# Dividing Social Networks into Two Communities Using the Maximum Likelihood Method: Application to ESG

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**Abstract.** This article explores the application of the Maximum Likelihood Estimation method (MLE) for community detection in environmental, social, and governance (ESG) networks. ESG factors are important in assessing the sustainability and ethical impact of investments. By understanding the structure of social networks that discuss and promote ESG practices, we can gain important insights. It proposes a probabilistic framework for identifying community structures by dividing the network into two distinct groups based on connectivity patterns using the MLE method. The network structure is analyzed, and the method identifies groups of united organizations such as companies, investors, and NGOs with similar ESG orientations and interaction patterns. The results reveal important insights into how ESG information flows within and between these communities, highlighting key influencers and central nodes whose connections play a key role in the diffusion of ESG practices. These conclusions can be important in developing targeted communication strategies, identifying potential opportunities for cooperation, and forming informed investment decisions. By providing a solid framework for analyzing ESG networks, this paper is relevant to a broader understanding of ESG dynamics and supports the development of a more sustainable and interconnected global ecosystem.

**Key Words:** Maximum likelihood method; Community detection; ESG networks; Social networks.

## 1 Introduction

Environmental, social and governance (ESG) factors are increasingly important in assessing the sustainability and ethical impact of investments. Covers a wide range of issues including environmental protection, social justice and corporate governance through ESG. These factors are important to investors, companies and other stakeholders working to align their practices and portfolios to sustainable and ethical standards. For this reason, it is interesting to understand the spread and impact of ESG information on social media.

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Social media, both online and offline, is important in shaping ESG-related thinking and behavior. They facilitate the exchange of information, ideas and experiences between various stakeholders, including companies, investors, non-governmental organizations (NGOs) and the general public. Analyzing the structure of these networks can provide valuable insights into how ESG-related information spreads and how different entities interact. Community detection is a key problem in social networks and aims to identify groups of nodes (entities) that are more connected to each other than the rest of the network. By studying these communities, we can understand the importance of the basic structure of the network and the dynamics of information flow. One of the main methods for community detection is the Maximum Likelihood Estimation (MLE) method, which uses a probabilistic basis to partition the network into cohesive groups based on observed connectivity patterns.

The maximum likelihood method involves estimating the parameters that maximize the likelihood of the observed network structure. In the context of community detection, this method can be used to divide networks into two communities, where each community represents a cluster of nodes with a high probability of being interconnected. This approach is important in identifying key groups within the network and understanding the relationships between these groups.

In this article, we use a maximum likelihood method to divide ESG-related social networks into two communities. We aim to explore how this approach reveals distinct clusters of entities focusing on different aspects of ESG, and to understand the implications of these clusters for information dissemination and influence.

First of all, we will start with the theoretical foundations of the Maximum Likelihood method and its application in community detection. We then describe the process of creating ESG networks from various data sources, such as social media platforms, corporate reports, and investor relations. We then use a maximum likelihood method to divide these networks into two communities and analyze the resulting clusters through ESG focuses and connectivity patterns. The results of our analysis can provide important insights into the community structure of ESG networks. By identifying different communities, we can better understand how ESG information is communicated within and between these groups, identify key influencers and explore potential opportunities for targeted communication. Many methods have been developed for community detection, each with its own advantages and disadvantages. In the article, we focus on using the maximum likelihood method to divide social networks into two communities. This method is based on a probabilistic framework, where the network is assumed to consist of two communities with distinct connectivity patterns. By estimating the parameters that maximize the probability of the observed connections, the method allows to identify cohesive groups of nodes within a network.

Our approach is inspired by several key works in the field of network science. Newman and Girvan [1] proposed a method for finding and evaluating community structure in networks, laying the foundation for many subsequent studies in the field. Fortunato [2] provided a comprehensive review of community detection methods, highlighting the importance of probabilistic models in capturing the complex structure of social networks. Nowicki and Snijders [3] introduced a method for estimating and predicting stochastic block structures in networks, which has been widely used in the analysis of community structure. Peixoto [4] developed efficient Monte Carlo and greedy heuristic algorithms for inferring stochastic block models, further advancing the field of community detection. The authors in [5, 6] validate the algorithm using synthetic network data with known community structure, demonstrating its ability to accurately recover the underlying block assignments. They also apply the algorithm to real-world networks, showing its effectiveness in detecting meaningful community structure. Peel et al. [7] investigate the impact of metadata on community detection algorithms in networks. Metadata refers to additional information about nodes in a network, such as their attributes or labels, which can provide valuable context for understanding the network's structure.

