Optimizing the accuracy of an industrial robot: a model for improving positional accuracy

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Abstract. This article investigates methods for improving the positional accuracy of industrial robots, focusing on their movement dynamics, trajectory management, and fundamental principles of precise positioning. The research examines nine critical points within the gripper's working zone, shaped as a parallelepiped, which is essential for defining the accuracy limits of industrial robots. The study addresses the challenge of aligning the capture device with the ninth point by proposing a solution that involves precise movements along the diagonals of the parallelepiped to enhance accuracy. The article provides a detailed analysis of the construction and mechanics of industrial robots, emphasizing how different link configurations impact performance. It highlights the benefits of fewer links, which tend to maintain stable positional accuracy, while also discussing how an increase in the number of links leads to trajectory variations. These variations affect both robots with mobile bases and fixed-base robots, impacting practical applications such as machining where managing these trajectories is crucial to avoid collisions and ensure smooth operation. Additionally, the research explores the role of various coordinate systems in shaping the working zone and basic movements of industrial robots. It covers rectangular, cylindrical, spherical, and angular coordinate systems, each offering different perspectives on the robot's operational area. This comprehensive analysis aims to address the complexities associated with enhancing positional accuracy in industrial robotics.

1 Introduction

In the rapidly advancing field of industrial automation, achieving precision within robotic systems is essential for operational excellence. This study provides an in-depth exploration focused on improving the crucial aspect of positional accuracy in industrial robotics.

At the heart of efficient and effective automated manufacturing processes lies the ability to precisely manage and fine-tune the movements, trajectories, and overall positional accuracy of industrial robots within their designated operational zones. This article embarks

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on an in-depth investigation to analyse and illuminate the complex aspects of achieving this precision.

The core of this study involves a detailed examination of nine distinct reference points within the parallelepiped-shaped working zone of the robot's gripper. These points are recognized as crucial elements that significantly affect and determine the accuracy limits of industrial robots. The primary focus of this research is to address the challenge of precisely aligning the capture device with the ninth reference point. The proposed solution involves a comprehensive framework that highlights the importance of accurate movements along the intersecting diagonals of the parallelepiped to significantly enhance positional accuracy.

A central theme of this exploration is the detailed analysis of the construction and mechanical attributes of industrial robots (IR). The study carefully evaluates how different link configurations impact the operational efficiency of these robotic systems. It emphasizes the benefits of using robots with fewer links, which tend to maintain consistent positional accuracy even in simpler setups. Conversely, the research also addresses how an increase in the number of links leads to greater trajectory variations, affecting the robot's movement dynamics.

These trajectory variations have significant implications not only for robots with mobile bases but also for fixed-base robots and their working arm movements. In practical industrial scenarios, such as machining, these variations can lead to potential collisions with technological barriers, making it essential to make timely and well-informed decisions to avoid disruptions and maintain operational integrity.

Furthermore, the article explores the intricate world of coordinate systems used by industrial robots, detailing their impact on the configuration of the working zone and the basic principles of movement. It discusses various coordinate systems—rectangular, cylindrical, spherical, and angular—each providing a unique perspective on the robot's operational area and contributing to a deeper understanding of positional accuracy.

Comprehensive in its scope, this article seeks to unravel the intricate tapestry underlying positional accuracy within industrial robots. Through its thorough exploration of trajectory variations and their tangible ramifications, it endeavours to construct a comprehensive blueprint for comprehending, refining, and optimizing the precision of industrial robotic systems.

2 Materials and methods

Industrial robots (IR) participate in technological processes with six or fewer links based on their real construction. The fewer the number of links, the simpler it is to control the robot. The number of links significantly affects the simplicity of robot control until the number of links is considered [1]. The differentiation becomes challenging when the number of links is three or fewer, but the variation in the number of links affects the trajectory significantly. However, concerning the manipulator, positional accuracy is almost the same in both trajectories.

The difference between trajectories is not only essential in the movement of an IR but also induces noticeable deficiencies in the working hand movement of a robot that does not primarily involve manipulator motion. For instance, in a mechanical assembly area, an IR may encounter technological interference when placing a part into the assembly or retrieving a processed part from the work area. As a result, a person who decides to accept or reject the alternative solution may differ.

The coordinate system of an IR describes the shape of the working zone and its primary movements. In the Cartesian coordinate system, widely used in the phase space, the following coordinate systems are prevalent: rectangular, cylindrical, spherical, and polar.

The four main representations of the IR's working zone are associated with the coordinate system names.

In the rectangular coordinate system, the working zone for operative robots has a parallelepiped shape. In the cylindrical coordinate system, the working zone appears as cylinders with heights h and radius r, and R multiple internal cylinders can be nested. In the spherical coordinate system, the working zone for operative robots consists of concentric figures with the radius of the cylindrical axis r and the radius of the sphere R. Here, the axis of rotation of the cylinder passes through the centre of the sphere.

