Seismic resistance calculation in multi-story structures under vibration forces

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Abstract. The article is devoted to developing methods for solving the problem of seismic vibrations using a continuous spatial plate model of a multi-story building developed within the framework of the bi-moment theory of thick plate structures. A method for dynamic spatial calculations for seismic resistance of multi-story buildings under longitudinal seismic influences is proposed. Formulas are given for determining the reduced elastic moduli. The values of eigenfrequencies, displacements, and stresses are determined within the framework of the resonance method.

1 Introduction

When constructing a plate model of a building, two main problems arise. The first problem is related to determining the reduced moduli of elasticity and density of multi-story buildings as a continuous three-dimensional body. The second problem is to determine the boundary conditions on the side faces and at the top end of the building. Many engineering structures can be represented by beam and plate models. For example, multi-story and high-rise buildings, hydro-technical structures, and chimneys. The issues of the modeling dynamic behavior of buildings and structures under seismic impacts are relevant and complex problems in the sciences of structural dynamics and mechanics of a deformable rigid body.

It should be noted that the problems of seismic resistance of buildings and structures remain one of the most pressing problems in the dynamics of structures and are being successfully developed by many authors.

The study in [1] is devoted to solving the problem of determining the natural frequencies of multi-story buildings. The influence of the ratio of the rigidities of the horizontal and vertical elements of the building on the value of the first frequency of natural vibrations is shown. An analytical formula is proposed for determining the frequency of the first natural oscillations. Reference [2] presents a method for calculating the seismic resistance of multi-story monolithic concrete buildings using the dynamic method. The

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numerical solution of the equations of motion is obtained using the explicit scheme of the finite difference method.

Article [3] presents the results of a numerical study of the behavior of a reinforced concrete 9-story frame building designed for construction in an area with a calculated seismicity of 7 points. In [4], the seismic characteristics of a 14-story office building in Nanjing, China, were studied. Due to its plan, the vertical disturbances in the structural system were evaluated using response spectrum method, and elastic and elastoplastic time analysis.

Article [5] presents various types of shear walls in buildings. Namely, concrete shear walls, silica concrete shear walls, steel plate shear walls, and steel-silica concrete composite shear walls are considered elevator walls in 22- and 52-story tall buildings. Article [6] proposed a technique for assessing damage to loose non-structural components by analyzing rigid blocks under seismic loading. Article [7] is devoted to studying the direct influence of the frequency content of seismic load on high-rise buildings.

In [8-10], the influence of the moisture content of loess soil and soil foundations on sedimentary deformations was studied. Subsidence of loess soils occurs due to a decrease in soil strength parameters and transformation of the stressed state of soils.

References [11,12] investigated the influence of linear and nonlinear models of interaction between the foundation and soil on the vibrations of a multi-story building under seismic influence. Articles [13, 14] are devoted to the theoretical calculation of the box-shaped structure of large-panel buildings for dynamic impacts, considering the spatial work of transverse and longitudinal walls under dynamic impacts specified by the movement of the base according to a sinusoidal law.

Articles [15-17] are devoted to dynamic calculations of elements of box-shaped buildings for seismic resistance, considering the spatial work of box-shaped elements. In this case, the dynamic effect is specified as harmonic oscillations of the base movements according to a sinusoidal law. The equations of motion are given for each of the plate and beam elements of the box-shaped building structure based on the Kirchhoff-Love theory.

In [18-20], a method and algorithm for numerically solving the problem of seismic resistance of multi-story buildings with transverse and longitudinal vibrations were developed within the framework of a plate model. Methods for determining displacements, stresses and forces, moments and bi-moments are proposed. A methodology, algorithm, and programs for the numerical calculation of displacements, accelerations and stresses of a multi-story building have been developed using an explicit scheme of the finite difference method.

This article also relates to solving the problem of seismic resistance of multi-story buildings under seismic influences. A method for calculating the natural frequencies, displacements and stresses of multi-story buildings under shear and longitudinal seismic vibrations is proposed.

