On numerical and approximate analytical modeling of single- and two-photon Young's experiment using the photon coordinate wave function

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Abstract. In modern areas of photonics, the physical description of the interaction of photons with matter in the control, transmission and registration of single-photon and two-photon states implemented in practice is of great importance. An appropriate acceptable description may be faced with the need to take into account various kinds of interference effects associated with these states. Meanwhile, even the most “simple” case of single-photon interference in Young's experiment requires the use of a rather complex apparatus of quantum electrodynamics. This article explains one- and two-photon interference in Young's thought experiment based on the photon wave function (PWF) in coordinate representation. This explanation is illustrated by two examples of wavelengths: 10.6 µm and 1.5 cm. For both examples, two approaches to PWF modeling are used: “purely quantum-mechanical” and “quasi-classical”. In the first approach, a 6-component coordinate PWF is constructed using a spherically symmetric momentum distribution in a wave packet, followed by numerical and approximate analytical calculations. In the second approach, a one-component “quasi-classical” PWF is constructed, which corresponds to either electric dipole radiation or simulated spherically symmetric radiation. In all cases, the same pronounced interference pattern was obtained, which allows us to conclude that not only the quantum-mechanical coordinate PWF is able to explain the phenomena of one- and two-photon interference, but also a much simpler “quasi-classical” PWF. This conclusion sheds light on the theoretical interpretation of the measurement of the coordinate PWF in some recent experiments.

1 Introduction

One of the important tasks of photonics is the theoretical explanation of the interaction of photons with various structures of condensed matter, and for radiation in a very wide range

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of the spectrum: from the nano-range to its long-wavelength part, in fact already microwave.
In particular, the directions in which one-photon states are considered, as well as states of
two or more photons, including entangled ones, become important. Since the control of
photons, their transmission and detection are associated with possible interference effects, it
is important to have at your disposal the most illustrative and as simple as possible, but at the
same time accurate description of elementary processes upon which could be relied
considering the complex interactions of photons with matter. As such an elementary process,
in this article we consider a thought experiment that could be carried out according to the
scheme of Young’s experiment, and give a description of its expected one-photon and two-
photon interference, using the PWF in the coordinate representation, constructed and justified
in the previous authors works (see [1, 2] and references there]. For the examples, we use the
radiation with 10.6 µm wavelength by 2 ns duration, and with 1.5 cm wavelength by 0.2 ns
duration. For both examples of radiation, we apply two approaches to constructing the PWF
in the coordinate representation: “purely quantum-mechanical” and the so-called “quasi-
classical”. In the first approach, we build a PWF (wave packet) within the framework of
photon quantum mechanics – the “primary” quantization of an electromagnetic field by
specifying a spherically symmetric photon momentum distribution with subsequent
numerical and approximate analytical calculations. In the second approach, the PWF in the
coordinate representation is built on the basis of “quasi-classical” formulas for the strengths
and energy density of the electromagnetic field as applied to electric dipole radiation or
simulated spherically symmetric radiation.
Generally speaking, the construction of the coordinate PWF, despite numerous arguments
of the impossibility of its justification [3–9], is periodically discussed, and its various variants
are proposed [10–17]. In this article we will rely in the first approach, on our [1, 2, 18–21]
method of its construction – within the framework of photon quantum mechanics – in the
form of a 6-component wave packet, which is the superposition (integral) of generalized
eigenfunctions of the operators of momentum, energy, and helicity of photon in the bivector
representation. This method does not use the second quantization, but is, in fact, the result of
the “primary” quantization of the classical electromagnetic field.
In the second approach, to construct the 1-
component “quasi-classical” PWF, we use as
in [22, 23] somewhat generalized formulas of classical electrodynamics for the energy
density and energy flux density of the electromagnetic field, dividing them by energy of the
photon to obtain the probability density and the flux density of the probability of detecting a
photon in space and time.
The obtained results of modeling the PWF and its application to explaining this thought
experiment in the scheme of Young’s experiment allow us to conclude that not only the PWF
constructed within the framework of the primary quantization of the electromagnetic field is
able to explain one-photon and two-photon interference phenomena, but also the “quasi-
classical” PWF since the positions of the interference maxima and minima of the graphs of
the probability density of photon detection at the corresponding points on the screen coincide
in both approaches with very high accuracy. These results also allow us to conclude that the
measurements of the “spatial” PWF carried out (see, for example, [24–28]) in the experiment
make sense to relate and compare exactly with the 1-component “quasi-classical” PWF,
although less theoretically substantiated than the 6-component coordinate PWF, since to set
the latter only at one spatial point at one time point, 12 real numbers are required, which are
obviously practically impossible to measure experimentally.

2 Quantum mechanical coordinate photon wave function

The PWF in coordinate representation, constructed and substantiated in a number of previous
authors works [1, 2, 18–21] on the basis of Maxwell’s equations in the Majorana form [29],
can be written in the form of a wave packet as an integral over the entire space of the photon wave vector \( k = p/\hbar \):

\[
\Psi(r, t) = \int b(k, +1) \Psi_{k, +1}(r, t) \, d^3k + \int b(k, -1) \Psi_{k, -1}(r, t) \, d^3k
\]  

(1)

where “dimensionless plane monochromatic circularly polarized vectors”

\[
\Psi_{k, +1}(r, t) = (2\pi)^{-3/2} e_{+1}(k) \, e^{i(kr - kct)} \begin{pmatrix} 1 \\ 0 \end{pmatrix}
\]

(2)

\[
\Psi_{k, -1}(r, t) = (2\pi)^{-3/2} e_{-1}(k) \, e^{i(kr - kct)} \begin{pmatrix} 0 \\ 1 \end{pmatrix}
\]

(3)

satisfy the orthonormalization condition

\[
\int d^3r \, \Psi_{k, \lambda'}^{+}(r, t) \Psi_{k, \lambda}(r, t) = \delta_{\lambda \lambda'} \delta(k' - k)
\]

(4)

forming the basis used in formula (1), since they are generalized eigenfunctions of the complete set of mutually commuting momentum, energy, and helicity operators. For a photon, the eigenvalues of the helicity operator are known to be \( \lambda = \pm 1 \). In (2) and (3), the complex polarization vectors \( e_{\pm 1}(k) \) satisfy a number of orthonormal relations [1, 2, 18–21] and are generally written as follows:

\[
e_{\lambda}(k) = [e_I(k) + i \lambda e_{II}(k)]/\sqrt{2}
\]

(5)

where \( e_I(k) \) and \( e_{II}(k) \) are real mutually perpendicular vectors, equal in modulus to one, form the right triple of vectors with the vector \( n = k/k \); for relation (4) to take place, the vector \( e_I(k) \) must not change its direction when the direction of the vector \( k \) changes to the opposite (that is, equality \( e_I(k) = e_I(-k) \) must be fulfilled for any \( k \)).

