Numerical modeling of the wastewater self-purification process in a two-dimensional approximation using biofilters

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Abstract. A two-dimensional modification of the mathematical models of Streeter-Phelps, Monod, Dobbins-Driesnek and Kemp using drip and high-load biofilters is considered. Based on the results of numerical calculations, their influence on the process of treating wastewater entering the Selenga River is studied. Numerical implementation is carried out using the implicit difference method.

1 Introduction

Baikal is the largest freshwater lake in Eurasia. More than three hundred rivers flow into it, including the river. Selenga. Due to anthropogenic impact on the tributaries, “conditionally” treated wastewater, including petroleum products, enters the lake.

2 Materials and methods

The proposed mathematical model of the self-purification process is studied using the example of the Selenga River. The river bed is modeled by a rectangular channel through which an ideal incompressible fluid flows. The fluid flow is considered in a two-dimensional approximation and is characterized by two spatial coordinates x and y, and time t.

The discharge of organic matter (petroleum products, L=0.749 mg/l [1]) into the river is carried out for 2 days and then the pollution moves along the river with water flows at a speed of 129.6 km/day. Along with the change in L (mg/l), the concentration of microorganisms X (mg/l) and oxygen deficiency D (mg/l) change. These processes are described by the above modification of models [2-4] taking into account the influence of microorganisms, the aeration process, oxidation of the substrate with oxygen initiated by photosynthesis, and the flow of pollutants from bottom sediments.

The numerical implementation of the model is carried out using the implicit difference method [5-6].

A section of the Selenga River with a length of 250 km and a width of 1.5 km is being considered [7].

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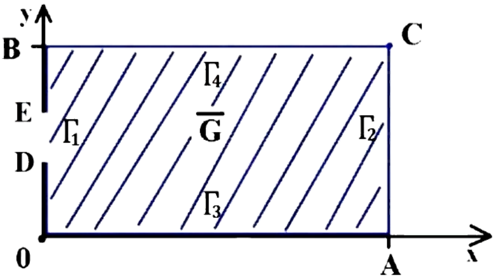

Fig. 1. Calculation area $\overline{G}$.

The following notations are introduced:

$$\mathbf{W} = (L, X, D)^T, \mathbf{S} = (S_1, S_2, S_3)^T, S_1 = -k_1 L - k_3 L - kL - \frac{\mu_{\max} XL}{Y(K_L + L)} + J_B,$$
$$S_2 = \frac{\mu_{\max} XL}{K_L + L}, S_3 = k_1 L - k_2 D - \frac{\mu_{\max} XL}{K_L + L} + J_1 + J_2 - J_3.$$

In area:

$$\overline{G} = G \bigcup \Gamma, G = \{(x,y) \mid 0 < x < A, 0 < y < B\}, t \in [0,T];$$
$$\Gamma = \Gamma_1 \bigcup \Gamma_2 \bigcup \Gamma_3 \bigcup \Gamma_4, \Gamma_1 = \Gamma_1^{(1)} \bigcup \Gamma_1^{(2)} \bigcup \Gamma_1^{(3)}, \quad \text{where}$$
$$\Gamma_1^{(1)} = \{(x,y) \mid x = 0, 0 \le y < D\}, \Gamma_1^{(2)} = \{(x,y) \mid x = 0, D \le y \le E\},$$
$$\Gamma_1^{(3)} = \{(x,y) \mid x = 0, E < y \le B\}; \Gamma_2 = \{(x,y) \mid x = A, A < y < C\},$$
$$\Gamma_3 = \{(x,y) \mid 0 < x \le A, y = 0\}, \Gamma_4 = \{(x,y) \mid B < x \le C, y = B\}$$

We are looking for a $\mathbf{W} \in C_1^{2,2}(\overline{G})$ solution to the original problem that satisfies the equation:

$$\frac{\partial \mathbf{W}}{\partial t} + U \frac{\partial \mathbf{W}}{\partial x} + V \frac{\partial \mathbf{W}}{\partial y} = \mathbf{S} + D_f \left(\frac{\partial^2 \mathbf{W}}{\partial x^2} + \frac{\partial^2 \mathbf{W}}{\partial y^2} \right) \tag{1}$$