The directional phase of the IR's interaction with the working zone has been verified with a functional-parametric compatibility model [2,3]. The content of this model includes the working zone R (WZ) and the directional phase Ω (DP), and the determinant constructs based on them are as follows:

$$\Delta^1 = \begin{bmatrix} \dfrac{\partial \Omega_1}{\partial x} & \dfrac{\partial \Omega_1}{\partial y} \\ \dfrac{\partial R_1}{\partial x} & \dfrac{\partial R_1}{\partial y} \end{bmatrix} \quad \Delta^2 = \begin{bmatrix} \dfrac{\partial \Omega_1}{\partial y} & \dfrac{\partial \Omega_1}{\partial z} \\ \dfrac{\partial R_1}{\partial y} & \dfrac{\partial R_1}{\partial z} \end{bmatrix} \quad \Delta^3 = \begin{bmatrix} \dfrac{\partial \Omega_1}{\partial z} & \dfrac{\partial \Omega_1}{\partial x} \\ \dfrac{\partial R_1}{\partial z} & \dfrac{\partial R_1}{\partial x} \end{bmatrix}$$

The general determinant form needs to be considered:

$$\Delta = \Delta^1 \Delta^2 \Delta^3 \tag{1}$$

$\Delta \neq 0$ in this case, it should be divisible. Thus, the working zone aligns with the directional phase, or it does not match.

Despite the advantages of this model, it does not address the issue of enhancing the positional accuracy of industrial robots (IR). The IR's movement within the working zone, as outlined in previous studies [4], aligns with the ninth point of the parallelepiped. However, detailed information on this specific alignment is lacking.

To improve the positional accuracy of the IR, it is essential to design internal drawings of the parallelepiped within the working zones that correspond to their respective coordinate systems. The key to enhancing positional accuracy lies in focusing on the intersection point of the parallelepiped's main diagonals. If the movement structure relies on the other eight points of the parallelepiped, it can lead to inaccuracies and variations in the technological interference scenarios, compromising the overall precision of the IR's operation.

In [4], the lengths of the sides of the parallelepiped are given for the cylindrical and spherical coordinate systems. According to this, for the cylindrical coordinate system, the internal drawing of the parallelepiped aligns with the axes of the cylindrical coordinates.

$$x = \frac{\sqrt{8R^2 + r^2} - 3r}{4},$$

$$y = \frac{\sqrt{8R^2 - 2r\sqrt{8R^2 + r^2} - 2r^2}}{4}, \tag{2}$$

$$z = H.$$

For a spherical coordinate system

$$x = \frac{2\sqrt{3R^2 - r^2} - r\sqrt{3R^2 + r^2}}{3},$$

$$y = \frac{2\sqrt{3R^2 - r^2} - r\sqrt{3R^2 + r^2}}{3},$$ (3)

$$z = \frac{\sqrt{3R^2 + r^2} - 2r}{3}.$$

For the cylindrical coordinate system, the lengths of the sides of the parallelepiped are determined straightforwardly. This is because the working zone associated with this coordinate system has a parallelepiped shape. The lengths of the sides of the parallelepiped for the cylindrical coordinate system are obtained through operations performed in the spherical coordinate system [5].

To establish the mathematical model of the movement for the parallelepiped based on the given lengths, it is necessary to formulate the measurements of the side lengths as constraints using algebraic expressions.

For the movement of the parallelepiped, the following six parameters are essential:

$$X_g = [x_A, y_A, z_A, \theta, \phi, \psi]^T$$ (4)

x_A, y_A, z_A - Cartesian coordinate system;

θ, ϕ, Ψ - Euler angles defining the target;

A - characteristic material point of the robotic structure.

To increase the positional accuracy of the gripping device, it is necessary to limit its movement. Limitations reduce the number of control parameters.

The "K-controllability" method, which minimizes the number of control parameters presented in [6,7], was applied to the matrix characterizing the behaviour of the capture device, and as a result, let (k-n) exit from the process.

The "K-controllability" method, which minimizes the number of control parameters presented in the literature [8,9,10], was applied to the matrix characterizing the movement of the gripping device. As a result, let $(n-k)$ the constraints leave the process.

Here, it should be $k \leq n$.

Remaining parameters: $X_k = [u_1, u_2, ..., u_k]^T$.

In this context, let's present the following equation(5):

$$\tau_g = J_k \ddot{X}_k + A_k$$ (5)

Here

J_k is a diagonal matrix with a multiplicity of k eigenvalues

A_k - a matching shortened vector.