Lancichinetti and Fortunato [8] begin by introducing the concept of community detection and discussing its importance in understanding the structure and function of complex networks. They highlight the diversity of community detection algorithms, which can be categorized into several classes based on their underlying principles and techniques. Newman [9] explores the relationship between two popular approaches for community detection: modularity optimization and maximum likelihood methods.

Abdushukurov and Zakhidov [10] explore methods for dividing social networks into two and three groups. The focus of the study is on identifying effective techniques for partitioning nodes in a network into cohesive groups based on their connectivity patterns.

In conclusion, the application of the maximum likelihood method to ESG networks offers a robust framework for analyzing the community structure of these networks. By revealing the underlying patterns of connectivity and influence, this approach provides valuable insights that can help stakeholders make more informed decisions and foster a more interconnected and sustainable ESG ecosystem.

## 2 Materials and Methods

### 2.1 Data collection

For research purposes, we construct ESG-related social networks from diverse data to ensure comprehensive coverage of environmental, social, and governance (ESG) related entities and interactions. Key data sources include:

1. Social media platforms: we collected data from platforms such as Facebook, Twitter and Telegram, where entities frequently discuss ESG issues. Identifying relevant hashtags, keywords and user accounts for interaction mapping and networking.
2. Corporate reports and data: we used information from annual reports, company websites, and corporate data available in financial databases. These documents provide information on partnerships, joint ventures and other collaborations related to ESG initiatives.
3. Investor networks: information on investor relations, including investor conferences, shareholder meetings and information on ESG-focused mutual funds, is collected from financial news sources and investor platforms.

### 2.2 Network construction

The collected data were used to construct an ESG network, which is represented in the form of a graph $G = (V, E)$, where $V$ represents a set of nodes (objects) and $E$ denotes a set. edges (interactions or relationships between objects). Nodes in the network include companies, investors, NGOs and individuals active in ESG discussions. Edges represent different types of interactions, such as collaborations, notes, and co-authoring.

1. Node Identification: entities were identified based on their participation in ESG-related activities. For social media data, we identify nodes through interactions with user profiles and related hashtags. Nodes for corporate and investor data were identified based on mentions in reports and disclosures.
2. Edge Formation: edges are formed based on the frequency and type of interactions between nodes. For social media, margins represent mentions, retweets, and replies. Edges for corporate and investor data represent partnerships, collaborations, and joint investments in ESG projects.

3. Preprocessing: the raw data were preprocessed to remove noise and irrelevant data. Duplicate recordings were combined and spontaneous loops and isolated nodes were removed to focus on meaningful interactions.

### 2.3 Maximum likelihood method

A maximum likelihood method was used to identify communities in the ESG network. This method involves estimating the parameters that maximize the probability of observing a given network structure under a given probabilistic model.

1. Model assumptions: we adopted a stochastic block model (SBM) in which the network is divided into two communities. Each team is more likely to be internally connected and less likely to be externally connected.

2. Parameter estimation: the parameters to be estimated include within-community connectivity probability $p_{in}$ and inter-community connectivity probability $p_{out}$. These parameters were iteratively changed to maximize the likelihood function.

3. Likelihood function: the likelihood function for SBM is given by $L_{\Pi}$:

$$L_{\Pi} = \prod_{k=1}^{2} p_{in}^{m_k} (1-p_{in})^{\frac{n_k(n_k-1)}{2}-m_k} \prod_{i\in S_k} p_{out}^{\frac{1}{2}\sum_{j\notin S_k} E(i,j)} (1-p_{out})^{\frac{1}{2}\left(n-n_k-\sum_{j\notin S_k} E(i,j)\right)} \quad (1)$$

Taking the logarithm of the probability function $L_{\Pi}$ in (2) and simplifying it, we get

$$l_{\Pi} = (m_1+m_2)\log p_{in} + \left(\frac{n_1^2+n_2^2-n}{2} - (m_1+m_2)\right)\log(1-p_{in}) + \left(m-(m_1+m_2)\right)\log p_{out} +$$

$$\left(n_1 n_2 - m + (m_1+m_2)\right)\log(1-p_{out}) \quad (2)$$

The partition Π , where the function $l_{\Pi}$ achieves its maximum among all possible partitions, is considered optimal. Please note that there is still uncertainty regarding the selection of probabilities $p_{in}$ and $p_{out}$. The function $l_{\Pi} = l_{\Pi}(p_{in}, p_{out})$ is a function of the arguments $p_{in}$ and $p_{out}$. By maximizing $l_{\Pi}$ with respect to $p_{in}$ and $p_{out}$, one can then apply these values in numerical calculations.