With the corresponding initial

$$\mathbf{W}(x,y,0) = \mathbf{W}^0(x,y), \, (x,y) \in \overline{G} \tag{2}$$

And boundary conditions:

$$\mathbf{W}(0,y,t) = \mathbf{W}^{left}, \; y \in \Gamma_1, 0 < t \le T;$$
$$\frac{\partial \mathbf{W}(A,y,t)}{\partial x} = \bar{0}, \; y \in \Gamma_2, 0 < t \le T;$$

$$\begin{aligned} &\frac{\partial \mathbf{W}(x,0,t)}{\partial y} = \bar{0},\ x \in \Gamma_3, 0 < t \le T; \\ &\frac{\partial \mathbf{W}(x,B,t)}{\partial y} = \bar{0},\ x \in \Gamma_4, 0 < t \le T; \end{aligned} \tag{3}$$

Where $\mathbf{W}^0(x,y) = (0,1,0)^T$,

$$\mathbf{W}^{left} = \begin{cases} L(0,y,t) = \begin{cases} 0,749,\ y \in \Gamma_1^{(2)}, \\ 0,\ y \in \Gamma_1^{(1)} \bigcup \Gamma_1^{(3)}, \end{cases} \quad 0 < t \le 2, \\ L(0,y,t) = 0,\ y \in \Gamma_1, 2 < t \le T; \\ X(0,y,t) = 1,\ y \in \Gamma_1, 0 \le t \le T; \\ D(0,y,t) = 0,\ y \in \Gamma_1, 0 \le t \le T; \end{cases}$$

k – is the rate constant for the removal of organic matter (drip and high-load biofilters are used as a treatment facility [8]). The meaning of the remaining parameters from (1)-(3) is given in [9-10].

3 Results

The issues of approximation, stability and convergence of the used implicit difference method are investigated.

The results of numerical calculations without and with the use of biofilters are presented in Figures 2-7 in the form of graphs of changes in the concentrations of substrate L and oxygen deficiency D.

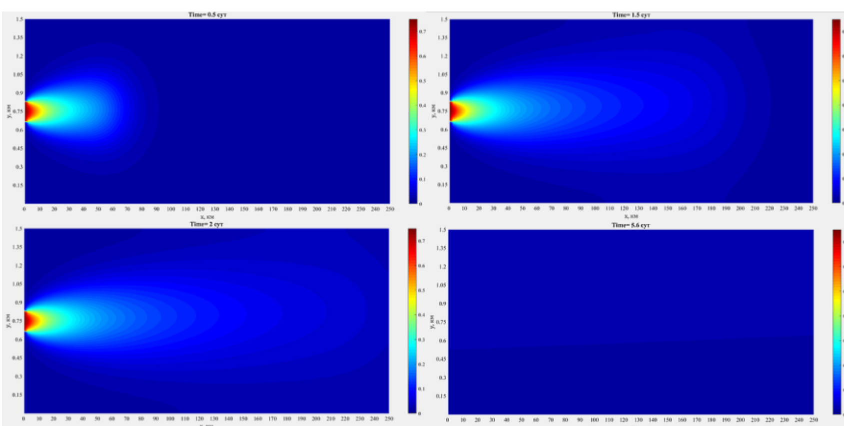

Fig. 2. Distribution of substrate concentration L (mg/l) without using a biofilter.

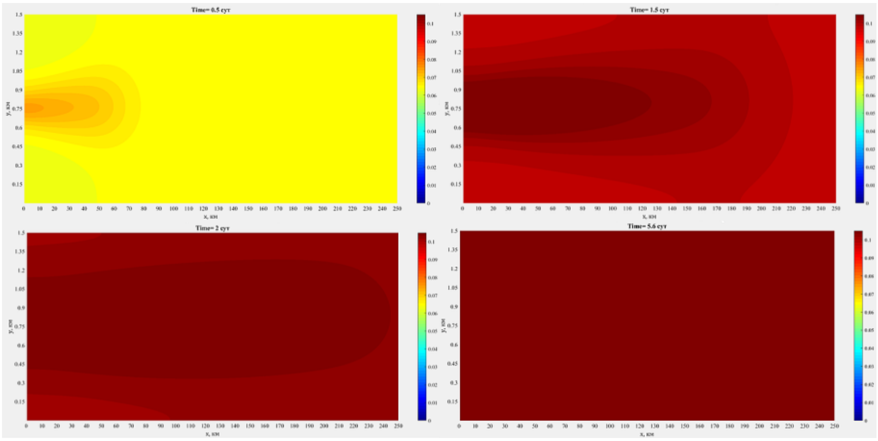

Fig. 3. Distribution of oxygen deficiency D (mg/l) without using a biofilter.