2 Formulation and solution of the problem of seismic resistance of a multi-story building within the framework of a plate model under longitudinal vibrations

The problem of longitudinal vibrations of a multi-story building is a symmetric problem of the bi-moment theory of plate structures, was developed in [15-18]. The system of equations for longitudinal vibrations of multi-story and high-rise buildings is described by nine unknown kinematic functions, determined by the following formulas:

$$\overline{\psi}_k = \frac{1}{2h}\int_{-h}^{h} u_k dz, \quad \overline{\beta}_k = \frac{1}{2h^3}\int_{-h}^{h} u_k z^2 dz, \quad (k=1,2),$$

$$\overline{r} = \frac{1}{2h^2}\int_{-h}^{h} u_3 z dz, \quad \overline{\gamma} = \frac{1}{2h^4}\int_{-h}^{h} u_3 z^3 dz. \tag{1}$$

Based on generalized functions (1), longitudinal and shear forces are calculated N_{11}, N_{12}, N_{22} from stress σ_{11}, σ_{12}, σ_{22} according to the following formulas:

$$N_{11} = E_{11}H\frac{\partial\overline{\psi}_1}{\partial x_1} + E_{12}H\frac{\partial\overline{\psi}_2}{\partial x_2} + 2E_{13}\overline{W},$$

$$N_{22} = E_{12}H\frac{\partial\overline{\psi}_1}{\partial x_1} + E_{22}H\frac{\partial\overline{\psi}_2}{\partial x_2} + 2E_{23}\overline{W}, \quad N_{12} = N_{21} = G_{12}\left(H\frac{\partial\overline{\psi}_2}{\partial x_1} + H\frac{\partial\overline{\psi}_2}{\partial x_1}\right). \tag{2}$$

In contrast to traditional approaches, concepts were introduced and expressions were constructed for bi-moments generated during shear-longitudinal vibrations of a plate model of a building. Longitudinal and shear bi-moments T_{11}, T_{22}, T_{12} from stress σ_{11}, σ_{12}, σ_{22}, are defined as:

$$T_{11} = H\left(E_{11}\frac{\partial\overline{\beta}_1}{\partial x_1} + E_{12}\frac{\partial\overline{\beta}_2}{\partial x_2} + E_{13}\frac{2\overline{W}-4\overline{r}}{H}\right),$$

$$T_{12} = T_{21} = HG_{12}\left(\frac{\partial\overline{\beta}_2}{\partial x_1} + \frac{\partial\overline{\beta}_1}{\partial x_2}\right), \quad T_{22} = H\left(E_{12}\frac{\partial\overline{\beta}_1}{\partial x_1} + E_{22}\frac{\partial\overline{\beta}_2}{\partial x_2} + E_{23}\frac{2\overline{W}-4\overline{r}}{H}\right). \tag{3}$$

Specific transverse bi-moments of the plate model of the building were introduced, $\overline{p}_{13}$, $\overline{p}_{23}$ and $\overline{\tau}_{13}$, $\overline{\tau}_{23}$ from shear stresses σ_{13}, σ_{23}, determined by formulas:

$$\overline{p}_{k3} = G_{k3}\left(\frac{\partial\overline{r}}{\partial x_k} + \frac{2(\overline{u}_k - \overline{\psi}_k)}{H}\right), \quad \overline{\tau}_{k3} = G_{k3}\left(\frac{\partial\overline{\gamma}}{\partial x_k} + \frac{2(\overline{u}_k - 3\overline{\beta}_k)}{H}\right), \quad (k=1,2). \tag{4}$$

The concept and formulas of specific normal bi-moments are also introduced $\overline{p}_{33}$ and $\overline{\tau}_{33}$ from normal stress σ_{33} according to the following formulas:

$$\overline{p}_{33} = E_{31}\frac{\partial\overline{\psi}_1}{\partial x_1} + E_{32}\frac{\partial\overline{\psi}_2}{\partial x_2} + E_{33}\frac{2\overline{W}}{H}, \quad \overline{\tau}_{33} = E_{31}\frac{\partial\overline{\beta}_1}{\partial x_1} + E_{32}\frac{\partial\overline{\beta}_2}{\partial x_2} + E_{33}\frac{2\overline{W}-4\overline{r}}{H}. \tag{5}$$