We calculate the particular derivatives of (2) with respect to $p_{in}$ and $p_{out}$:

$$\frac{\partial l_{\Pi}}{\partial p_{in}} = \frac{m_1+m_2}{p_{in}} - \frac{\frac{n_1^2+n_2^2-n}{2} - (m_1+m_2)}{1-p_{in}}$$

$$\frac{\partial l_{\Pi}}{\partial p_{out}} = \frac{m-(m_1+m_2)}{p_{out}} - \frac{n_1 n_2 - m + (m_1+m_2)}{1-p_{out}}$$

*Statement 1*. For a fixed partition Π, the function $l_{\Pi} = l_{\Pi}(p_{in}, p_{out})$ reaches its maximum at

$$p_{in} = \frac{2(m_1+m_2)}{n_1^2+n_2^2-n}, p_{out} = \frac{m-(m_1+m_2)}{n_1 n_2}$$

When dividing a social network into two communities, different divisions may occur depending on the number of participants in the community. For example, when a social network with n=8 is divided into two communities, there are three types of division:

$$b_1 = (4;4), b_2 = (3;5), b_3 = (2;6)$$

We can find a formula that determines the number of types of division of a social network into two teams, depending on the number of participants:

$$c(n) = \frac{n}{2} - \left(\frac{3}{2}\right)^{\frac{1-(-1)^n}{2}} \quad (3)$$

where *n* is the number of participants in the social network.

After determining $c(n)$, we can determine the appearance of division types:

$$b_k = \left(\frac{2n-(4k-3)+(-1)^n}{4}; \frac{2n+(4k-3)-(-1)^n}{4}\right)$$

where $k = 1,2,\dots,c(n)$.

Depending on the number of connections between participants, we will consider which type of division has the highest probability

If the number of edges is $m \in \left[\frac{(n-1)^2}{4} + \frac{8k^2-12k+19}{8} - \frac{(4k-3)(-1)^n}{8}; \frac{(n-1)^2}{4} + \frac{8k^2+4k+15}{8} - \frac{(4k+1)(-1)^n}{8}\right)$, the maximum likelihood of $b_k$-type division is greater than the maximum likelihood of other types of divisions.

*Statement 2*. If the number of edges of a social network with *n* participants is equal to $\frac{n(n-1)}{2}$, then the maximum likelihood of all types of divisions is equal to 0.

## 2.4 Application to ESG Networks

Applying the maximum likelihood approach to ESG (Environmental, Social and Governance) networks involves several steps, each of which is designed to better understand and analyze the relationships and interactions within these networks. ESG networks consist of various organizations such as companies, investors, non-governmental organizations (NGOs) and individuals who deal with situations and actions related to sustainability and ethical governance. By applying a maximum likelihood approach to these networks, we can uncover meaningful societal structures and gain insight into how ESG-related information and practices diffuse and affect stakeholders.

### *2.4.1 Data sources and network construction*

*Social media platforms.* Social media is a rich source of ESG-related interactions. Platforms such as Twitter, Telegram and Facebook provide information on how entities discuss and disseminate ESG topics. By collecting tweets, messages and interactions that contain specific ESG-related keywords and hashtags, we can identify the network of entities participating in these discussions.

*Corporate reports and disclosures*. Companies often publish sustainability reports, annual reports, and other information that highlight their ESG initiatives and partnerships. These documents can be analyzed to identify connections between companies based on common projects, partnerships and joint ventures focused on ESG goals.

*Investor networks*. Investors and mutual funds play a crucial role in promoting ESG practices by directing capital to sustainable and ethical companies. Information about investor relations such as investment portfolios, shareholder meetings and ESG-focused funds can help create a network of investors and companies connected through ESG investments.

### *2.4.2 Network construction*

*Nodes:* nodes in the ESG network can include companies, investors, NGOs, regulatory bodies, and individuals who are active in the ESG domain. Nodes in an ESG network may include companies, investors, NGOs, regulators, and individuals active in the ESG domain.