The wave function (1) makes it possible [1, 2, 18–21] to calculate the photon detection probability density normalized to the unit probability

\[
\rho_p(r, t) = \Psi^+(r, t) \Psi(r, t)
\]

(6)

In formula (1), the coefficients \( b(k, \lambda) \), generally speaking, can be set arbitrarily (or calculated), but they must also satisfy the orthonormality condition, which can be combined into a chain of general relations, taking into account (6)

\[
\langle \Psi | \Psi \rangle = \int d^3r \rho_p(r, t) = \int d^3k \rho_p(k) = \int \sum_\lambda |b(k, \lambda)|^2 \, d^3k = 1
\]

(7)

The PWF in the momentum representation is also [18, 20, 21] the 6-component function:

\[
\Psi(k, t) = e^{-i kct} \left\{ b(k, +1) e_{+1}(k) \begin{pmatrix} 1 \\ 0 \end{pmatrix} + b(k, -1) e_{-1}(k) \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}
\]

(8)

Formulas (1) and (8) show that the PWFs in the coordinate and momentum representations are related to each other by the usual Fourier transform for quantum mechanics.
It can be shown [30] that the PWF in the form of a plane monochromatic wave, linearly or circularly polarized, normalized to one photon in a certain volume $V$, determines the probability density and the probability flux density that satisfy the continuity equation, which remains invariant under Lorentz transformations, that is, it is relativistically invariant equation.

3 Quantum-mechanical explanation of Young's experiment by approximate analytical calculations in the framework of photon wave function modeling

The explanation of Young’s experiment in classical electrodynamics is reduced to finding the phase difference of monochromatic waves emitted by both slits and incident on a given point P of the second screen. A similar situation arises with the probability amplitudes of quantum electrodynamics in the framework of the second quantization of the electromagnetic field. In [31], the idea was put forward that such a phase $\varphi = kr - kct$ is present in each term of the wave photon function (1), taking into account (2) and (3). If the radiation is sufficiently monochromatic then in formula (6) for the probability density, obviously, there is a single term proportional to the cosine of the phase difference $\delta = \varphi_1 - \varphi_2 = k (r_1 - r_2)$ of these waves emitted by two slits. This also provides an explanation for the presence of interference fringes, as does the explanation that takes place in classical electrodynamics. Let’s look at this explanation in more detail.

Let’s put the coefficients $b(k, \pm 1)$ equal

$$b(k, \pm 1) = b(k, \pm 1) = \frac{\alpha}{\sqrt{\pi}} \exp \left[ -\alpha^2 (k - k_0)^2 \right]$$

which realize a propagating spherical wave corresponding to the state of a photon with zero average momentum and its mean modulus $\hbar k_0$. The choice of (9) forms a delta-like function, which allows one to single out a monochromatic wave in the decomposition (1) in terms of plane waves, and also gives a fairly simple analytical formula for the photon wave function [21, 32]. The main ideas according to [21] of this derivation are as follows. Let $\theta_r$ and $\varphi_r$ define the radius vector $r$ in the spherical coordinate system of the configuration space. Assuming $\varphi_r = \frac{\pi}{2}$, we decompose the factor $\exp(ikr)$ in (2), (3) taking into account the geometry of Young's experiment (see Fig. 1), into the Maclaurin series near the modulus-small parameter $\cos \theta_r$, bearing in mind that in this experiment the angle values are in the region $\theta_r = \frac{\pi}{2}$:

$$e^{ikr} = e^{i(k_x x + k_y y + k_z z)} = e^{ikr \sin \theta \sin \varphi (1 + ikr \cos \theta \cos \theta_r)}$$

where, therefore, the Cartesian coordinates of the vector $r$ are assumed to be:

$$x = r \sin \theta_r \cos \varphi_r = 0, \quad y = r \sin \theta_r \sin \varphi_r \approx r, \quad z = r \cos \theta_r$$

and $\theta, \varphi$ define the wave vector $k$ in the spherical coordinate system.
Fig. 1. Geometry of single-photon and two-photon Young’s experiment.

The following vectors were chosen as polarization vectors (5) [1, 2, 21, 22]:

\[
e_{\text{I}}(k) = \begin{pmatrix} 1 - (1 - \cos \theta) \cos^2 \varphi \\ -(1 - \cos \theta) \sin \varphi \cos \varphi \\ -\sin \theta \cos \varphi \end{pmatrix}, \quad e_{\text{II}}(k) = \begin{pmatrix} -(1 - \cos \theta) \sin \varphi \cos \varphi \\ \cos \theta + (1 - \cos \theta) \cos^2 \varphi \\ -\sin \theta \sin \varphi \end{pmatrix}
\]

at \( 0 \leq \theta \leq \frac{\pi}{2} \) (12)

\[
e_{\text{I}}(k) = \begin{pmatrix} 1 - (1 + \cos \theta) \cos^2 \varphi \\ -(1 + \cos \theta) \sin \varphi \cos \varphi \\ \sin \theta \cos \varphi \end{pmatrix}, \quad e_{\text{II}}(k) = \begin{pmatrix} (1 + \cos \theta) \sin \varphi \cos \varphi \\ \cos \theta - (1 + \cos \theta) \cos^2 \varphi \\ -\sin \theta \sin \varphi \end{pmatrix}
\]

at \( \frac{\pi}{2} < \theta \leq \pi \) (13)

where the Cartesian components of the vectors \( e_{\text{I,II}}(k) \) are specified. Substituting (2), (3), (5), (10), (12), (13) into (1) and integrating over the angles \( \theta \) and \( \varphi \) we obtain

\[
\Psi(r, t) = \int_{0}^{\infty} \frac{k b(k, \pm 1)}{r \sqrt{\pi}} e^{-ik_{0}ct} \left[ \begin{pmatrix} \sin kr \\ \cos kr \cos \theta_{r} \\ -\coskr \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} \sin kr \\ \cos kr \cos \theta_{r} \\ i \coskr \end{pmatrix} \right] dk
\]

where we have neglected the terms proportional to \( 1/r^2 \), having in mind the radiation in the wave zone. In (14), the coefficient \( b(k, \pm 1) \) has not yet been specified in a specific form. Substituting expression (9) for it at \( \alpha \rightarrow \infty \), which ultimately corresponds to the separation of a monochromatic wave by this method, we find

\[
\Psi(r, t) = \frac{k_{0} e^{-ik_{0}ct}}{r \sqrt{\pi}} \left[ \begin{pmatrix} \sin k_{0}r \\ \cos k_{0}r \cos \theta_{r} \\ -\cos k_{0}r \cos \theta_{r} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} \sin k_{0}r \\ \cos k_{0}r \cos \theta_{r} \\ i \cos k_{0}r \end{pmatrix} \right]
\]

