Calculation and manufacture of multilayer composite structures

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Abstract. A brief overview of the work done to solve the problems of bending of composite plates and shells is presented. A mathematical model has been developed that takes into account the physically nonlinear properties of the layer`s material. The seams between the layers have finite longitudinal stiffness. Seams can be continuous or discrete (anchors). The proposed mathematical model makes it possible to take into account the creep properties of the aging material. The reliability of calculations is substantiated by comparison with experimental data and known calculations. Taking into account the temperature mode made it possible to develop a technology for manufacturing multilayer structures with predicted properties of the material of the layers. Based on the results presented, methods for calculating multilayer structures and technologies for their manufacture have been developed. It is proposed to create products from cement-based composites with unique planned characteristics, oriented to the intended purpose of the manufactured structures. The intended purpose of products can be achieved by using a material with special characteristics in each layer. To protect against rodents and microorganisms, light antiseptics can be used in the process of making the composite. The solution of these issues in thin-walled structures is associated with the development of technologies for creating multilayer composite systems. The results of the creation of composites are presented.

1 Introduction

Recently, a large number of materials have been created that meet increased requirements for the reliability of structures, including strength, thermal conductivity, resistance to aggressive environment, etc. It is possible to achieve the desired combination of the required properties by selecting and alternating layers in composite shells. Structures of two or more monolithic elements have been known for a long time and have found very wide application in all areas of activity. These are all kinds of load-bearing and enclosing structures, multilayer elements in buildings and mechanisms, steel-concrete pipes with high tightness and strength, and much more. If the work of the seams is taken into account when calculating multilayer structures, then such systems are classified as composite [1, 2].

Two-layer composite beams, plates and shells are often made from a concrete layer, reinforced on the side of the tension zone with a steel sheet that only takes membrane forces.

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Three-layer and multi-layer beams and slabs are arranged in such a way that layers of a more rigid material strengthen the stretched area of the structure. Such systems are used in the construction of foundations and bases of industrial construction projects with increased requirements for thermal insulation. They are also used in foundations for residential buildings erected on permafrost.

2 Main points

Further development of the theory of composite plates led to the decision to physically nonlinear problems of bending considered designs. To implement the mathematical model, a system of resolving differential equations was built taking into account the nonlinear properties of the material of each layer. The layers are connected by flexible shear links, which are described by nonlinear relationships, since the stiffness coefficient of the seams depends on the magnitude of the shear stresses. Comparison of the results of solving the problem with experimental data (in the inelastic area) confirmed the effectiveness of the proposed method [3].

The tasks were considered taking into account the isotropy and anisotropy of the properties of the materials of the structural layers and shear seams. Kirchhoff - Love hypothesis is fulfilled for a separate layer, but not for the package as a whole. The layers are interconnected by elastic - yielding shear links, which allow one layer to shift with respect to another. Cross-links are absolutely rigid. The load normal to the surface of the package is distributed according to an arbitrary principle. The total number of layers is n + 1, and the seams are n. Nonlinear equations are derived for active elastic-plastic deformations.

To record the relationship between stresses and strains, the dependences of the deformation theory are used. The material of the layers is compressed, the coefficient of transverse deformation is variable.

The influence of the stiffness of the anchorage of the plate layers on the behavior of the composite package was also investigated [2].

As a result, a system of nonlinear differential equations of equilibrium, continuity and seam work has been developed. The total number of equations is 2 (n + 1). The number of the sought functions is also 2 (n + 1): the deflection function W (x, y), n + 1 functions of efforts $\varphi^i(x,y)$ and n shear functions $T^i(x,y)$:

$$D_{11}^*\left(\frac{\partial^4 W}{\partial x^4}+\frac{\partial^4 W}{\partial y^4}\right)+2(D_{12}^*+2D_{33}^*)\frac{\partial^4 W}{\partial x^2\partial y^2}+2[\frac{\partial D_{11}^*}{\partial x}\frac{\partial^3 W}{\partial x^3}+$$
$$+\frac{\partial}{\partial x}(D_{12}^*+2D_{33}^*)\frac{\partial^3 W}{\partial x\partial y^2}+\frac{\partial D_{11}^*}{\partial y}\frac{\partial^3 W}{\partial y^3}+\frac{\partial}{\partial y}(D_{12}^*+2D_{33}^*)\frac{\partial^3 W}{\partial x^2\partial y}]+$$
$$+\left(\frac{\partial^2 D_{11}^*}{\partial x^2}+\frac{\partial^2 D_{12}^*}{\partial y^2}\right)\frac{\partial^2 W}{\partial x^2}+\left(\frac{\partial^2 D_{11}^*}{\partial y^2}+\frac{\partial^2 D_{12}^*}{\partial x^2}\right)\frac{\partial^2 W}{\partial y^2}+4\frac{\partial^2 D_{33}^*}{\partial x\partial y}\frac{\partial^2 W}{\partial x\partial y}=$$
$$=q+\sum_{i=1}^{n}c^i\left(\frac{\partial^2 T^i}{\partial x^2}+\frac{\partial^2 T^i}{\partial y^2}\right)+\sum_{i=1}^{n+1}\left(\frac{\partial^2 M_1^i}{\partial x^2}+2\frac{\partial^2 M_{12}^i}{\partial x\partial y}+\frac{\partial^2 M_2^i}{\partial y^2}\right) \quad (1)$$