*Edges:* edges represent a variety of interactions such as collaboration, collaboration, social media interactions (mentions, retweets, replies) and co-authorship in ESG-related publications.

*Data collection:* ESG networks can be built from a variety of sources, including social media platforms (Twitter, LinkedIn and other platforms where entities discuss ESG issues); Corporate reporting and information (communications based on shared sustainability goals and collaborative projects); Investor networks (relationships between investors and companies based on ESG criteria).

### *2.4.3 Implications for ESG practices*

Targeted communication. By understanding community composition, organizations can tailor their communication strategies to effectively reach and influence specific groups within the ESG network [11-14, 16]. For example, companies can identify key influencers and partner with them to promote ESG initiatives.

Opportunities for collaboration. Identifying communities that are close to each other can reveal potential opportunities for collaboration. Entities in the same community may share similar ESG goals, making them suitable partners for joint projects and initiatives.

Investment strategies. Investors can use the results of community detection to better assess the ESG impact of their portfolios. Understanding network dynamics and key influencers can guide investment decisions towards more sustainable and impactful ventures.

Applying the maximum likelihood method to ESG networks provides a solid basis for analyzing the structure and dynamics of these networks [15, 17-20]. By unlocking societal structures and understanding the flow of ESG information, stakeholders can improve their strategies for promoting sustainability and ethical practices, ultimately contributing to a more sustainable and interconnected world.

## 3 Results

Let us look at a social network of 28 participants. We will consider the issue of dividing this social network into two:

Then $n = 28$, $m\epsilon[28; 378]$. We count the number of types of division (Fig. 1):

$$c(28) = \frac{28}{2} - \left(\frac{3}{2}\right)^{\frac{1-(-1)^{28}}{2}} = 13.$$

We write the view of division types:

$$b_k = (15 - k; 13 + k),$$

where $k = 1,2,\ldots,13$.

We compute the maximum likelihood of all partition types when $m\epsilon[28; 378]$.

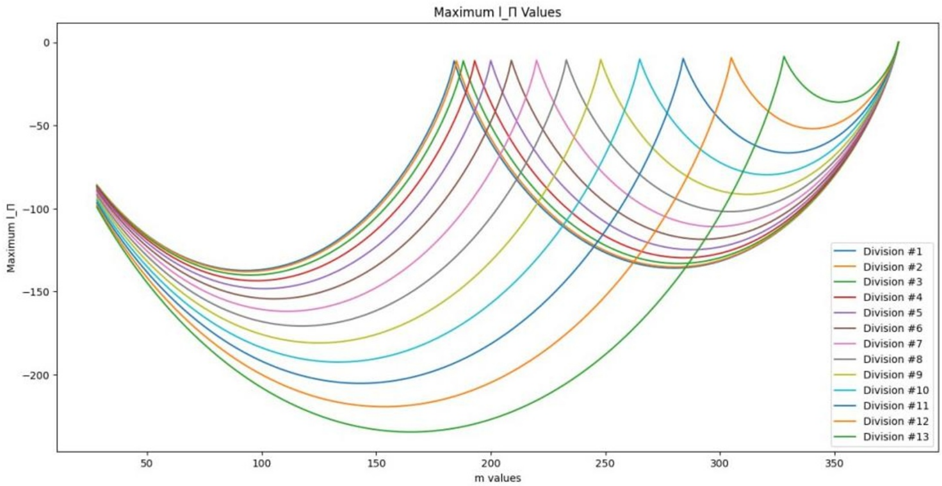


**Fig. 1.** A graph of $l_{\Pi}$ values of all types of divisions.

Source: compiled on the basis of the author's research.

We will make Table 1 based on the model above.

