve.

The length of the curvilinear section of the roll contact curve is determined by the following formula:
\[ l_1 = \int_{-\varphi_1 + \varphi_3}^{\theta_1} \sqrt{r_1^2 + r_1'{}^2} \ d(\theta_1 + \varphi_3). \]  
\hspace{1cm} (28)

Let us transform this formula considering expression \( \frac{r_1'}{r_1} = \tan \mu_1 \)
\[ l_1 = \int_{-\varphi_1 + \varphi_3}^{\theta_1} \frac{r_1'}{\sin \mu_1} \ d(\theta_1 + \varphi_3). \]  
\hspace{1cm} (28)

From expressions (1) and (15), we find
\[ \tan \mu_1 = \frac{a \sin(\theta_1 + \varphi_3)}{(a + \cos(\theta_1 + \varphi_3)) \cos(\theta_1 + \varphi_3)}. \]  
\hspace{1cm} (28)
or assuming that \( \cos \mu_1 \approx 1 \) or \( \tan \alpha_1 \approx \sin \mu_1 \), since \( \frac{a}{a + \cos(\theta_1 + \varphi_3)} < 1 \), we obtain:

\[
\sin \mu_1 = \frac{a \sin(\theta_1 + \varphi_3)}{(a + \cos(\theta_1 + \varphi_3)) \cos(\theta_1 + \varphi_3)}.
\]

(29).

After substituting this expression into equality (28) and integrating it, we obtain:

\[
l_1 = R(\varphi_1 - \varphi_3).
\]

(30)

From Figure 2, it follows that

\[
l_2 = R(\sin \varphi_2 + \sin \varphi_3).
\]

(31)

Using formulas (27), (30), and (31), we find the specific pressure

\[
Q_{\varphi} = \frac{n_{\text{max}}}{3(\varphi_1 + \sin \varphi_2)} \left( \frac{(\varphi_1 - \varphi_3)(1 + mk)(1 + a)(1 - \alpha)}{(1 + mk)(1 + a)(1 - \alpha)} \right) - (2a + 1)(\varphi_1 - \varphi_3)^2(1 - \alpha) + 2(1 + a)(1 + mk)\alpha + \frac{1}{4} (10 - (\varphi_2 + \varphi_3)^2(\varphi_2 + \varphi_3)).
\]

(32)

The moments of normal and shear elementary forces relative to the pole are determined by the following expressions \([1,14]\):

\[
dM_n = nrr'd\theta, \quad dM_p = prr'd\theta, \quad dM_t = tr^2d\theta.
\]

The main moment of the curvilinear section of the roll contact curve is determined by integrating the sum of these expressions on segment \([-\varphi_1 + \varphi_3;0] \):

\[
M_1 = \int_{-\varphi_1 + \varphi_3}^{0} ((n_1 + p_1) r_1 r'_1 + t_1 r_1^2) d(\theta_1 + \varphi_3)
\]

or taking into account expressions (1), (3), (4), (5), and (15)

\[
M_1 = \frac{R^2 n_{\text{max}}}{(1 + mk)^2} \int_{-\varphi_1 + \varphi_3}^{0} \left( 1 + \frac{a}{\cos(\theta_1 + \varphi_3)} \right)^2 \left( 1 - \frac{(\theta_1 + \varphi_3)^2}{(-\varphi_1 + \varphi_3)^2} \right) \times
\]

\[
x \left( C + \tan(\theta_1 + \varphi_3) + \frac{a \sin(\theta_1 + \varphi_3)}{1 - \alpha \cos^2(\theta_1 + \varphi_3)} \right) d(\theta_1 + \varphi_3).
\]

After integrating these expressions, substitution of limits, and some transformations, we have

\[
M_1 = \frac{R^2 n_{\text{max}} (\varphi_1 - \varphi_3)}{12(1 + mk)} \left( 4Ca - 3(\varphi_1 - \varphi_3) - \frac{\alpha}{1 - \alpha} mk(\varphi_1 - \varphi_3) \right)
\]

(32)

The main moment of the straight section of the roll contact curve is determined by the following integral:

\[
M_2 = \int_{\varphi_2 + \varphi_3}^{\varphi_2 + \varphi_3} (n_2 r_2 r'_2 + t_2 r_2^2) d(\theta_2 + \varphi_3).
\]

(33)
After substitution of expressions $r_2$, $n_2$, $\tau_2$ and $r_2'$ from equalities (2), (6), (7), and (24), into integral (33), integration, and transformation, we determine:

$$M_2 = \frac{3R^2n_{\text{max}}}{8} (2 + (\varphi_2 + \varphi_3))^2.$$  \hspace{1cm} (34)

From dependencies (32) and (34), we find the main moment of all elementary forces relative to the pole of the roll in the following form:

$$M = \frac{R^2n_{\text{max}}}{24} \left( \frac{2(\varphi_1 - \varphi_3)}{1 + mk} \left( 4Ca - 3(\varphi_1 - \varphi_3) - \frac{\alpha}{1 - \alpha} mk(\varphi_1 - \varphi_3) \right) + 9(2 + (\varphi_2 + \varphi_3)^2) \right).$$ \hspace{1cm} (35)

From equality (10), it follows that

$$M_{\text{rot}} = M_{fr} - M.$$  

In roll mechanisms, the friction moment in the roll supports $M_{fr}$ depends on the type of roller transmission mechanism.

With known torque $M_{\text{rot}}$, we can find the force $N$ required to rotate the rolls using the following formula [24]:

$$N = (1,15 - 1,25) \frac{M_{\text{rot}}\omega}{975\eta i},$$ \hspace{1cm} (36)

where $\omega$ – is the angular velocity of the roll, $i$ – is the total gear ratio of the drive mechanism; $\eta$ – is the overall efficiency of the drive mechanism.

### 4 Conclusions

Based on the analysis of roller squeezing mechanisms, analytical dependencies (27), (32), (35), and (36) were obtained to determine the energy-power parameters of roller squeezing of semi-finished leather products after dyeing. These dependencies were determined taking into account the hydraulic pressure during squeezing.

As a result of the force analysis, it was established that the energy-power parameters are most influenced by the thickness of the leather layer and the radii of the rolls.

With a decrease in the thickness of leather and the radius of the roll, the gripping force can be reduced due to a decrease in the extent of the contact zone. A decrease in the roll radius reduces the torque and force required to rotate the roll.

### References

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