Using the concept of forces and bi-moments (2)-(5), equations for shear and longitudinal vibrations of a plate model of a multi-story building are derived. The system of differential equations of seismic vibrations of a multi-story building relative to longitudinal and shear forces (2) is constructed in the following form:

$$\frac{\partial N_{11}}{\partial x_1} + \frac{\partial N_{12}}{\partial x_2} = \rho H\ddot{\overline{\psi}}_1,$$

$$\frac{\partial N_{21}}{\partial x_1} + \frac{\partial N_{22}}{\partial x_2} = \rho H\ddot{\overline{\psi}}_2. \tag{6}$$

The system of differential equations of seismic vibrations of a multi-story building relative to longitudinal and shear bi-moments (3), (4) is constructed as:

$$\frac{\partial T_{11}}{\partial x_1} + \frac{\partial T_{12}}{\partial x_2} - 4\overline{p}_{13} = \rho H \ddot{\overline{\beta}}_1,$$

$$\frac{\partial T_{12}}{\partial x_1} + \frac{\partial T_{22}}{\partial x_2} - 4\overline{p}_{23} = \rho H \ddot{\overline{\beta}}_2. \tag{7}$$

The system of differential equations of seismic vibrations of a multi-story building relative to the intensities of transverse longitudinal and shear bi-moments (3), (4) and (5) is constructed in the form:

$$\frac{\partial \overline{p}_{13}}{\partial x_1} + \frac{\partial \overline{p}_{23}}{\partial x_2} - \frac{2\overline{p}_{33}}{H} = \rho \ddot{\overline{r}}, \tag{8}$$

$$\frac{\partial \overline{\tau}_{13}}{\partial x_1} + \frac{\partial \overline{\tau}_{23}}{\partial x_2} - \frac{6\overline{\tau}_{33}}{H} = \rho \ddot{\overline{\gamma}}. \tag{9}$$

It should be noted that the systems of four equations (5) - (9) contain new six unknown functions $\overline{u}_1$, $\overline{u}_2$, $\overline{\beta}_1$ $\overline{\beta}_2$, $\overline{r}$, $\overline{\gamma}$.

Thus, the systems of six equations (5)-(9) contain nine unknown functions $\overline{u}_1$, $\overline{u}_2$, $\overline{\beta}_1$ $\overline{\beta}_2$, $\overline{r}$, $\overline{\gamma}$, $\overline{\psi}_1$, $\overline{\psi}_2$, $\overline{W}$. To ensure that the system of equations is closed, three more equations are constructed to determine generalized displacements $\overline{u}_1$, $\overline{u}_2$, $\overline{W}$, rewritten as follows:

$$\overline{u}_k = \frac{1}{4}\left(21\overline{\beta}_k - 3\overline{\psi}_k\right) - \frac{1}{20} H \frac{\partial \overline{W}}{\partial x_k} \quad (k=1,2), \tag{10}$$

$$\overline{W} = \frac{1}{2}\left(21\overline{\gamma} - 7\overline{r}\right) - \frac{1}{30} H\left(\frac{E_{31}}{E_{33}}\frac{\partial \overline{u}_1}{\partial x_1} + \frac{E_{32}}{E_{33}}\frac{\partial \overline{u}_2}{\partial x_2}\right). \tag{11}$$

To formulate the boundary conditions of the problem of longitudinal vibrations for multi-story buildings (10) - (11), we introduce the intensities of bi-moments $\overline{\sigma}_{11}$, $\overline{\sigma}_{22}$, $\overline{\sigma}_{12}$, $\overline{\sigma}_{11}^*$, $\overline{\sigma}_{22}^*$, determined by formulas obtained in [13-18, 21]. Bi-moments $\overline{\sigma}_{11}, \overline{\sigma}_{12}, \overline{\sigma}_{22}$ have the following expressions:

$$\overline{\sigma}_{11} = \left(E_{11} - \frac{E_{13}}{E_{33}}E_{31}\right)\frac{\partial \overline{u}_1}{\partial x_1} + \left(E_{12} - \frac{E_{13}}{E_{33}}E_{32}\right)\frac{\partial \overline{u}_2}{\partial x_2},$$

$$\overline{\sigma}_{22} = \left(E_{21} - \frac{E_{23}}{E_{33}}E_{31}\right)\frac{\partial \overline{u}_1}{\partial x_1} + \left(E_{22} - \frac{E_{23}}{E_{33}}E_{32}\right)\frac{\partial \overline{u}_2}{\partial x_2}, \tag{12}$$

$$\overline{\sigma}_{12} = G_{12}\left(\frac{\partial \overline{u}_1}{\partial x_2} + \frac{\partial \overline{u}_2}{\partial x_1}\right).$$