Motion without constraints, that is the complete case of equation (5).

$$\tau_g = J\ddot{q} + A$$ (6)

We equate equations (5) and (6).

$$J_k \ddot{X}_k + A_k = J\ddot{q} + A$$ (7)

$$\ddot{q} = J_k \ddot{X}_k J^{-1} - J^{-1}(A_k - A)$$ (8)

Equation (3) represents the movement that improves the positional accuracy of the robotic structure.

By applying constraints on reactions, forces, and moments, the solution is obtained through the general theorem of dynamics, and it can be described as follows:

$$H\ddot{q} + h = P + DR_A \qquad (9)$$

The appearance of the reaction vector R_A and Matrix D

$$D = [D_{1[n\times3]} \vdots D_{2[n\times3]}], \qquad R_A = \begin{bmatrix} F_{A(3\times1)} \\ \cdots\cdots \\ M_{A(3\times1)} \end{bmatrix}. \qquad (10)$$

In equation (9), $H = H(q)$ $\qquad (n\times n)$ − inertial matrix.

$h = h(q,\dot{q}),$ $\qquad (n\times1)$ - dimensional centrifugal and correlation force vector;

$P = P(t),$ $\qquad (n\times1)$ - moment vector on dimensioned hinges;

$\ddot{q}$ $\qquad (n\times1)$ - acceleration vector in dimensional generalized coordinates;

D $\qquad (n\times6)$ - matrix defining the dimensional response vector;

R_A $\qquad (n\times1)$ - the reaction vector of the measuring device.

Taking into account the conditions imposed on the movement of the gripper and the configuration of the industrial robot, additional conditions related to the reaction vector Ra are derived. In particular, the scalar condition is expressed in the form of the following matrix [5]: $6-(n-k)$

$$ER_A = 0 \qquad (11)$$

E $\qquad (6-n+k)\times6$ - dimensional matrix.

Equations (8), (9), and (11) define the complete mathematical model of the robotic system.

The orientation phase of a robotic system should be designed to optimize its interaction with the working zone, moving beyond simplistic geometric shapes such as cylinders, parallelepipeds, or spheres. For improved positional accuracy, it is crucial to employ an orientation phase that facilitates a more effective engagement with the working zone, preferably through a parallelepiped or an internally configured design, as detailed in previous research [11]. In the optimal design, the internal structure of the component aligns such that the centre of mass coincides with the centre of the joint, as illustrated in Figure 1.

When the orientation phase adopts a cylindrical form, the radius of the cylinder's cross-section should match the radius of the sphere encompassing the part. Under these conditions, the part's configuration will exhibit a sectional profile akin to that of a parallelepiped, which defines the robot's working zone, as demonstrated in Figure 2. This approach ensures that the working zone's characteristics are adequately represented, enhancing the overall interaction between the robot and the component.
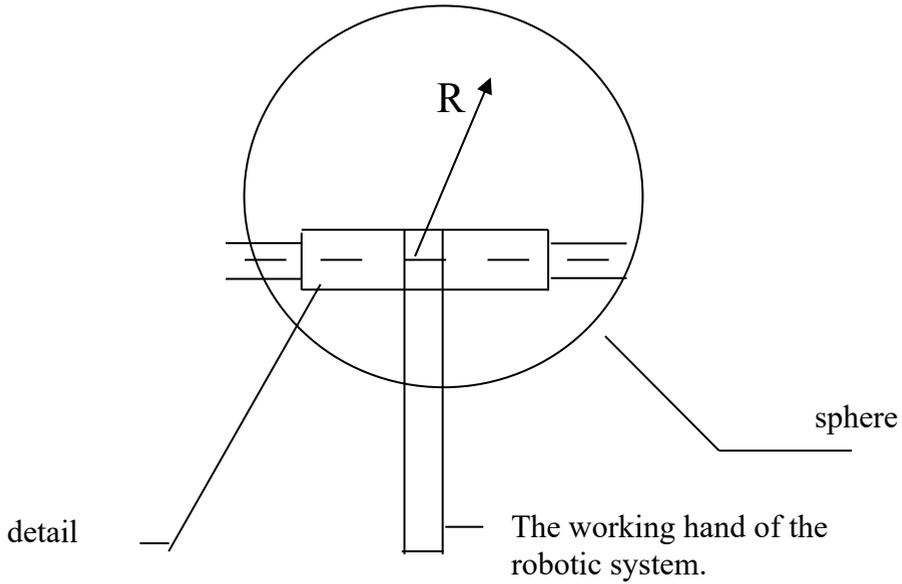
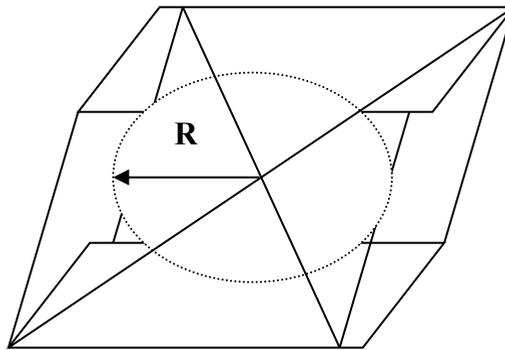**Fig. 1.** Internal placement of the part within the joint.