**Table 1.** Twenty-eight participant social network division types.

| $b_k$ | $(n_1; n_2)$ | $m\epsilon$ |
|---|---|---|
| $b_1$ | $(14; 14)$ | [28, 184] |
| $b_2$ | $(15; 13)$ | [185, 187] |
| $b_3$ | $(16; 12)$ | [188, 192] |
| $b_4$ | $(17; 11)$ | [193, 199] |
| $b_5$ | $(18; 10)$ | [200, 208] |
| $b_6$ | $(19; 9)$ | [209, 219] |
| $b_7$ | $(20; 8)$ | [220, 232] |
| $b_8$ | $(21; 7)$ | [233, 247] |
| $b_9$ | $(22; 6)$ | [248, 264] |
| $b_{10}$ | $(23; 5)$ | [265, 283] |
| $b_{11}$ | $(24; 4)$ | [284, 304] |
| $b_{12}$ | $(25; 3)$ | [305, 327] |
| $b_{13}$ | $(26; 2)$ | [328, 377] |

If we want to divide this social network into two communities and let there be 17 participants in the first of these communities and 11 participants in the second. If it is possible to obtain the number of connections ($m$) in the social network in the interval [28, 184], then the maximum likelihood of division of type $b_1(14; 14)$ will be higher than the maximum likelihood of other types of divisions.

We take for example a graph representation of a social network with 28 participants and the number of connections between them *m*=90 (Fig. 2).

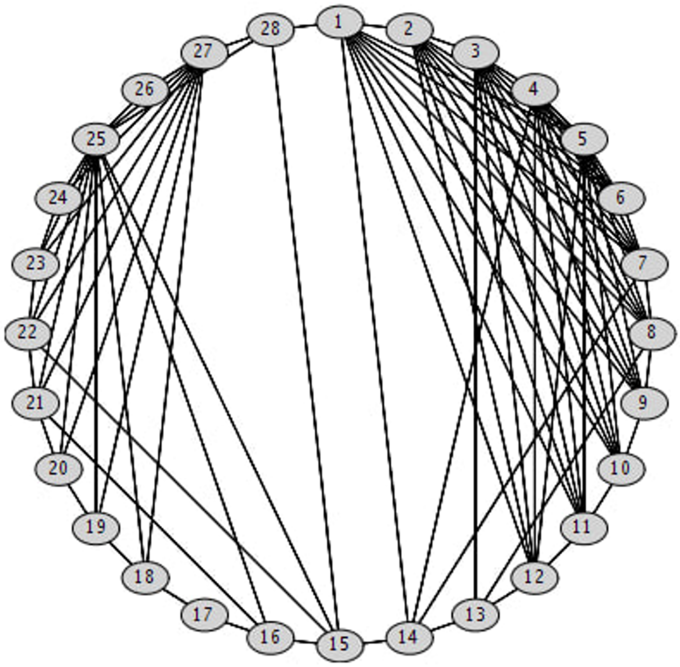


**Fig. 2.** A social network with 28 vertices and 90 edges.

Note: $b_k$ we divide into teams according to the types of division.
Source: compiled on the basis of the author's research

The highest maximum likelihood for the type of partition $b_1$ is as follows (Fig. 3).

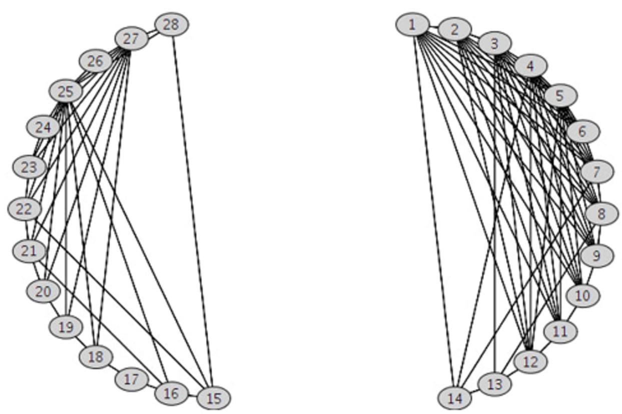


**Fig. 3.** To divide the social network into communities by partitioning $b_1$.

where $n_1 = 14$, $n_2 = 14$, $m_1 = 55$ and $m_2 = 33$.

We calculate $l_\Pi$ according to formula (2).

$$l_\Pi = 88 \log p_{in} + 94 \log(1 - p_{in}) + 2 \log p_{out} + 194 \log(1 - p_{out})$$

We differentiate it by $p_{in}$ and $p_{out}$ and equate it to zero.

$$\begin{cases} \dfrac{88}{p_{in}} + \dfrac{94}{1-p_{in}} = 0 \\ \dfrac{2}{p_{out}} + \dfrac{194}{1-p_{out}} = 0 \end{cases}, \quad \begin{cases} p_{in} = \dfrac{44}{91} \\ p_{out} = \dfrac{1}{98} \end{cases}$$

We do a calculation by entering the discovered values into $l_{\Pi}$.

$$l_{\Pi}\left(\frac{44}{91};\frac{1}{98}\right) \approx -137.2135638$$

The highest maximum likelihood for the type of partition $b_2$ is as follows (Fig. 4).