Here D_{mn}^* - is the single cylindrical stiffness of the layer package; ; M_{mn}^i - bending moments in the middle of the i-th layer; $c^i = a^{i+1}+b^i$ - the distance between the middle surfaces of the layers on both sides of the i-th seam.

Continuity equations for the middle surface of the i-th layer are presented in the form [4]:

$$B_{11}^{*i}\left(\frac{\partial^4\varphi^i}{\partial x^4}+\frac{\partial^4\varphi^i}{\partial y^4}\right)+2\left(\frac{B_{33}^{*i}}{2}-B_{12}^{*i}\right)\frac{\partial^4\varphi^i}{\partial x^2\partial y^2}+2[\frac{\partial B_{11}^{*i}}{\partial x}\frac{\partial^3\varphi^i}{\partial x^3}+$$

$$+\frac{\partial}{\partial x}\left(\frac{B_{33}^{*i}}{2}-B_{12}^{*i}\right)\frac{\partial^3\varphi^i}{\partial x\partial y^2}+\frac{\partial B_{11}^{*i}}{\partial y}\frac{\partial^3\varphi^i}{\partial y^3}+\frac{\partial}{\partial y}(\frac{B_{33}^{*i}}{2}-B_{12}^{*i})\frac{\partial^3\varphi^i}{\partial x^2\partial y}]+$$
$$+\left(\frac{\partial^2 B_{11}^{*i}}{\partial y^2}-\frac{\partial^2 B_{12}^{*i}}{\partial x^2}\right)\frac{\partial^2\varphi^i}{\partial y^2}+\left(\frac{\partial^2 B_{11}^{*i}}{\partial x^2}-\frac{\partial^2 B_{12}^{*i}}{\partial y^2}\right)\frac{\partial^2\varphi^i}{\partial x^2}+\frac{\partial^2 B_{33}^{*i}}{\partial x\partial y}\frac{\partial^2\varphi^i}{\partial x\partial y}-$$
$$-\frac{\partial^2}{\partial y^2}\left[\left(B_{11}^{*i}-B_{12}^{*i}\right)(T^i-T^{i-1})\right]-\frac{\partial^2}{\partial x^2}\left[\left(B_{11}^{*i}-B_{12}^{*i}\right)(T^i-T^{i-1})\right]=$$
$$=\frac{\partial^2\varepsilon_1^i}{\partial y^2}+\frac{\partial^2\varepsilon_2^i}{\partial x^2}+2\frac{\partial^2\gamma_{12}^i}{\partial x\partial y}(2)$$

To describe the joint operation of layers on both sides of the i-th seam, it was taken into account that the characteristics of the shear stiffness of the seam are considered isotropic in the seam plane. Cross-links are absolutely rigid:

$$\frac{\partial}{\partial x}\frac{\partial T^i}{\eta^i\partial x}+\frac{\partial}{\partial y}\frac{\partial T^i}{\eta^i\partial y}+\frac{1}{\eta^i}\left(\frac{\partial^2 T^i}{\partial x^2}+\frac{\partial^2 T^i}{\partial y^2}\right)=c^i\left(\frac{\partial^2 W}{\partial x^2}+\frac{\partial^2 W}{\partial y^2}\right)+$$
$$+\varepsilon_x^{i+1}-\varepsilon_x^i+\varepsilon_y^{i+1}-\varepsilon_y^i(3)$$

The obtained equations of the mathematical model were transformed into algebraic ones by the asymptotic expansion of the determined functions into sinusoidal series. In this case, the procedure for orthogonalization of the Bubnov - Galerkin method is implemented. The system of algebraic equations was solved by the iterative method according to the Seidel scheme [3].