Based on Hooke's law and expressions (18), we obtain the following expressions for bi-moments $\overline{\sigma}^*_{11}$, $\overline{\sigma}^*_{22}$:

$$\overline{\sigma}^*_{11} = -E_{11}H\frac{\partial^2\overline{W}}{\partial x_1^2} - E_{12}H\frac{\partial^2\overline{W}}{\partial x_2^2} + E_{13}\frac{60\left(7\overline{W} + 42\overline{r} - 105\overline{\gamma}\right)}{H},$$

$$\overline{\sigma}^*_{22} = -E_{12}H\frac{\partial^2\overline{W}}{\partial x_1^2} - E_{22}H\frac{\partial^2\overline{W}}{\partial x_2^2} + E_{23}\frac{60\left(7\overline{W} + 42\overline{r} - 105\overline{\gamma}\right)}{H}. \tag{13}$$

At the base of the plate model of a multi-story building, the boundary conditions for bending and shear vibrations have the form:

$$\overline{\psi}_1 = u_0(t), \ \overline{\psi}_2 = 0, \ \overline{\beta}_1 = \frac{1}{3}u_0(t), \ \overline{\beta}_2 = 0, \overline{u}_1 = u_0(t), \ \overline{u}_2 = 0, \ \overline{r} = 0, \ \overline{\gamma} = 0, \ \overline{W} = 0. \tag{14}$$

Where $u_0(t)$ –is the law of motion of the base.

Let us write down the boundary conditions on the free side and top faces of the multi-story building. On the side faces of the building, we set zero conditions for forces, moments, and bi-moments:

$$N_{11} = 0, \ N_{12} = 0, \ T_{11} = 0, \ T_{12} = 0, \ \overline{p}_{13} = 0, \ \overline{\tau}_{13} = 0,$$

$$\overline{\sigma}_{11} = 0; \ \overline{\sigma}_{12} = 0 \ \overline{\sigma}^*_{11} = 0. \tag{15}$$

On the free upper face, zero conditions for forces and bi-moments are also specified:

$$N_{12} = 0, \ N_{22} = 0, \ T_{12} = 0, \ T_{22} = 0, \ \overline{p}_{23} = 0, \ \overline{\tau}_{23} = 0,$$

$$\overline{\sigma}_{12} = 0; \ \overline{\sigma}_{22} = 0 \ \overline{\sigma}^*_{22} = 0. \tag{16}$$

Initial conditions, i.e. the values of the sought-for functions for longitudinal vibrations of the building are considered zero:

$$\overline{\psi}_1 = 0, \quad \overline{\psi}_2 = 0, \quad \overline{\beta}_1 = 0, \quad \overline{\beta}_2 = 0, \ \overline{r} = 0, \quad \overline{\gamma} = 0, \quad \overline{u}_1 = 0, \quad \overline{u}_2 = 0, \quad \overline{W} = 0,$$

$$\dot{\overline{\psi}}_1 = 0, \quad \dot{\overline{\psi}}_2 = 0, \quad \dot{\overline{\beta}}_1 = 0, \quad \dot{\overline{\beta}}_2 = 0, \ \dot{\overline{r}} = 0, \quad \dot{\overline{\gamma}} = 0, \quad \dot{\overline{u}}_1 = 0, \quad \dot{\overline{u}}_2 = 0, \quad \dot{\overline{W}} = 0. \tag{17}$$

3 Solution method

The problem is solved using the explicit scheme of the finite difference method.

When constructing finite-difference expressions for the derivatives of generalized displacements along spatial coordinates, we will use the formulas of central difference schemes.

To approximate the first derivatives of generalized functions with respect to central points, we have the following expressions:

$$\frac{\partial f_{i,j}^k}{\partial x_1} = \frac{f_{i+1,j}^k - f_{i-1,j}^k}{2\Delta x_1}, \quad \frac{\partial f_{i,j}^k}{\partial x_2} = \frac{f_{i,j+1}^k - f_{i,j-1}^k}{2\Delta x_2}, \tag{18}$$

where $\Delta x_1 = \dfrac{a}{N}$, $\Delta x_2 = \dfrac{b}{M}$ – is the grid method calculation step, N, M – are the numbers of divisions per grid.