Figures 2 and 3 show graphs of changes in L and D at time points t=0.5, t=1.5, t=2 and t=5.6 (in days). During the first 2 days, pollution enters the river and spreads at a speed of 129.6 km/day throughout the entire area under consideration, gradually decreasing from 0.749 mg/l to 0.05 mg/l as it moves away from the source of pollution. In addition, the concentration of organic matter is influenced by oxidation reactions, consumption of organic substances by microorganisms and sedimentation of the substrate to the bottom of the river. As a result, by day 5.6 the L value drops to 0.025 mg/l. In Fig. Figure 3 shows the dynamics of changes in oxygen deficiency. Over the course of 2 days, an oxidation reaction occurs with oxygen consumption, as a result of which the oxygen deficiency increases to 0.118 mg/l. After the supply of pollution stops, D changes little and decreases over time to 0.104 mg/l.

In the case of using a drip biofilter (Figures 4 and 5), the pollution does not spread so intensively and the L concentration drops to a value of 0.025 mg/l in 2.46 days. The concentration of oxygen deficiency at the time point of 2.46 days varies from 0.102 mg/l to 0.108 mg/l.

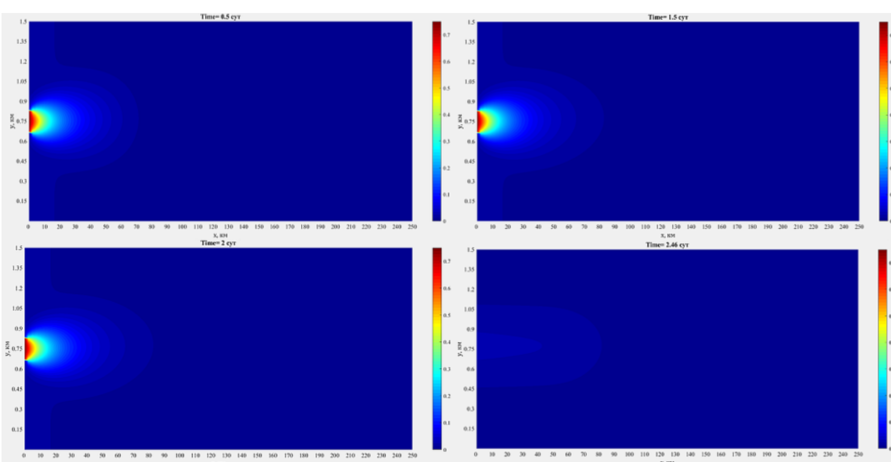

Fig. 4. Distribution of substrate concentration L (mg/l) using a trickling biofilter.

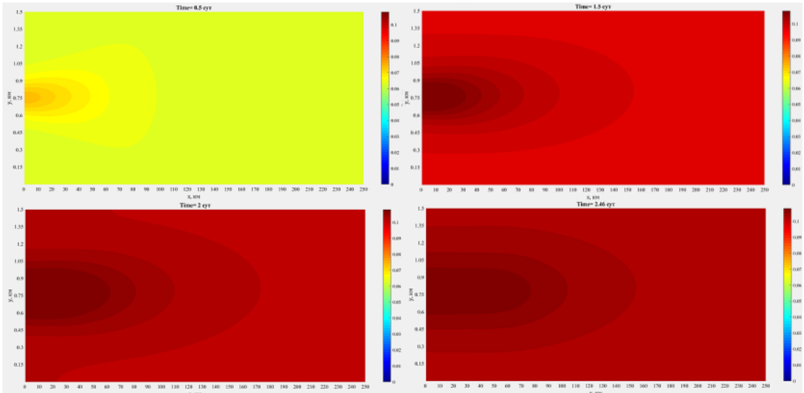

Fig. 5. Distribution of oxygen deficiency D (mg/l) using a trickling biofilter.

When using a high-load biofilter (Figure 6), the area of pollution distribution along the river section is slightly smaller than when using a drip biofilter. The drop in organic matter concentration from the initial value of 0.749 mg/l to 0.025 mg/l occurs in 2.3 days. At this point in time, the amount of oxygen deficiency varies from 0.101 mg/l to 0.109 mg/l.