Fig. 2. Internal configuration of the robotic system's working zone with the orientation phase.

The centre of the sphere is defined as the intersection point of the parallelepiped's main diagonals, and its radius is determined as follows:

$$R = \frac{d}{2} = \frac{1}{2}\sqrt{x_1^2 + x_2^2 + x_3^2}$$

The centre R is at the intersection of the main diagonals of the inner parallelepiped, which is the ninth point of the parallelepiped.

3 Results and discussion

The figure representing the working zone of the industrial robot in a geometrical view, the movement of the robot's grasping device towards the parallelepiped along the space is determined using the following formula:

$$
\begin{cases}
\dfrac{dx(t)}{dt} = \pm \dfrac{v\Delta_{23}}{\sqrt{\Delta_{12}^2 + \Delta_{23}^2 + \Delta_{31}^2}}, & x(0) = x_{\text{м}}. \\[4mm]
\dfrac{dy(t)}{dt} = \pm \dfrac{v\Delta_{31}}{\sqrt{\Delta_{12}^2 + \Delta_{23}^2 + \Delta_{31}^2}}, & y(0) = y_{\text{м}}. \quad (13) \\[4mm]
\dfrac{dz(t)}{dt} = \pm \dfrac{v\Delta_{12}}{\sqrt{\Delta_{12}^2 + \Delta_{23}^2 + \Delta_{31}^2}}, & z(0) = z_{\text{м}}.
\end{cases}
$$

To increase the positional accuracy of the robot, the boundary problem along with the initial problem for moving the grasping device towards the ninth point of the parallelepiped is put into the equation. The lengths of the ninth point are involved in the boundary value problem.

And, as a result, the system of equations will look like this:

$$
\begin{cases}
\dfrac{dx(t)}{dt} = \pm \dfrac{v\Delta_{23}}{\sqrt{\Delta_{12}^2 + \Delta_{23}^2 + \Delta_{31}^2}}, & x(0) = x_{\text{м}}. \\[4mm]
\dfrac{dy(t)}{dt} = \pm \dfrac{v\Delta_{31}}{\sqrt{\Delta_{12}^2 + \Delta_{23}^2 + \Delta_{31}^2}}, & y(0) = y_{\text{м}}. \quad (14) \\[4mm]
\dfrac{dz(t)}{dt} = \pm \dfrac{v\Delta_{12}}{\sqrt{\Delta_{12}^2 + \Delta_{23}^2 + \Delta_{31}^2}}, & z(0) = z_{\text{м}}.
\end{cases}
$$

$$
\begin{cases}
x(T) \le R \\
y(T) \le R \\
z(T) \le R
\end{cases}
\qquad (15)
$$

The initial and boundary value problems are formulated, and they are solved using the Runge-Kutta method. In performing the manipulative operation of the robotic system, certain coordinate displacements do not participate. This means that in the execution of the manipulative operation, the robotic arm provides a combination of trajectories in various forms.

4 Conclusions

In conclusion, the quest for precision in industrial robotics is an ongoing and critical pursuit within the landscape of automated manufacturing. This article has provided a comprehensive exploration of the complexities surrounding positional accuracy, focusing on movement dynamics, trajectory management, and foundational principles governing the precise positioning of industrial robots.

The model introduced in this study addresses the challenge of aligning the capture device with the ninth point within the parallelepiped of the gripper's working zone. The proposed solution framework emphasizes meticulous movements along the intersecting point of the parallelepiped's diagonals, showcasing a holistic approach to elevate positional accuracy.

A significant aspect of this exploration involves a detailed analysis of the construction and mechanics of industrial robots, with an emphasis on link configurations. The research highlights the operational advantages associated with fewer links, demonstrating consistent positional accuracy even in simpler robotic setups. However, it also recognizes the amplified trajectory variations that come with an increased number of links, impacting both mobile-base and fixed-base robots in practical industrial scenarios.

Furthermore, the study delves into the intricate landscape of coordinate systems employed by industrial robots, exploring rectangular, cylindrical, spherical, and angular systems. Each coordinate system provides unique insights into the robot's working area, influencing configurations and basic movements.

In practical terms, the article underscores the importance of understanding and refining trajectory variations to prevent collisions and uphold operational continuity, especially in critical operations like machining. The intricate interplay between trajectory distinctions, link configurations, and coordinate systems necessitates well-informed decision-making paradigms to mitigate potential disruptions and ensure operational integrity.

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