When approximating the derivatives of stresses, forces, moments, and bi-moments, central finite-difference schemes are used in half steps, which have the following form:

$$\frac{\partial F_{i,j}^k}{\partial x_1} = \frac{F_{i+\frac{1}{2},j}^k - F_{i-\frac{1}{2},j}^k}{\Delta x_1}, \quad \frac{\partial F_{i,j}^k}{\partial x_2} = \frac{F_{i,j+\frac{1}{2}}^k - F_{i,j-\frac{1}{2}}^k}{\Delta x_2} \quad (i = 1,N; \ j = 1,M). \tag{19}$$

Where $\Delta x_1 = \dfrac{a}{N}$, $\Delta x_2 = \dfrac{b}{M}$.

Note that formulas (19) have the second-order of accuracy:

When approximating the condition that force factors on the side faces and edges of a multi-story building are zero, we use the following expressions:

$$F_{N+\frac{1}{2},j}^k + F_{N-\frac{1}{2},j}^k = 0 \ (j = 1,M); \quad F_{i,M+\frac{1}{2}}^k + F_{i,M-\frac{1}{2}}^k = 0 \ (i = 1,N). \tag{20}$$

When using formulas (19) and (20), it is necessary to approximate the derivatives of the generalized displacement functions at the central point between two points x_i and x_{i+1} or y_j and y_{j+1}. In these cases, formulas are used, replacing i – with $i-\frac{1}{2}$ and j – with $j-\frac{1}{2}$.

$$\frac{\partial f_{i-\frac{1}{2},j}^k}{\partial x_1} = \frac{f_{i,j}^k - f_{i-1,j}^k}{\Delta x_1}, \quad \frac{\partial f_{i,j-\frac{1}{2}}^k}{\partial x_2} = \frac{f_{i,j}^k - f_{i,j-1}^k}{\Delta x_2}, \quad (i = 1,N; \ j = 1,M), \tag{21}$$

$$\frac{\partial f_{i-\frac{1}{2},j}^k}{\partial x_1} = \frac{\partial}{\partial x_1}\left(\frac{f_{i,j}^k + f_{i-1,j}^k}{2}\right), \quad \frac{\partial f_{i,j-\frac{1}{2}}^k}{\partial x_2} = \frac{\partial}{\partial x_2}\left(\frac{f_{i,j}^k + f_{i,j-1}^k}{2}\right), \quad (i = 1,N; \ j = 1,M). \tag{22}$$

We present the second derivative of the function with respect to time within the framework of the finite difference method as:

$$\frac{\partial^2 f_{i,j}^k}{\partial t^2} = \frac{f_{i,j}^{k+1} - 2f_{i,j}^k + f_{i,j}^{k-1}}{\Delta t^2}, \tag{23}$$

where Δt – is the time step.

When implementing a numerical method for solving the problem, we select steps in spatial coordinates and time as follows:

$$\Delta x_1 = \frac{a}{N}, \ \Delta x_2 = \frac{b}{M}, \quad c\Delta t \le \min(\Delta x_1, \Delta x_2) \tag{24}$$

Based on the developed method for numerically solving the equation of shear and

longitudinal vibrations of a multi-story building, an algorithm and program for calculating a multi-story building for seismic resistance were created.

4 Numerical results and their analysis

Numerical calculations were conducted under the assumption that seismic motion of soil occurs in the direction of the OZ axis (along the width of the building) in the form of acceleration of the base of the building:

$$\ddot{u}_0(t) = a_0 \cos(\omega_0 t), \tag{25}$$

where $a_0 = k_c g$ - maximum acceleration and $\omega_0 = 2\pi \nu_0$ – circular frequency of the soil base, k_c and ν_0 - earthquake magnitude coefficient and natural frequency of external influence, respectively.

From here we get the movement of the building base in the form:

$$u_0(t) = \frac{A_0}{2}(1 - \cos(\omega_0 t)). \tag{26}$$

Here $A_0 = \frac{2a_0}{\omega_0^2}$ – is the amplitude of movement of the base. We take zero values as initial conditions.