Multiplying (15) by the necessary factor independent of the coordinates, we can restore the correct dimension of the approximate quantum mechanical PWF, which is initially violated by parametrization (9):

\[
\Psi_{\text{QM appr}}(r, t) = A e^{-ik_{0}ct} \left[ \begin{pmatrix} \sin k_{0}r \\ \cos k_{0}r \cos \theta_{r} \\ -\cos k_{0}r \cos \theta_{r} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} \sin k_{0}r \\ \cos k_{0}r \cos \theta_{r} \\ i \cos k_{0}r \end{pmatrix} \right]
\]
Then the substitution of PWF (16) into formula (6) gives the following approximate quantum-mechanical density of the probability of detecting the photon:

$$\rho^{\text{appr}}_{\text{QM}}(r, t) = \frac{2A^2}{r^2} \left( 1 + \cos^2 \theta_r \cos^2 k_0 r \right)$$

(17)

Let us now assume that photons are emitted from some initial single-photon source of (almost) monochromatic radiation, which, with probability $P_0$, reach either the hole $S_1$ or the hole $S_2$ on the first screen and, then, with the equal probability 1/2 fly through any one hole, and it doesn't matter if another hole is open or closed. Both of these holes thus become new (equiprobable if both are open) single-photon sources $S_1$ and $S_2$ on the first screen (see Fig. 2).

![Fig. 2. Geometry of one-photon and two-photon Young’s experiment.](image)

Further, we will assume that the holes $S_1$ and $S_2$ themselves, in this case, give the probability distributions in the entire space, respectively given by formula (17):

$$\rho^{\text{QM}}_{1, \text{appr}}(r_1, t) = \frac{2A^2}{r_1^2} \left( 1 + \cos^2 \theta_{r_1} \cos^2 k_0 r_1 \right), \quad \rho^{\text{QM}}_{2, \text{appr}}(r_2, t) = \frac{2A^2}{r_2^2} \left( 1 + \cos^2 \theta_{r_2} \cos^2 k_0 r_2 \right)$$

(18)

Consequently, the probability distributions of detecting the photon in the entire space when either only the hole $S_1$ or only $S_2$ is open, are equal also, respectively:

$$\tilde{\rho}^{\text{QM}}_{1, \text{appr}}(r_1, t) = (1/2) P_0 \rho^{\text{QM}}_{1, \text{appr}}(r_1, t), \quad \tilde{\rho}^{\text{QM}}_{2, \text{appr}}(r_2, t) = (1/2) P_0 \rho^{\text{QM}}_{2, \text{appr}}(r_2, t)$$

(19)

Let now both holes are open at once, then the probability distribution in the entire space will be given by the probability density

$$\tilde{\rho}^{\text{QM}}_{12, \text{appr}}(r_1, r_2, t) = P_0 \rho^{\text{QM}}_{12, \text{appr}}(r_1, r_2, t)$$

(20)

where $\rho^{\text{QM}}_{12, \text{appr}}(r_1, r_2, t)$ calculated by the general formula (6):

$$\rho^{\text{QM}}_{12, \text{appr}}(r_1, r_2, t) = \left[ \Psi^{\text{QM}}_{12, \text{appr}}(r_1, r_2, t) \right]^+ \Psi^{\text{QM}}_{12, \text{appr}}(r_1, r_2, t)$$

(21)
and the coordinate quantum-mechanical PWF itself (in this approximation), according to the superposition principle, in this case is equal to

\[
\Psi_{12,\text{appr}}^{(QM)}(\mathbf{r}, t) = \frac{1}{\sqrt{2}} \left[ \Psi_{1,\text{appr}}^{(QM)}(\mathbf{r}_1 + \frac{\mathbf{d}}{2}, t) + \Psi_{2,\text{appr}}^{(QM)}(\mathbf{r}_2 - \frac{\mathbf{d}}{2}, t) \right] = \Psi_{12,\text{appr}}^{(QM)}(\mathbf{r}_1, \mathbf{r}_2, t) = (22)
\]

\[
= A e^{-i k_0 c t} \frac{1}{r_1 \sqrt{2}} \left[ \begin{pmatrix} 1 \cos k_0 r_1 & \sin k_0 r_1 \cos \theta_1 \end{pmatrix} - i \cos k_0 r_1 \right] + i \cos k_0 r_1 \cos \theta_1
\]

\[
+ A e^{-i k_0 c t} \frac{1}{r_2 \sqrt{2}} \left[ \begin{pmatrix} 0 & 0 \\ 1 & -i \cos k_0 r_2 \cos \theta_2 \end{pmatrix} - i \cos k_0 r_2 \cos \theta_2 \right] (23)
\]

where the factor \(1/\sqrt{2}\) corresponds to the equal contribution of each open hole, but is approximate, since the terms in (22) are not orthogonal to each other: their scalar product is not equal to zero, in the general case. Nevertheless, as calculations show, for many special cases of certain radiation and a “spherical” momentum distribution similar to formula (9), normalized to a unit probability (according to (7)), this scalar product gives typical values at least two orders (on magnitude) less than unity, which, in turn, is equal to the scalar products of each (in brackets) individual term of formula (22) itself with itself. Since formula (16) is approximate, it is impossible to perform analytical integration over the entire coordinate or momentum space of the probability density determined by the PWF function (22) and, thus, as a result, find the exact normalization factor instead of \(1/\sqrt{2}\), that explains its approximate use in (22), taking into account what has been said. A similar situation arises with the use of the same factor below in formulas (51) and (52).