The method of variable parameters of elasticity in the form of I.A. Birger was used in the calculations. This allowed the solution of a nonlinear problem to be reduced to a sequence of solutions to linear problems. At the same time, in the process of calculations, variable coefficients of the stiffness of the material of the layers of the composite structure were calculated.

The calculation was reduced to fixing variable stiffness coefficients at each stage. At the beginning of the calculation, elastic values of mechanical characteristics were introduced $\mathbf{E_{c1}^i=E_0^i,\ \nu_{c1}^i=\nu_0^i}$ **.**

The system of linear equations (1), (2), (3) was solved by the Bubnov - Galerkin method and the Seidel iterative method to determine the stress - strain state of the structure. The stresses in the i-th layer and the i-th seam were determined by the expressions **[4].** The intensity of normal stresses was calculated and, taking into account the properties of the material of the layers of the plate, the secant modulus was determined $\mathbf{E_{c2}^i(x,y,z)}$ and the lateral deformation coefficient $\mathbf{\nu_{c2}^i(x,y,z)}$**.** Next, a new linear bending problem was solved and the deformed state of the plate was determined. After determining $\mathbf{\sigma_{in2}^i(x,y,z)}$ by dependence $\mathbf{\sigma_{in}^i-\varepsilon_{in}^i}$ were calculated $\mathbf{E_{c3}^i(x,y,z)}$ etc.

The calculation was carried out up to the specified difference between the subsequent and previous values**:**

$$\frac{\boldsymbol{\sigma_{inj}^i-\sigma_{inj-1}^i}}{\boldsymbol{\sigma_{inj}^i}}\leq\Delta,$$

after which the problem was considered solved.

To assess the reliability of solving the problem of bending of the considered structure, taking into account the physically nonlinear properties of the material of the middle layer, a comparison with experiment was carried out.

Calculations revealed a change in stresses up to 25% due to the physical nonlinearity of the properties of the layer materials. The dependence of the strength of the composite shell

revealed on the stiffness of the connecting seams, and not on their type (using glue or anchors) [2].

As a further development of the mathematical modeling of the bending of composite structures, the account of plastic deformations and creep was introduced. In a viscoelastic material under a constant load, deformations increase (accumulate) over time. This leads to a qualitative and significant quantitative change in the stress-strain state: a decrease in stiffness, a redistribution of forces between structural elements, etc.

To describe these phenomena, it was proposed to record the relationship between stresses and deformations for an aging material. On the basis of the theory of an elastic-creeping body, the possibility of an analytical description of fast-flowing creep is shown. Using the technique of N.Kh. Arutyunyan and A.A. Zevin, new creep kernels were constructed in relation to the aging material [5]:

$$K_1(t,\tau) = Q(t,\tau) + B(t-\tau) + \int_\tau^t Q(s)B(s-\tau)ds \quad (4)$$

$$K_2(t,\tau) = Q(t,\tau) + B(t-\tau) + Q(\tau)\int_\tau^t B(t-s)ds\,.$$

Here $\boldsymbol{K_1(t,\tau)}$, $\boldsymbol{K_2(t,\tau)}$ – are the creep kernels of the aging material; $\boldsymbol{Q(t,\tau)}$ – is the regular part of the creep kernel; $\boldsymbol{B(t-\tau)}$ – is the difference part of the creep kernel; $\boldsymbol{t-\tau}$ – is the difference argument; $\boldsymbol{t}$ – considered moment in time; $\boldsymbol{\tau}$ – is an intermediate moment in time; $\boldsymbol{s}$ – is an intermediate argument.

A weakly singular kernel satisfying all the necessary requirements was used as the difference component:

$$\boldsymbol{B(t-\tau) = \chi\, e^{-\rho(t-\tau)^\alpha}\,(t-\tau)^{\vartheta-1}\,\frac{1}{\Gamma(\vartheta)}} \quad (5)$$

where $B(t-\tau)$ – is the difference part of the creep kernel; $\Gamma(\vartheta)$ – gamma function; χ, ρ, ϑ – core parameters [$\chi > 0$; $\rho > 0$; $\vartheta \in (0;1)$; $\alpha \in (0;1)$]; $t-\tau$ – is the difference argument; t – considered moment in time; τ – intermediate time.