Amplitude of external influence A_0 depends on the magnitude of the earthquake, which is determined from the condition $A_0 \omega_0^2 = 2k_c g$, where k_c, g are the seismicity coefficient and gravitational acceleration, respectively. From here, we find

$$A_0 = \frac{2k_c g}{\omega_0^2}.$$

Note that the seismicity coefficients for seven-magnitude, eight-magnitude and nine-magnitude earthquakes are equal $k_c = 0.1, \ 0.2, \ 0.4$ respectively.

Calculations were made for various mechanical and geometric data of multi-story buildings, bases and soil foundation, and the load terms of the equation for their transverse vibrations.

The geometric characteristics of the room panels and the external dimensions of the buildings must be specified as initial data. To obtain specific numerical results, the mechanical characteristics of the structure of the considered plate model of a multi-story building must also be known.

We assume that the external walls consist of reinforced concrete with an elastic modulus $E = 20000\,\text{MPa}$, density $\rho = 2500\,\text{kg/m}^3$ Poisson's ratio $\nu = 0.3$.

We consider the internal walls to consist of expanded clay concrete with the following physical characteristics: modulus of elasticity $E = 7500\,\text{MPa}$ density $\rho = 1200\,\text{kg/m}^3$ Poisson's ratio $\nu = 0.3$.

The results of calculations of forced vibrations of a building within the framework of a thick plate model are presented for the following dimensions of building slabs:

$$h_1 = 0.40\,m,\ h_2 = 0.25\,m,\ h_{floor} = 0.2\,м,\ a_1 = 5\,m,\ b_1 = 3\,m.$$

The height and length of a multi-story building are assumed to be $b = nb_1$ and $a = 30\,m$, respectively, and building width H varies.

The height for a nine-story, twelve-story and sixteen-story buildings is assumed to be $a = 30\,m$, and $a = 30\,m$, $a = 51\,m$ respectively.

Using the initial data, the values of the reduced moduli of elasticity, shear, and density, given in Table 1 and Table 2, of multi-story buildings were determined, calculated using the formulas given in [13-18, 21].

Table 1. Coefficients of elastic moduli of the continuum model of the building for given initial data.

Thickness	Coefficient of modulus of elasticity of the continuum model of the building						
$H\,(m)$	ξ_0	ξ_{11}	ξ_{12}	ξ_{13}	ξ_{22}	ξ_{23}	ξ_{33}
11	0.102	0.129	0.095	0.067	0.164	0.05	0.102
13	0.101	0.114	0.081	0.067	0.149	0.05	0.102
15	0.100	0.103	0.070	0.067	0.138	0.05	0.102

Table 2. Reduced elastic moduli of the continuum model of multi-story buildings with given initial data.

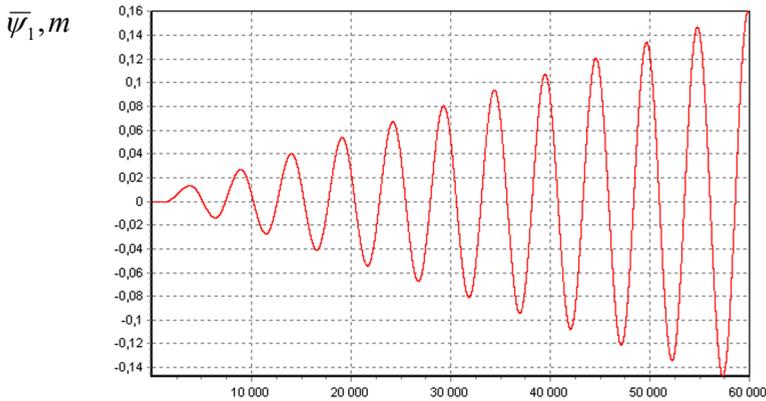| Thickness	Modules of elasticity of a continuum model of a building					
H (m)	E_1	E_2	E_3	G_{12}	G_{13}	G_{23}
11	1545	1965	1220	458.2	320.0	240.0
13	1369	1789	1220	387.7	320.0	240.0
15	1240	1660	1220	336.0	320.0	240.0

We present the results of calculations of the frequency, periods of oscillations, and displacement of points for twenty-, twenty-four, and twenty-eight-story buildings with longitudinal vibrations obtained using the resonance method. Let us present graphs of changes in generalized normal and longitudinal displacements near the resonant regime, from which the values of natural frequencies are determined.