Formulas (20) and (21) take into account the interference of both terms in (22), (23). If interference is not taken into account, then the “total” probability density of detecting the photon in all space, after it has passed through any one hole, with both open, is equal to the arithmetic sum of both densities (19):

\[
\tilde{\rho}_{\text{sum, appr}}^{(QM)}(\mathbf{r}_1, \mathbf{r}_2, t) = (1/2) P_0 \left[ \rho_{1,\text{appr}}^{(QM)}(\mathbf{r}_1, t) + \rho_{2,\text{appr}}^{(QM)}(\mathbf{r}_2, t) \right] (24)
\]

Having written the probability density of detecting the photon using formulas (20), (21), it is easy to make sure that the result of interference is determined by the interference term, which, after transformations and neglecting the summand term, which includes the product \(\cos \theta_1 \cos \theta_2\), reduces to the expression

\[
\tilde{\rho}_{\text{int, appr}}^{(QM)} = \frac{2 A^2}{r_1 r_2} P_0 \left[ \sin(k_0 r_1) \sin(k_0 r_2) + \cos(k_0 r_1) \cos(k_0 r_2) \right] (25)
\]

In the geometry of Young's experiment, usually considered in classical electrodynamics, it is assumed that \(r_1 + r_2 \approx 2 \ell\), \(r_2 - r_1 = \Delta\), where \(\Delta\) is the difference between the optical paths (in vacuum or air) of the rays emanating from both holes. In the same approximation \(\Delta \approx z d / \ell\), where \(z\) is the coordinate of the point P on the second screen, measured from the (middle) line of symmetry of the interference pattern. Taking into account these relations, the
interference term (25) takes exactly the same form, up to a normalization factor, as in classical electrodynamics:

\[
\rho_{\text{int, appr}}^{(QM)} = \frac{2A^2}{r_1r_2}P_0 \cos(k_0\Delta) = \frac{2A^2}{r_1r_2}P_0 \cos \delta
\]  

(26)

where \( \delta = \frac{2\pi\Delta}{\lambda_0} \) is the phase difference of two interfering beams from the point of view of classical electrodynamics, \( \lambda_0 = \frac{2\pi}{k_0} \) is the wavelength of the corresponding (almost) monochromatic radiation.

Thus, with the help of the coordinate PWF obtained in this paragraph in a rather rough approximation, we can explain the interference effects in Young's experiment on a single basis – within the framework of quantum mechanics – both for the particles with mass (if we also carry out a description of this experiment for them using the corresponding coordinate wave function) and for the photons, and using only the “primary” quantization, without resorting to the “secondary”.

Below, for specific examples, we will directly use formula (24), which is more accurate than it would be obtained after neglecting the term containing \( \cos \theta_{r_1} \cos \theta_{r_2} \), made in the derivation of (25). In addition, in the future, when presenting and comparing the results of specific calculations, we will assume \( P_0 = 1 \), thereby omitting this unknown probability from consideration, which is of no fundamental importance for our analysis.

4 Explanation of Young’s experiment using the quasi-classical photon wave function in the electric dipole approximation

A quasi-classical, in a certain sense, description of radiation can be introduced by modifying the purely classical formulas of electrodynamics, passing to complex expressions for the strengths of the electromagnetic field and modifying the formulas for the energy density and energy flux density accordingly. In this section, for example, we assume that the oscillations of some electric dipole occur along the axis \( z \) with a cyclic frequency \( \omega_0 = k_0c \), and its field radiated into the wave zone is described in a spherical coordinate system by the strengths of the electric and magnetic fields:

\[
E_r = E_{\phi_r} = 0, \quad E_{\theta_r} = C \sin \theta_r \exp \left[ -i(\omega_0 t - k_0 r) \right] / r
\]

\[
H_r = H_{\theta_r} = 0, \quad H_{\phi_r} = C \sin \theta_r \exp \left[ -i(\omega_0 t - k_0 r) \right] / r
\]

(27)

(28)

Accordingly, we define the energy density and energy flux density (in the CGS system) as

\[
\rho_E^{(\text{quasi})} = \frac{1}{8\pi} \left[ |E|^2 + |H|^2 \right], \quad \sigma_{\text{quasi}} = \frac{c}{4\pi} \text{Re} \left[ E^*H \right]
\]

(29)

With this definition, it is easy to check that these quantities satisfy the continuity equation

\[
\frac{\partial \rho_E}{\partial t} + \text{div} \sigma = 0
\]

(30)

taking place in the absence of charges. If the radiation is almost monochromatic and, ideally, is described by a plane wave, then characterizing with its help the one photon with linear or circular polarization, this radiation can be described, within the framework of photon quantum mechanics \([1, 2, 18–21]\), by the photon wave function in the form of “plane
monochromatic wave---by the bivector normalized to one photon in a certain volume $V$ [18, 20]. Then one can also introduce the probability density $\rho_P$ of detecting the photon in the vicinity of a given point $\mathbf{r}$ at any time $t$ and the corresponding probability flux density $\mathbf{j}_P$, according to the formulas:

$$\rho_P = \rho_E / \hbar \omega_0, \quad \mathbf{j}_P = \sigma / \hbar \omega_0$$

(31)

The introduced quantities satisfy the continuity equation

$$\partial \rho_P / \partial t + \text{div} \mathbf{j}_P = 0$$

(32)

which is relativistically invariant [30], which was also mentioned above.

Substituting (27), (28), we obtain:

$$\rho^{(\text{quasi})}_E = \frac{C^2 \sin^2 \theta_r}{4 \pi r^2}, \quad \sigma^{(\text{quasi})} = \frac{C^2 \sin^2 \theta_r}{4 \pi r^2} c \mathbf{e}_r = \rho^{(\text{quasi})}_E c \mathbf{e}_r$$

(33)

$$\rho^{(\text{quasi})}_P = \frac{C^2 \sin^2 \theta_r}{4 \pi \hbar k_0 r^2}, \quad \mathbf{j}^{(\text{quasi})}_P = \frac{C^2 \sin^2 \theta_r}{4 \pi \hbar k_0 r^2} c \mathbf{e}_r = \rho^{(\text{quasi})}_P c \mathbf{e}_r$$

(34)

where $\mathbf{e}_r$ is unit vector along radius vector $\mathbf{r}$.

Formulas (27), (28) describe the “infinite” radiation process in time. Applying them to single-photon radiation, we set the coefficients $C$ in (27), (28) and, at the same time, $A$ in (16) – (18) in the same Gaussian form:

$$A = A_0 \exp \left[ -\frac{(t-r/c)^2}{\tau^2} \right], \quad C = C_0 \exp \left[ -\frac{(t-r/c)^2}{\tau^2} \right]$$

(35)

where $\tau$ is the “radiation time”; $r/c$ is the delay time of “signal arrival” to the observation point $P$ from the origin of coordinates, in which the center of the dipole or another source of the corresponding radiation is located. In (35) it is assumed that $\tau >> T_0$, where $T_0 = 2\pi / \omega_0 = \lambda_0 / c$ is the period of oscillations of the electromagnetic field intensity corresponding to the average frequency of radiation.