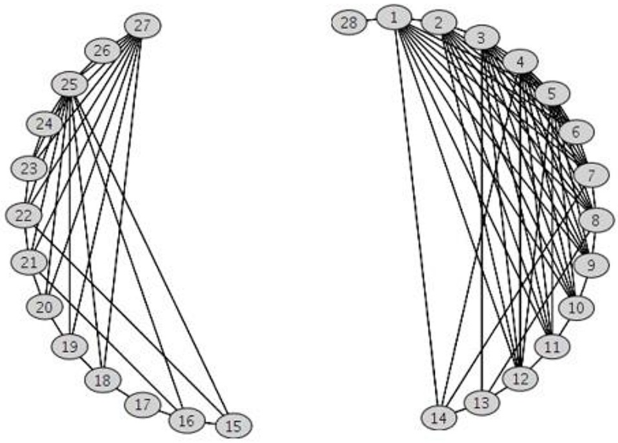


**Fig. 4.** To divide the social network into communities by partitioning $b_2$.

Compiled on the basis of the author's research

where $n_1 = 15$, $n_2 = 13$, $m_1 = 56$ and $m_2 = 30$.

We calculate $l_{\Pi}$ according to formula (2).

$$l_{\Pi} = 86\log p_{in} + 97\log(1-p_{in}) + 4\log p_{out} + 191\log(1-p_{out})$$

We differentiate it by $p_{in}$ and $p_{out}$ and equate it to zero.

$$\begin{cases} \dfrac{86}{p_{in}} + \dfrac{97}{1-p_{in}} = 0 \\ \dfrac{4}{p_{out}} + \dfrac{191}{1-p_{out}} = 0 \end{cases}, \quad \begin{cases} p_{in} = \dfrac{86}{183} \\ p_{out} = \dfrac{4}{195} \end{cases}$$

We do a calculation by entering the discovered values into $l_{\Pi}$.

$$l_{\Pi}\left(\frac{86}{183};\frac{4}{195}\right) \approx -146.0206453$$

Table 2 displays the remaining division types.

**Table 2.** $l_{\Pi}$ values of $b_k$ type divisions.

| $\boldsymbol{b_k}$ | $(\boldsymbol{n_1};\boldsymbol{n_2})$ | $l_{\Pi}$ | $l_{\Pi}(p_{in};p_{out})$ value |
|---|---|---|---|
| $b_3$ | $(16;12)$ | $85\log p_{in} + 101\log(1-p_{in}) + 5\log p_{out} + 187\log(1-p_{out})$ | -151.4109628 |
| $b_4$ | $(17;11)$ | $83\log p_{in} + 108\log(1-p_{in}) + 7\log p_{out} + 180\log(1-p_{out})$ | -160.6139890 |
| $b_5$ | $(18;10)$ | $83\log p_{in} + 115\log(1-p_{in}) + 7\log p_{out} + 173\log(1-p_{out})$ | -164.2373232 |
| $b_6$ | $(19;9)$ | $81\log p_{in} + 126\log(1-p_{in}) + 9\log p_{out} + 162\log(1-p_{out})$ | -173.8097290 |
| $b_7$ | $(20;8)$ | $79\log p_{in} + 139\log(1-p_{in}) + 11\log p_{out} + 149\log(1-p_{out})$ | -182.8046295 |
| $b_8$ | $(21;7)$ | $77\log p_{in} + 154\log(1-p_{in}) + 13\log p_{out} + 134\log(1-p_{out})$ | -190.9734883 |
| $b_9$ | $(22;6)$ | $76\log p_{in} + 170\log(1-p_{in}) + 14\log p_{out} + 118\log(1-p_{out})$ | -196.7323552 |
| $b_{10}$ | $(23;5)$ | $76\log p_{in} + 187\log(1-p_{in}) + 14\log p_{out} + 101\log(1-p_{out})$ | -200.7166852 |
| $b_{11}$ | $(24;4)$ | $77\log p_{in} + 205\log(1-p_{in}) + 13\log p_{out} + 83\log(1-p_{out})$ | -203.3970450 |

| | | | |
|---|---|---|---|
| $b_{12}$ | $(25;3)$ | $83\log p_{in} + 220\log(1-p_{in}) + 7\log p_{out} + 68\log(1-p_{out})$ | -201.1629230 |
| $b_{12}$ | $(26;2)$ | $86\log p_{in} + 240\log(1-p_{in}) + 4\log p_{out} + 48\log(1-p_{out})$ | -202.2031846 |

According to statement 1, when the number of edges is 378, the maximum likelihood of all types of divisions is 0 (Fig. 5).