Graphs of changes in displacement and stress values were obtained for longitudinal vibrations of multi-story buildings in the resonant case during an earthquake of magnitude 7. Now, we present the results of calculations of the displacement and stress of points of twenty-, twenty-four, and twenty-eight-storey buildings in the vicinity of the beating state and the resonant mode.

Let us present the results of calculations of natural frequencies, displacements and stresses for a 12-story building.
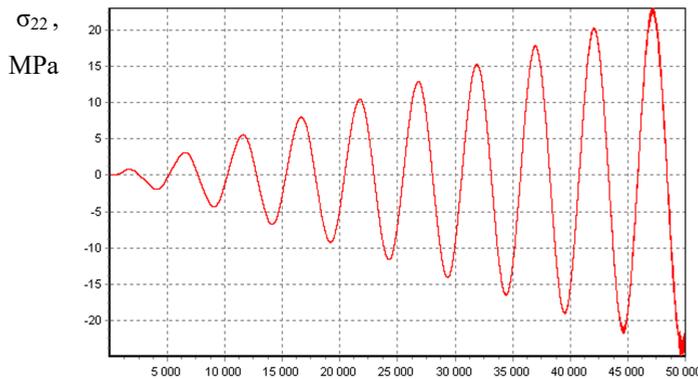
Using the resonance method, the values of the natural frequency of a twelve-story building are calculated depending on three values of the building width $H = 11\,m$, $H = 13\,m$ and $H = 15\,m$.

Figure 1 shows the graphical change in normal displacement values φ_1 at dimensionless time $\tau = \dfrac{ct}{H}$ in the middle of the top floor of a twelve-story building. As can be seen from Figure 1, when the frequency values of the external influence are very close to the value of the natural frequency $\nu_0 = p_1 = 5.02\,Hz$, then an infinite increase in the value of normal displacement is observed.

Fig. 1. Graph of changes in normal displacement $\overline{\psi}_1$ in the resonant case $v_0 = p_1 = 5.02$ Hz in time in the middle of the upper level of a twelve-story building.

Figure 2 shows a graph characterizing changes in the maximum normal stress σ_{22} near the resonant mode when $v_0 = p_1 = 5.02$ Hz, in the middle of the lower part of the first floor of a twelve-story building at time t. As can be seen from Figure 2, when the frequency values of external impact v_0 are very close to the natural frequency value p_1, then there is an infinite increase in stress σ_{22}.

Fig. 2. Graph of changes in normal stress σ_{22} in the resonant case over time in the middle of the first floor of a twelve-story building.

Based on the resonance method, the first three values of natural frequencies and the corresponding periods of natural oscillations were found depending on different values of the building width *H*.

Table 3 shows three values of natural frequencies p_1, p_2, p_3 and periods of natural oscillations T_1, T_2, T_3 of nine-, twelve-, sixteen-story buildings.
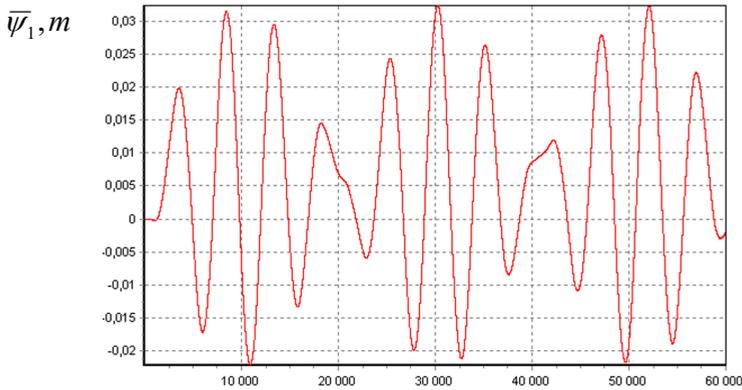
Table 3. First three natural frequencies p_1, p_2, p_3 and periods of natural oscillations T_1, T_2, T_3 of nine-, twelve-, sixteen-story buildings, depending on three building widths.