The constant $C_0$ in (35) can be found as follows. Assuming that one photon is emitted for the entire time of radiation $(-\infty, \infty)$ the average energy of which is $E_{ph} = \hbar \omega_0 = \hbar k_0 c$, we equate this value to the total energy of radiation passing through a sphere of large radius $r = R >> c \tau$, taking into account that the energy flux density $\sigma^{\text{quasi}}$ is determined in (33):

$$\hbar \omega_0 = \hbar k_0 c = \int_{-\infty}^{\infty} dt \int_{0}^{2\pi} d\phi \int_{0}^{\pi} d\theta_r \sin \theta_r \cdot R^2 \left| \sigma^{\text{quasi}} \right| = \frac{C_0^2 \tau c \sqrt{2\pi}}{3}$$

(36)

Obviously, $C_0^2$ can also be found by requiring that one photon will necessarily pass through the same sphere during the entire radiation time, that is, instead of the energy flux density in (36), it is possible to integrate the probability flux density $\mathbf{j}^{(\text{quasi})}_P$ from (34) over the entire surface of the sphere, equating the result to the total unit probability:
From (36) or (37), and also (35), we find:

\[ C_0 = \sqrt{\frac{3\hbar k_0}{\pi r^2}} \quad C = \sqrt{\frac{3\hbar k_0}{\pi r^2}} \exp \left[ -\frac{(t-r/c)^2}{\tau^2} \right] \]  

(38)

Then, using the formula for \( \rho_{P}^{(\text{quasi})} \) from (34), we find the “quasi-classical” probability density of detecting a photon in the wave zone in the electric dipole approximation of single-photon source radiation:

\[ \rho_{P,\text{dip}}^{(\text{quasi})} = \frac{3\sin^2\theta_r}{4\pi c r^2 \tau^2 \sqrt{2\pi}} \exp \left[ -\frac{2(t-r/c)^2}{\tau^2} \right] \]  

(39)

Extracting the square root from (39), we introduce into consideration the modulus of the 1-component “quasi-classical” PWF in the dipole approximation in the wave zone:

\[ \Psi_{\text{dip}}^{(\text{quasi})}(r, \theta, t) = \sqrt{\rho_{P,\text{dip}}^{(\text{quasi})}} = \sqrt{\frac{3}{4\pi c r^2 \tau \sqrt{2\pi}}} \exp \left[ -(t-r/c)^2/\tau^2 \right] \sin\theta_r \]  

(40)

Finally, adding to the modulus (40) the phase factor \( \exp \left[ -i(\omega_0 t - k_0 r) / r \right] \), contained in the strengths of “quasi-classical” electromagnetic field (27), (28), we obtain the entire 1-component “quasi-classical” PWF in the electric dipole approximation, in the wave zone

\[ \Psi_{\text{dip}}^{(\text{quasi})}(r, \theta, t) = \sqrt{\frac{3}{4\pi c r^2 \tau \sqrt{2\pi}}} \exp \left[ -(t-r/c)^2/\tau^2 - ik_0(ct-r) \right] \sin\theta_r \]  

(41)

Although this function does not reflect the polarization of the photon, but, as will be seen below in its specific applications, it describes one-photon interference quite well, and can also be used to construct the wave function that describes two-photon interference appearing in experiment of Young’s kind. This function main advantages are: 1) it is normalized to the total unit probability of the photon passing through a sphere of a sufficiently large radius including a single-photon source, for the entire time from \(-\infty\) to \(\infty\); 2) this function, up to the term proportional to \(1/r^3\), vanishing at \(r \to \infty\), satisfies the wave equation

\[ \Delta \Psi_{\text{dip}}^{(\text{quasi})} - \frac{1}{c^2} \frac{\partial^2 \Psi_{\text{dip}}^{(\text{quasi})}}{\partial t^2} = 0 \]  

(42)

which is known to be a special case of the Klein-Gordon-Fock equation for a particle with zero mass; 3) wave function (41) describes the probability flux density diverging from the source along radial lines, defined in (34), which, together with the probability density (39), satisfies the continuity equation

\[ \frac{\partial \rho_{P,\text{dip}}^{(\text{quasi})}}{\partial t} + \text{div} \mathbf{j}_{P,\text{dip}}^{(\text{quasi})} = 0 \]  

(43)
Describing the one-photon Young experiment using (39) – (41) in the same way as it was done in section 3 using formulas (16) – (25), we obtain the formula for the probability density of detecting the photon in the case of two open holes, which is completely similar to the known ones formulas of classical electrodynamics for coherent continuous monochromatic radiation:

\[
\tilde{\rho}_{12,\text{dp}}(r_1, r_2, t) = \tilde{\rho}_{1,\text{dp}}(r_1, t) + \tilde{\rho}_{2,\text{dp}}(r_2, t) + 2 \sqrt{\tilde{\rho}_{1,\text{dp}}(r_1, t) \tilde{\rho}_{2,\text{dp}}(r_2, t) \cos \delta}
\]

where in the interference term the phase difference of two wave functions propagating from open holes and arriving at the observation point P on the second screen is equal to the same value as in (26): \[\delta = 2\pi \Delta / \lambda_0 = k_0 (r_2 - r_1)\].

5 Illustration of single-photon interference in Young’s thought experiment with radiation with 10.6 µm wavelength by 2 ns duration using analytical calculations

5.1 Using quasi-classical PWF in the electric dipole approximation

For the convenience of comparing the results, obtained to illustrate the theoretical formulas given above, we will start with the application of quasi-classical formulas based on (39), (41). Namely, we will assume that the holes \(S_1\) and \(S_2\) themselves give the probability distributions of detecting a photon in the entire space, respectively, given in this case by formula (39):

\[
\rho_{\text{p1,dp}}^{(\text{quasi})}(r_1, t) = \frac{2\tilde{C}_1}{r_1^2} \sin^2 \theta_{r_1}, \quad \rho_{\text{p2,dp}}^{(\text{quasi})}(r_2, t) = \frac{2\tilde{C}_2}{r_2^2} \sin^2 \theta_{r_2}
\]

where, to simplify the notation, instead of coefficient \(C\) specified in (38), the \(\tilde{C}_1, \tilde{C}_2\) are introduced playing role similar as \(A\) in (17), which for a certain distance \(r\) we will model again in the Gaussian form:

\[
\tilde{C} \equiv \tilde{C}_0 \exp \left[-(t-r/c)^2 / \tau^2\right] = \sqrt{\frac{3}{8\pi c \tau}} \exp \left[-(t-r/c)^2 / \tau^2\right], \quad \tilde{C}_0 = \sqrt{\frac{3}{8\pi c \tau}}
\]