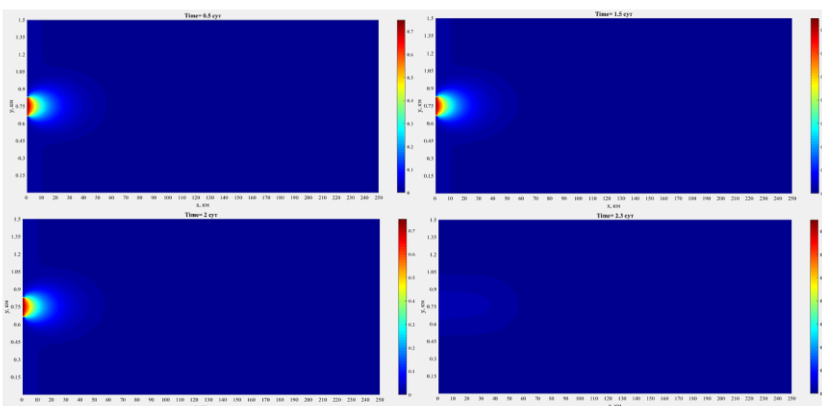

Fig. 6. Distribution of substrate concentration L (mg/l) using a high-load biofilter.

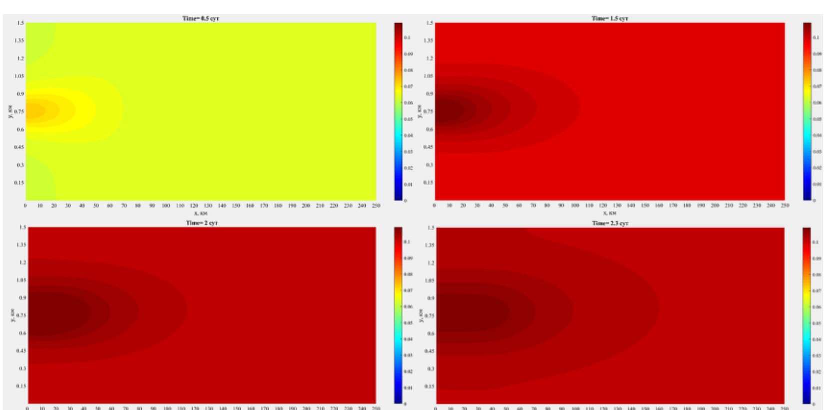

Fig. 7. . Distribution of oxygen deficiency D (mg/l) using a high-load biofilter.

4 Discussion

The use of both types of biofilters affects the speed of the self-purification process of wastewater entering the Selenga River. At the same time, the use of a drip biofilter accelerates the purification of incoming pollution by 2.28 times, and in the case when purification is carried out by a high-load biofilter - by 2.43 times.

This is explained by the fact that the concentration of the substrate entering the river decreases due to purification with biofilters and the self-purification of the river occurs faster due to the speed of the reservoir, as well as the processes of oxidation, consumption of organic substances by microorganisms and sedimentation of the substrate to the river bottom.

5 Conclusion

When using biofilters, the purification process of the Selenga River is accelerated by an average of 2.35 times. This is due to the fact that the concentration of the substrate entering the river decreases after purification with a biofilter, and due to the fairly high speed of the river (129.6 km/day), part of the organic matter is washed out from the area under consideration, and the remaining part of the pollution is removed due to self-purification mechanisms reservoir

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References

1. I.E. Afonina, iPolytech Journal, **8**, **67** (2012)
2. V.A. Vavilin, Nonlinear models of biological treatment and self-purification processes in rivers, Moscow (1983)
3. M.D. Mikhailov, Bulletin of TSU. Mathematics and Mechanics, 1 (2010)
4. A.V. Amzarakova, *Application of a biofilter in numerical modeling of the process of self-purification of wastewater*, collection of proceedings of the XIX International Conference of Students, Postgraduate Students and Young Scientists, April 26-29, 2022, Tomsk, 3 (2022)
5. N.N. Merkulova, M.D. Mikhailov, Methods of approximate calculations, Tomsk (2014)
6. A.A. Samarsky, Introduction to the theory of difference schemes, Moscow (1971)
7. O.I. Ivanova, Izvestia USGU, **2**, **62** (2021)
8. S.V. Yakovlev, Yu.V. Voronov, Biological filters, Moscow (1982)
9. V.E. Alekhin, Analysis and improvement of mathematical models for forecasting the environmental reliability of public water supply sources, Moscow (2006)
10. A.S. Davydov, Mathematical modeling and numerical study of the process of self-purification of a river reservoir, Tomsk (2019)