№	Number of floors	H, m	p_1, Hz	p_2, Hz	p_3, Hz	T_1, sec	T_2, sec	T_3, sec
1	9	11	7.70	20.61	33.91	0.1298	0.0485	0.0295
		13	7.63	20.31	33.25	0.1311	0.0492	0.0301
		15	7.58	20.12	32.82	0.1319	0.0497	0.0304
2	12	11	5.02	15.31	28.16	0.1992	0.0653	0.0355
		13	4.97	15.22	27.81	0.2012	0.0657	0.0359
		15	4.93	15.15	27.42	0.2028	0.0660	0.0365
3	16	13	3.52	11.51	23.02	0.2841	0.0868	0.0434
		15	3.44	11.41	22.83	0.2907	0.0876	0.0438
		18	3.36	11.31	22.44	0.2976	0.0884	0.0446

We present the results of calculations of displacements and stresses during forced vibrations of a twelve-story building with a transverse dimension $H=13$ *m*. In this case, the values of the frequency and amplitude of the eight-point external impact $(k_c = 0.2)$ are
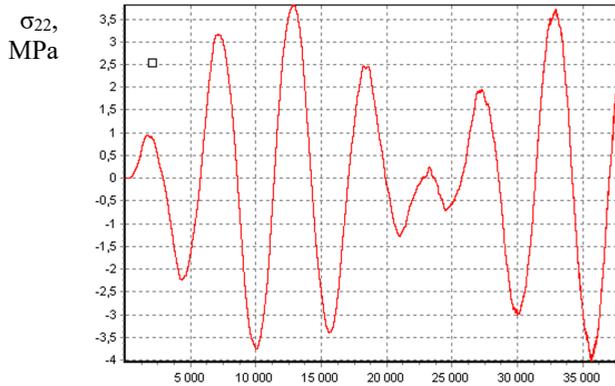
$$\nu_0 = 3.95 \text{ Hz} \text{ and } A_0 = \frac{2k_c g}{\omega_0^2} = \frac{2 \cdot 0.2 \cdot 9.81}{24.806^2} = 0.00638 \ m.$$

Figure 3 shows a graph of changes in normal displacement values $\widetilde{\psi}_1$ at time t in the middle of the top floor of a twelve-story building. As established (Figure 3), in the middle of the ninth floor of the building the maximum value of normal displacement φ_1 turned out to be $\widetilde{\psi}_1 = 3,5 \, sm$.

Fig. 3. Graph of changes in displacements $\overline{\psi}_1, m$ in time in the middle of a twelve-story building with a transverse dimension for the frequency of oscillations $\nu_0 = 3.95 \, Hz$.

Figure 4 shows a graph characterizing changes in the maximum normal stress σ_{22} in the middle of the lower part of the first floor of a twelve-story building in time t. As established (Figure 4), in the middle of the first floor of the building the stress value component $\sigma_{22} = 3,75 \, MPa$ is the maximum.

Fig. 4. Graph of changes in normal stress in time in the middle of the first floor of a twelve-story building.

Note that in the calculations for a nine-story and twelve-story building, the natural frequency is determined for three options for the building width: *H=11m H=13m, H=15m.* For a sixteen-story building, the frequency of natural vibrations is determined for three options for building widths *H=13m H=15m, H=18m.*

In conclusion, we note that during an earthquake magnitude 7 points, large displacements occur in high-rise buildings. The proposed plate model of the building correctly reflects the vibration shape of the building under seismic influences.

When implementing the numerical method for solving the problem, the steps in spatial coordinates are: $\dfrac{c\Delta t}{\Delta x_1} = \dfrac{c\Delta t}{\Delta x_2} = \dfrac{1}{4}$.

5 Conclusion

Note that the problem of seismic vibrations of multi-story buildings was formulated and numerically solved within the framework of a continuum plate model. Differential equations are approximated by finite difference expressions and the effectiveness of this method is shown.

For various options of geometric dimensions, based on the application of the resonance method, the first three values of natural frequency and displacements, accelerations, and stresses under longitudinal forced vibrations of a multi-story building with a number of floors ranging from nine to sixteen floors were determined.

Based on the analysis of numerical results, it was established that the plate model is appropriate for describing the dynamic behavior and calculating the stress-strain state of multi-story buildings under seismic impacts.

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