With these notations, the wave function (41) in the electric dipole approximation of the quasi-classical approach takes the following simplified form

\[
\Psi_{\text{dp}}^{(\text{quasi})}(r, \theta, t) = \frac{\tilde{C}_0}{r} \exp \left[-(t-r/c)^2 / \tau^2 - i k_0 (ct-r) \sin \theta_r\right]
\]

Consequently, the probability distributions of detecting the photon in the entire space when either only the hole \(S_1\) or only \(S_2\) is open, are equal also, respectively:

\[
\tilde{\rho}_{\text{p1,dp}}^{(\text{quasi})}(r_1, t) = (1/2) P_0 \rho_{\text{p1,dp}}^{(\text{quasi})}(r_1, t), \quad \tilde{\rho}_{\text{p2,dp}}^{(\text{quasi})}(r_2, t) = (1/2) P_0 \rho_{\text{p2,dp}}^{(\text{quasi})}(r_2, t)
\]

Let now both holes are open at once, then the probability distribution in the entire space will be given by the probability density

\[
\tilde{\rho}_{\text{dp12}}^{(\text{quasi})}(r_1, r_2, t) = P_0 \rho_{\text{dp12}}^{(\text{quasi})}(r_1, r_2, t)
\]
where \( \rho^{(\text{quasi})}_{\text{P12,dip}}(\mathbf{r}_1, \mathbf{r}_2, t) \) calculated by the general formula (6):

\[
\rho^{(\text{quasi})}_{\text{P12,dip}}(\mathbf{r}_1, \mathbf{r}_2, t) = \left[ \Psi^{(\text{quasi})}_{12,\text{dip}}(\mathbf{r}_1, \mathbf{r}_2, t) \right]^* \Psi^{(\text{quasi})}_{12,\text{dip}}(\mathbf{r}_1, \mathbf{r}_2, t) \tag{50}
\]

and the coordinate “quasi-classical” PWF itself (in this approximation), according to the superposition principle, in this case is equal to

\[
\Psi^{(\text{quasi})}_{12,\text{dip}}(\mathbf{r}, t) = \frac{1}{\sqrt{2}} \left[ \Psi^{(\text{quasi})}_{12,\text{dip}}(\mathbf{r}_1 - \frac{\mathbf{d}}{2}, t) + \Psi^{(\text{quasi})}_{12,\text{dip}}(\mathbf{r}_2 - \frac{\mathbf{d}}{2}, t) \right] \equiv \Psi^{(\text{quasi})}_{12,\text{dip}}(\mathbf{r}_1, \mathbf{r}_2, t) = (51)
\]

\[
= \frac{\sim C_0 e^{-(t-r_1/c)^2/r^2 - i k_0 (ct-r_1)}}{r_1 \sqrt{2}} \sin \theta_1 + \frac{\sim C_0 e^{-(t-r_2/c)^2/r^2 - i k_0 (ct-r_2)}}{r_2 \sqrt{2}} \sin \theta_2, \tag{52}
\]

where the multiplier \( 1/\sqrt{2} \) still corresponds to the equal contribution of each open hole.

Formulas (49) and (50) take into account the interference of both terms in (51), (52). If interference is not taken into account, then the “total” probability density of detecting the photon in all space, after it has passed through any one hole, with both open, is equal to the arithmetic sum of both densities (48):

\[
\tilde{\rho}_{\text{sum,dip}}^{(\text{quasi})}(\mathbf{r}_1, \mathbf{r}_2, t) = (1/2) \rho_0 \left[ \rho^{(\text{quasi})}_{\text{P1,dip}}(\mathbf{r}_1, t) + \rho^{(\text{quasi})}_{\text{P2,dip}}(\mathbf{r}_2, t) \right] \tag{53}
\]

Assuming \( \rho_0 = 1 \), as mentioned above, we apply formulas (45) – (53) to photon emission with wavelength \( \lambda_0 = 10.6 \, \mu \text{m} \), duration \( \tau = 2 \, \text{ns} \).

Fig. 3 shows the photon detection probability density curves calculated in the quasi-classical approach in the electric dipole approximation for radiation with a wavelength \( \lambda_0 = 10.6 \, \mu \text{m} \), duration \( \tau = 2 \, \text{ns} \), when the distance between the holes is set equal to \( d = 100 \, \mu \text{m} \), and the distance \( \ell \) between the screens and the moment of observation of the interference pattern \( t \) are represented by two cases: (a) \( \ell = c \tau / 400 \approx 0.15 \, \text{cm} \), \( t = \tau = 2 \, \text{ns} \); (b) \( \ell = 3c \tau \approx 180 \, \text{cm} \), \( t = \ell / c + \tau / 4 = 6.5 \, \text{ns} \).

![Fig. 3. Distribution of the quasi-classical (in the electric dipole approximation) probability density on the screen as a function of z-coordinate of the point P, obtained by formula (47) for radiation with \( \lambda_0 = 10.6 \, \mu \text{m} \), \( \tau = 2 \, \text{ns} \), at \( d = 100 \, \mu \text{m} \): (a) at \( \ell \approx 0.15 \, \text{cm} \), \( t = \tau = 2 \, \text{ns} \); curve 1 is the contribution from only \( S_1 \); curve 2 is the contribution from only \( S_2 \); curve “1+2” – “arithmetic” sum of contributions \( S_1 \) and \( S_2 \); curve “12” is the result of interference obtained by formula (49); (b) at \( \ell \approx 180 \, \text{cm} \),](https://doi.org/10.1051/e3sconf/202447402026)
\( t = 6.5 \text{ ns} \): curve “1,2” – contribution from only \( S_1 \) or \( S_2 \); curve “1+2” – “arithmetic” sum of \( S_1 \) and \( S_2 \); curve “12” is the result of interference obtained by formula (49).

The value \( \ell \equiv 0.15 \text{ cm} \) is taken at selected \( d = 100 \mu \text{m} \) in order to in Fig. 3a it was possible to distinguish from each other the contributions to the interference only from one first open hole (“upper” \( S_1 \), curve 1, the first formula (48)) and only from the second open hole (“lower” \( S_2 \), curve 2, the second formula (48)). Their “arithmetic” sum is shown as the “1+2” curve (formula (53)), and the result of clearly expressed interference is the “12” curve (formula (49)). With an increase in \( d \), the number of interference peaks rapidly increases, and with a decrease in \( d \), it decreases down to one central peak. On Fig. 3b, the contributions from each individual hole \( S_1 \) and \( S_2 \) are already indistinguishable from each other, so each of them is shown by the “1,2” curve.