The core was used as a regular component, the basis of which - the creep measure had the form:

$$\boldsymbol{Q(t,\tau) = -\frac{\partial}{\partial\tau}\left[C_0 + Ae^{-\beta\tau}\right]\left[1 - e^{-\gamma(t-\tau)}\right] = \sum_{k=1}^{3} a_k e^{-\gamma_k t} e^{\alpha_k \tau}} \quad (6)$$

$$a_1 = \beta A,\quad a_2 = C_0\gamma,\quad a_3 = A(\gamma-\beta),$$
$$\alpha_1 = -\beta,\quad \alpha_2 = \gamma,\quad \alpha_3 = \gamma-\beta,\quad \gamma_1 = 0,\quad \gamma_2 = \gamma_3 = \gamma.$$

Here $Q(t,\tau)$ – regular part of the creep core; $t-\tau$ – is the difference argument; t – considered moment in time; τ – intermediate time.

After transformations, a formula was obtained that was used to calculate the bending of a composite structure, taking into account the visco-elastic properties of the aging material:

$$\boldsymbol{K_r(t,\tau) = Q(t,\tau) + B(t-\tau) + \frac{\chi}{\Gamma(\vartheta)}\sum_{k=1}^{3}\left[a_k\, e^{\alpha_k\tau-\gamma_k t}\int_0^{t-\tau} e^{\eta_{rk}z-\rho z^\alpha}\, z^{\vartheta-1}dz\right.} \quad (7)$$

where r = 1, 2; $\eta_{1k} = \alpha_k$; $\eta_{2k} = \gamma_k$, a_k, χ – required parameters.

For numerical implementation, the exponential functions in the integrand were expanded into a Maclaurin power series.

3 Conclusion

The formation of a new creep core by adding the regular and difference cores has led to the possibility of describing the rapidly flowing creep of an aging material. The use of a weakly singular expression in the difference part as applied to an aging body made it possible to simplify the numerical implementation without involving special techniques.

The possibility of using the new relationships in real practice was proved by comparison with experimental creep curves of an aging material (Ross data) [5]. The solution of the problem of deformation in time of the composite plate showed that the displacements and stresses in the composite plate change, depending on the age of loading, by 50% or more.

The results obtained were used as a theoretical basis for the creation of structures from composite plates and shells.

4 Discussion

The work carried out made it possible to proceed to the development of materials and structures with predictable properties. As a result of the use of nanomaterials, a more precise adjustment of such structural properties as strength, deformability, durability, rest of it. Development of the technology for manufacturing products is based on the study of the effect of temperature on the formation of the properties of composites [1].

Composite materials are planned with the specified parameters, oriented to the purpose (target application). Depending on the purpose of the product, a specially selected composition and manufacturing technology will provide the necessary characteristics: strength, chemical resistance, plasticity, etc.

The solution of manufacturing issues is associated with the creation of new and modernization of existing installations, the selection of the composition of the composite, ensuring the specified quality of products at a lower cost compared to the existing analogues, which will ensure the commercialization of the project.

The proposed technology makes it possible to manufacture and carry out thermal insulation of production sites of oil and gas fields in permafrost conditions.

Samples of composite boards are shown in the figure 1.

Thermal insulation materials are often used in the construction of various types of roads. The latter include railways, motorways and other roads, runways, pedestrian zones, etc. Protection against freezing of road surfaces is one of the important issues. The material for thermal insulation will protect the road from the destructive effects of frost heaving of the soil.

a)

b)

c) d)

Fig. 1. Samples of composite boards.

a) – perlite, b) – expanded polystyrene, c) – expanded clay, d) – multilayer slab.

It is proposed to use composite plates based on cement-polymer materials with fillers for the manufacture of heat-shielding insulation elements for roadbeds for various purposes.

The influence of the composition and size of the composite on the properties and characteristics of products are summarized in the table 1.

Table 1. Composition and sizes of composite ingredient.

Material	**Indicators**			
	Specific gravity (kg/m^3)	**Coefficient of thermal conductivity (watt/(m*degree))**	**Particle size (mm)**	**Cost (rubles)**
Cement M 500	1350	0,25	5-50 *10^{-3}	7000 per ton
Sand (10% of humidity)	1500	0,97	0,1-1,5	1000 per ton
Perlite M76	76	0,05	0,16-1,25	2500 per m^3
Expanded polystyrene granules	15	0,037	2-2,5	2000 per m^3
Expanded clay 0-5 mm	450	0,2	0-5	1500 per m^3
Expanded clay 10-20 mm	300	0,16	10-20	1300 per m^3

The composite consists of particles of materials of various sizes: nanometers, microns, millimeters and centimeters. Nano-technologies implemented. This allows to obtain a material with the required characteristics of strength, water resistance and thermal insulation.

The solution of these issues in thin-walled structures is associated with the development of technologies for creating multilayer composite systems. The intended purpose of products can be achieved by using a material with special characteristics in each layer. Depending on the purpose, the characteristics of the layers can provide gas tightness, strength etc.

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