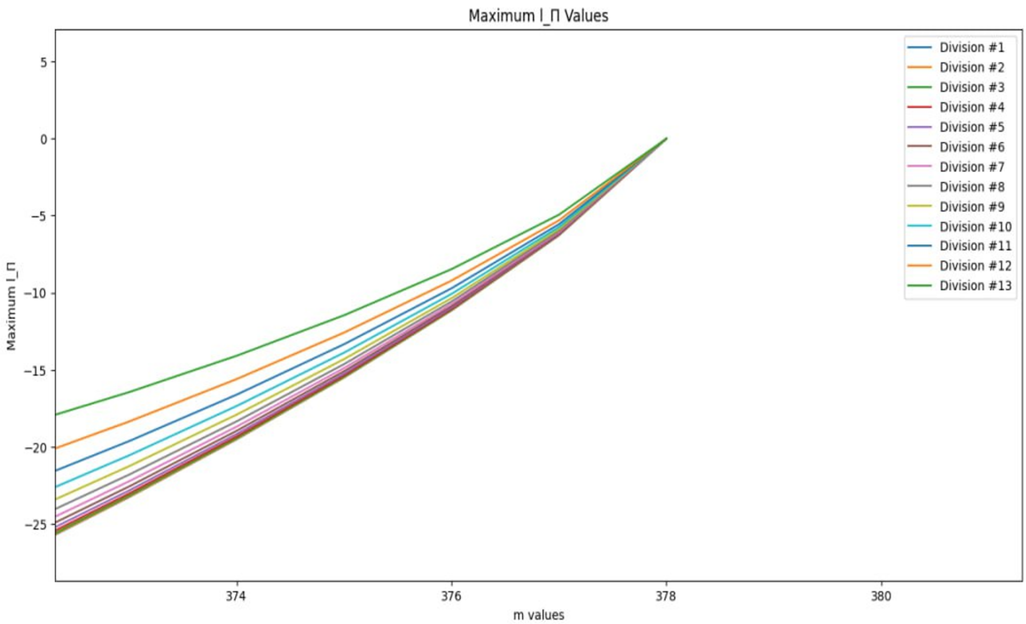


**Fig. 5.** Maximum probability of all types of splits when the number of edges is 378.

Source: compiled on the basis of the author's research.

These calculations show that if we want to divide a social network with n participants into two groups, we can predict which type of division is more likely depending on the number of edges. According to the intervals given above, we need to increase or decrease the number of edges in order to implement the type of partitioning that we need, that is, by establishing additional connections or removing connections between the participants.

Applying the maximum likelihood method to ESG networks reveals distinct communities:

- Community A: entities with a strong focus on environmental sustainability, including green investors, environmentally-conscious companies, and NGOs advocating for climate action.

- Community B: entities prioritizing social and governance aspects, such as labor rights organizations, companies with robust governance practices, and investors focused on social impact.

The analysis highlights how ESG information flows within and between these communities, indicating potential influencers and key players in the dissemination of ESG practices.

## 4 Discussion

Using the maximum likelihood approach in ESG industries has several important implications for stakeholders, including companies, investors, NGOs, and policy makers.

These implications increase our understanding of how ESG information moves through networks and inform strategies for promoting sustainability practices.

### 4.1 Advanced target communication

Understanding community dynamics: by identifying specific communities in ESG networks, organizations can tailor their communication strategies to effectively reach and engage specific groups. Knowing which objects are grouped together based on interaction patterns can help create targeted activities that match the values and interests of each community.

Influence key nodes: recognizing the key influencers in each community allows organizations to focus their efforts on these key nodes. Influencers have the power to amplify messages and catalyze broader engagement across the network, making them important partners in advancing ESG initiatives.

### 4.2 Identify opportunities for cooperation

Synergistic collaboration: organizations in the same community often share similar ESG goals and values. Identifying these clusters can identify potential partners for collaborative projects, joint ventures, and alliances. Such partnerships can leverage pooled resources and expertise to achieve greater impact in sustainability efforts.