### 5.2 Using approximate quantum mechanical PWF according to formula (16)

Since formula (16) is approximate and is valid only near the angle \( \theta_r = \pi / 2 \), it is incorrect to find the coefficient \( A_0 \) in formulas (16) – (18), (35) in the same way as \( C_0 \) in (35), (38). However, since the general form of formulas (17), (39), (45), taking into account (46), coincides at \( \theta_r = \pi / 2 \), we can set the coefficients \( A_0 \) equal to \( \widetilde{C}_0 \), that is, according to (46), we have formulas:

\[
A_0 = \widetilde{C}_0 = \sqrt{\frac{3}{8\pi c\tau \sqrt{2\pi}}} \quad (54)
\]

On Fig. 4 “against the background” of quasi-classical probability densities already shown earlier in Fig. 3 in the form of curves “12” (calculated by formula (49) in the electric dipole approximation, see “Qu-Cl” curves), the curves of the photon detecting probability density are shown, for comparison, calculated using the approximate quantum mechanical formula (20), for radiation with a wavelength \( \lambda_0 = 10.6 \mu \text{m} \), duration \( \tau = 2 \text{ ns} \).

![Fig. 4. Distribution of quasi-classical (in the electric dipole approximation, calculated by formula (49), “Qu-Cl” curve) and approximate quantum mechanical (calculated by formula (20), “Ap-QM” curve) probability densities on the screen depending on \( z \)-coordinates of point P for radiation with \( \lambda_0 = 10.6 \mu \text{m}, \tau = 2 \text{ ns} \), at \( d = 100 \mu \text{m}: \) (a) \( \ell \equiv 0.15 \text{ cm}, \ t = 2 \text{ ns} \); (b) \( \ell \equiv 180 \text{ cm}, \ t = 6.5 \text{ ns} \).]
The distance between the holes is set equal to \( d = 100 \mu m \), the distance between the screens \( \ell \), and the time of observation of the interference pattern \( t \) are represented by two cases, the same as in Fig. 3, namely: (a) \( \ell = c \tau / 400 \approx 0.15 \text{ cm}, \ t = \tau = 2 \text{ ns} \); (b) \( \ell = 3 c \tau \approx 180 \text{ cm}, \ t = \ell / c + \tau / 4 \approx 6.5 \text{ ns} \).

When analyzing Fig. 4, it should be borne in mind that the quantum mechanical formula (16) contains factors \( \sin k_0 r \) and \( \cos k_0 r \), which introduce additional very frequent oscillations into the corresponding curves, which therefore look very thick. In general both lines in each of Fig. 4a and 4b are very similar, and the positions of minima and maxima generally coincide with great accuracy, despite completely different approaches to obtaining the corresponding formulas. A noticeable difference begins to appear at large values of the \( z \) coordinate of the observation point \( P \), since formula (16) is valid only near \( \theta_r = \pi / 2 \).

### 5.3 Illustration of Two-Photon Interference in Young’s Thought Experience

Two-photon interference in the scheme of Young’s experiment can be carried out if the sources \( S_1 \) and \( S_2 \) on the first screen (see Fig. 2) will simultaneously emit individual photons described by wave functions of the same type, that is, the photons will be in the same single-particle or in some entangled state. In [22], a numerical simulation of the Young’s experiment was carried out, in which photons, after being simultaneously emitted by sources \( S_1 \) and \( S_2 \), are described by “spherical” wave packets of the form (1): with coefficients chosen in form

\[
b(k, \pm 1) = b(k, \pm 1) = A \exp \left[ -\alpha^2 (k - k_0)^2 / 2 - ikr_0 \right] \tag{55}
\]

satisfying the normalization condition (7), from which the coefficient \( A \) follows:

\[
A = \frac{\alpha \sqrt{\alpha}}{\sqrt{2\pi \left[ 1 + 2\alpha^2 k_0^2 \right] (1 + \text{erf}(\alpha k_0)) \sqrt{\pi} + 2\alpha k_0 \exp \left( -\alpha^2 k_0^2 \right) }} \tag{56}
\]

In [22], polarization vectors (5), (12), (13) were also used.

To simulate the two-photon interference in the stated “setting” of Young’s thought experiment, in [22] at the postulate level, it was assumed that the wave function of a system of two simultaneously emitted photons is equal to the sum of wave functions of the form (1) of each photon:

\[
\Psi_{12, \text{dual}}(r, t) = \Psi_1 \left( r_1 + \frac{d}{2}, t \right) + \Psi_2 \left( r_2 - \frac{d}{2}, t \right) = \Psi_{12, \text{dual}}(r_1, r_2, t) \tag{57}
\]

where the terms differ only by vectors \( r_{01} = d / 2 \) and \( r_{02} = -d / 2 \) of sources positions that determine for them the distinction of coefficients \( b(k, \pm 1) \), according to (55), and also the formula (57) does not contain the factor \( 1 / \sqrt{2} \), in contrast to (22), (51). In essence, such normalization in (57) approximately corresponds to the total number of registered photons, equal to 2, in the entire space. This correspondence would not be approximate, but exact, if both terms in (57) were orthogonal to each other, that is, their scalar product would be equal to zero. However, it is not equal to zero, but, as specific examples show, it is two orders of magnitude, at least less than unity – the total probability that is realized by the scalar products of each of these terms with itself according to (6) and (7).

This allows one to “postulate” relations (22), (51), in accordance with the general quantum-mechanical principle of superposition. Such the postulation does not change the
general interference pattern, but can only slightly affect the values of the interference maxima of the probability densities of the corresponding curves, but not their positions (as well as the positions of the minima) on the screen.

For completeness, we present from [22] in Fig. 5 the result of a numerical calculation (using (55), (5), (12), (13)) of the “density of the number of photons” which in [22] was assumed (not quite correctly) as the density of the probability of detecting a photon (any of the two) at simultaneous emission of both (single-photon) sources $S_1$ and $S_2$ – in the form of the curve “QM” as a function of the $z$ coordinate of the observation point $P$ on the screen (see Fig. 2) – for radiation with a wavelength $\lambda_0 = 1.5$ cm and duration $\tau = 0.200158$ ns at $d = 3$ cm, $\ell = ct = 6.00$ cm, at time $t = 0.2$ ns. The normalization in [22] is used according to (57), which corresponds to the approximate normalization “to the total number of photons”, equal to 2, as mentioned above.