Cross-community cooperation: understanding the relationships between different communities can facilitate cross-community cooperation. Increased communication between communities can lead to innovative solutions and the wider dissemination of best practices, increasing the overall effectiveness of ESG initiatives.

### 4.3 Providing information about investment strategies

ESG-oriented investment decisions: investors can use the results of community identification to improve their investment strategies. By identifying companies that are central to ESG communities, investors can prioritize investing in organizations that are leaders in sustainability practices and have strong influence in their industries.

Evaluating ESG impact: understanding the societal structure of ESG industries can help investors assess the potential impact of their portfolios. Investments in well-connected businesses within healthy communities can generate high ESG performance, contributing to both financial returns and positive social outcomes.

### 4.4 Policy and regulatory concepts

Effective policymaking: policymakers can leverage insights into the social fabric of ESG networks by designing targeted regulations and policies that address the specific needs and dynamics of diverse groups. Policies that take into account the interconnectedness and impact patterns within ESG communities are more effective and more widely adopted.

Monitoring and evaluation: the ability to identify and analyze communities within ESG networks provides a powerful tool for monitoring policy implementation and impact. Policymakers can monitor how rules and guidelines are adopted and propagated through the network, allowing for timely adjustments and improvements.

### 4.5 Strengthen ESG reporting and transparency

Improved reporting practices: companies can use an understanding of team structures to improve ESG reporting practices. By tailoring their reporting to the interests and concerns of specific communities, companies can increase transparency and build trust with stakeholders.

Benchmarking and best practices: identifying leading organizations in ESG communities allows benchmarking against best practices. Companies can learn from the strategies and initiatives of influential peers by adopting and adapting successful approaches to improve their ESG performance.

### 4.6 Encourage innovation and knowledge sharing

Centers of innovation: communities identified through the maximum likelihood method can serve as centers of innovation and knowledge sharing. Organizations within these communities can collaborate to develop and disseminate new technologies, practices and solutions that advance ESG goals.

Knowledge dissemination: understanding the pathways of ESG information flow enables more effective knowledge dissemination. Stakeholders can strategically share research, case studies and success stories within and across teams to maximize their impact and impact. Summary

Applying the maximum likelihood approach to ESG networks provides valuable insights into the societal structure and dynamics of these networks. By uncovering interconnected groups and understanding their interrelationships, stakeholders can improve strategies for promoting sustainability and ethical practices. These insights support targeted communication, effective collaboration, informed investment decisions and innovative policy-making, ultimately contributing to a more sustainable and interconnected ESG ecosystem.

## 5 Conclusion

This article explored the use of maximum likelihood for community detection in the context of ESG networks. Using this probabilistic framework, we divided the network into two distinct communities based on observed connectivity patterns, providing a deeper understanding of how ESG information reaches and impacts stakeholders.

Our analysis revealed several key points:

1. Community structure detection: the maximum likelihood method effectively identified clusters within ESG networks, thereby highlighting clusters of entities with similar ESG foci. This unit helps to understand the basic structure of ESG interactions and relationships.

2. Influence and information flow: by revealing the community structure, we identified key influencers and central nodes in each community. These entities play an important role in disseminating ESG information and shaping perceptions, making them important targets for communication and collaboration efforts.

3. Enhanced ESG strategies: information from community identification can inform targeted communication strategies, develop synergistic partnerships, and drive investment decisions. By tailoring approaches for specific communities, stakeholders can increase the impact and effectiveness of their ESG initiatives.

4. Policy and regulatory implications: understanding social dynamics in ESG sectors can help policymakers formulate targeted and effective regulations. Monitoring policy diffusion through these networks ensures timely adjustments and maximizes their positive impact.

5. Innovation and knowledge sharing: identifying communities within ESG networks can facilitate the diffusion of innovation and knowledge. Organizations in these communities can collaborate to develop and share best practices that lead to success in sustainability efforts.
In summary, applying a maximum likelihood approach to ESG networks offers a robust and insightful approach to analyzing community structures. By revealing patterns of interdependence and influence in these networks, stakeholders can make more informed decisions and develop more effective strategies to promote sustainability and ethical practices. This methodology not only expands our understanding of ESG dynamics, but also supports the creation of a more sustainable and interconnected global ecosystem.

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