![Fig. 5. Distribution of the quantum mechanical photons number density (curve “QM”) obtained numerically in [22] according to formulas (57), (55), (56), (5), (12), (13) and of corresponding quasi-classical (curve “Qu-Cl”) of photons number density on the screen depending on the $z$-coordinate of the point $P$ – with the simultaneous radiation of two photons by (single-photon) sources $S_1$ and $S_2$ with wavelength $\lambda_0 = 1.5$ cm, duration $\tau = 0.200158$ ns at $d = 3$ cm, $\ell = ct = 6.00$ cm, at time $t = 0.2$ ns. The figure is taken from the author's article [22].](image)

Note that in Fig. 5 we present a slightly corrected graph for the quasi-classical curve with respect to this curve given in [22] (during the construction of which a technical inaccuracy was made, somewhat distorting the behavior of this curve in the region of the second maxima).

Fig. 5 also shows the quasi-classical photons number density curve taken from [22] obtained in compliance with 1-component wave function which was constructed similarly to formula (45), applied to the case of spherically symmetric (not dipole!) radiation, in the form not depending on $\varphi, \theta:$

$$\Psi^{(\text{quasi sphere})}(r, \theta, t) = \frac{1}{\sqrt{2\pi r^2 c \tau \sqrt{2\pi}}} \exp\left[-(t - r/c)^2/\tau^2 - i k_0 (ct - r)\right]$$

(58)

Note that in Fig. 5 we present a slightly corrected graph for the quasi-classical curve with respect to this curve given in [22] (during the construction of which a technical inaccuracy was made, somewhat distorting the behavior of this curve in the region of the second maxima).

From Fig. 5 shows that, in general, both curves describe the interference in almost the same way, except for some “violations” from the “regular” form of the quantum mechanical distribution in the region of the first peaks (left and right), counting them from the central one, caused by a specific momentum spherically symmetric distribution (55), as well as the corresponding set of radiation parameters used, the geometry of the experiment, and the moment of picture fixation.
It should also be noted that the numerical calculation performed in [22] was available for its implementation on an ordinary home computer using the Maple mathematical software package only in a limited range of wavelengths and radiation times, so the value of 1.5 cm was used, which is somewhat not characteristic of the usual ideas about the photon.

6 Conclusions

Based on the results of this work, several conclusions can be drawn. First, within the framework of the constructed quantum mechanics of the photon [1, 2, 18–21], it is possible to successfully explain interference phenomena, in particular, single-photon and two-photon ones including those with entangled photon states, using the PWF in the coordinate representation and the corresponding quantum mechanical superposition principle. In [2], in particular, an example of constructing the wave function in the coordinate representation of two entangled photons emitted from a nonlinear crystal after spontaneous parametric scattering is given. Specific modeling and construction of a 6-component PWF can proceed from the corresponding variants of the description of electromagnetic radiation, for example, electric dipole one in classical electrodynamics, as it was done in [23], or by specifying of momentum distribution in the wave packet as it was done in the case of obtaining formula (16). With their help, one can either carry out numerical calculations of the interference pattern, as, for example, in [22, 33], or additionally use approximate analytical transformations to obtain the final applied formulas, as is done in this work.

Secondly, in view of such a rather complex correct quantum mechanical explanation of interference phenomena, it is quite reasonable to introduce into consideration the “quasiclassical” 1-component PWF in the coordinate representation, endowed with normalization to the total unit probability of detecting the photon in the entire space or, for example, passing by it a sphere of some large radius in infinite time, as it was done in the present work when formula (41) was obtained.

In particular, from this, and also from the fact that both used quantum mechanical approaches describe Young’s experiment in almost the same way, we can draw the third conclusion: the "mathematical structure" of the PWF is not so important, in terms of that this function is 6-component or “just” 1-component how its diverging wave general form is important. Obviously, this general form is directly related to the form of the postulated transition amplitudes used in the apparatus of second quantization, which follow from the Feynman amplitudes [34] of the form \( \exp(ikr)/r \), where \( r = |\mathbf{r}_2 - \mathbf{r}_1| \), \( k = p/h \). From the point of view of rigorous quantum mechanics, the interpretation of these amplitudes, as, for example, in the case of a nonrelativistic spinless particle, as a solution to the Schrödinger equation, is unjustified: a particle cannot simultaneously have a certain momentum \( \mathbf{p} \) and a certain coordinate, first \( \mathbf{r}_1 \) and then \( \mathbf{r}_2 \), which, however, is practically is clearly implied in the formulation of the meaning of this amplitude. In fact, this type of transition amplitude is, up to a factor \(-m/2\pi \hbar^2\) (where \( m \) is the mass of the particle), the Green's function corresponding to the stationary Schrödinger equation for a free particle, which the given Green's function itself does not satisfy at \( \mathbf{r}_2 \rightarrow \mathbf{r}_1 \). However, it satisfies this equation “in the wave zone”, at \( \mathbf{r}_2 \neq \mathbf{r}_1 \), that is, in this case it is yet a solution to the Schrödinger equation, describing, for example, a diverging spherical wave from the point \( \mathbf{r}_1 \). A similar situation also arises in the case of the PWF, which satisfies its own equation, the “stationary” equation of free photon motion (see [1, 2, 18–21]). Taking into account the different nature of the photon compared to the nonrelativistic spinless particle, for the photon there are more diverse
cases of the asymptotic dependence of its wave function on distance and time, although the
common factor \( \sim 1/r \) is also present in the obtained formula (16), which makes it possible
to explain the Young's experiment. An explicit form of the time Green's function
Corresponding to the time equation of motion of a free photon is given in [35].

In our opinion, the constructed quantum mechanics of the photon, presented in [1, 2, 18–
21], plays the role of the missing link between classical electrodynamics, which is usually
always referred to when explaining the photons wave properties in experiments type
Young's, and quantum electrodynamics using the apparatus secondary quantization, in which
there are many postulates that supplement the postulates of “ordinary” quantum mechanics,
for example, transition amplitudes or the Malus law. In particular, this theory, the quantum
mechanics of a photon, can be considered as a theory of “primary” quantization of the
electromagnetic field, in which the transition amplitudes arise naturally as solutions to the
Corresponding equations of photon motion, and the Malus law follows (see [21]) from
“simple” initial principles of quantum mechanics such as the superposition principle for the
PWF in the coordinate representation and the 6-component form of this function itself which
is the solution of the corresponding quantum mechanical equation.

In conclusion, we also note that at present, a growing number of theoretical papers are
devoted to various ongoing experiments with single photons and systems of several photons,
including those justifying the introduction of the photon wave function in coordinate
representation (see, for example, [37-40] and references there). Obviously, such works will
continue to serve as a theoretical basis for many areas of rapidly developing photonics. An
important role in it is played by the directions associated with the creation of quantum
computers.

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