Study of the Behavior of Block Replacement Model (BRM) Using Higher Order Markov Chain in Varying Inflationary Economy

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Abstract. Influence of inflation on block replacement of the computers in ICT industry by using Markov chain is spotlighted in the present paper. The concept of minor, major repairs, breakdown and failure have been discussed in detail. Replacement decisions for variable maintenance costs has been thoroughly investigated and the results are presented. Replacement age in years is calculated for forecasted inflation, rapid increase in inflation and sluggish increase in inflation.

Keywords: Block replacement, Markov Chain, Inflation, Regression, Rapid trend, Sluggish trend

1. Introduction

Maintenance is a routine and repetitive activity of grooming a machine or a facility in its normal operating condition to perform with high efficiency. Maintenance is a function to keep the equipment/machine condition by replacing or repairing some of the components of the machine. The idea of maintenance is a traditional process but the process of adopting new and sophisticated procedures takes a step ahead in inclusion of the maintenance in different stages of minor, major, break down and repair.

To maximize the availability, high cost machines need to be properly maintained during their entire life cycle. Due to the accompanying development of the services and safety requirements, it is to be mandated to analyze proper maintenance of equipment. Continuous breakdowns, bad operating conditions lead to lower product quality, high maintenance costs, safety issues and depleting revenues. The aim of maintenance is to overcome failures, assess the profitability of the equipment, good replacement decisions. This activity of maintenance includes repairs, minor and major services which cannot be computed in specific, various costs and influence of the economic variables such as inflation, costs etc.

The main objective of the present article is to find the replacement policy to determine an age at which the replacement will be most economical instead of continuing at increased maintenance costs which will be an effective decision to direct the organization for maximizing its profit (or minimizing cost). **The following reports were projected by the researchers for block replacement models:**
Nuthall et al (1983) discussed the impact of inflation on replacement costs with the impact of finance and hours of usage [1]. Felix Belzunce et al (2011) compared the block

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replacement policies with other policies in terms of unplanned failures, aging properties and analysis of the cost associated with it [2]. Nairu Fan (2014) detailed about intelligent memory block replacement in computer systems [3]. Jongmoo Choi (2014) discussed about a new buffer replacement scheme DEAR (detection-based adaptive replacement) which is used for effective caching of disk blocks in the operating system, implementation and performance measurement of the scheme [4].B. Srinivas (2014) explained about the group replacement model for the completely failed systems [5].Sericola (2005) discussed the importance of Markov Chains in various domains for solving problems and analyzing the systems [6].Chein et al (2007) presented an agereplacement model with minimal repair based on cumulative repair cost limit. In this they considered the complete repair cost data in order to decide whether to repair the unit or to replace [7]. Bagai et al (1994) discussed optimal replacement time under the age replacement policy for a system with minor repairs [8]. Archibald et al (1996) studied and compared the optimal age-Replacement, Standard Block Replacement and Modified Block Replacement (MBR) Policies with an inference of MBR policy is appreciably better than the remaining two [9].

However there is no much literature on block or group replacement model with Markov chain transition probabilities that is being used in many applications. And hence in the present paper, we discuss a block replacement model for a block of computers which consist of many intermediate minor, major repairs with less functional with decreased efficiency, repeated repairs, breakdown or failure.

The uniqueness of the present paper is that inflation is calculated using four different methods, continued with the case study onInfluence of inflation on block replacement decision using higher order Markov chain with four different prevailing conditions as low initial maintenance cost with big increments during later periods , high initial maintenance cost with small increments during later periods, Rapid up-trend in inflation (assumed inflation data) and Sluggish up-trend in inflation (assumed inflation data). As of our knowledge, there are few reports that explained all the above conditions in a case study.

2. Methodology:

2.1 FORECASTING INFLATION

In the present study, the half yearly inflation data for ***Computer and Computer based system*** in India from the second half of 2019 to the second half of 2023 which is calculated based on the Wholesale Price Index-WPI.

Table 1: Past data of (Half yearly) Inflation

Year	Inflation (ϕ) during	
	Jan. - June	July – Dec.
2018		-17.48
2019	-22.40	-1.18
2020	-16.44	-15.27
2021	10.76	6.03
2022	3.41	16.5
2023	20.38	

Inflation is forecasted using Time Series and Causal forecasting models to identify the underlying model that best fits the time series data. And also an error analysis is made as a measurement of accuracy. Moving average method, Weighted moving average method, Simple exponenetial smoothing method and regression methods are employed for the forecasting of inflation

Table 2: Forecasted inflation data forfuture time periods using regression model

Year	Time period (t)	Inflation rate (ϕ)	Forecasted inflation (F)
2018H-II	0	-17.48	**-18.56424**
2018H-I	0.5	-22.4	-14.04579
2019H-II	1	-1.18	**-17.41886**
2019H-I	1.5	-16.44	-12.90041
2020H-II	2	-15.27	**-0.490441**
2020H-I	2.5	10.76	4.0280094
2021H-II	3	6.03	**0.6549406**
2021H-I	3.5	3.41	5.1733906
2022H-II	4	16.59	**17.583359**
2018H-II	4.5	20.38	22.101809
2018H-I	5		**18.728741**
2019H-II	5.5		23.247191
2019H-I	6		**35.657159**
2020H-I	6.5		40.175609
2020H-II	7		**36.802541**
2021H-I	7.5		41.32
2021H-II	8		**53.73**
2022H-I	8.5		58.24
2022H-II	9		**54.87**

2.2. Influence of inflation on block replacement decision using higher order Markov chain

Second Order Markov Chain (SOMC) assumes that probability of next state depends on the probabilities of immediately two preceding states. Then we will have SOMC whose transition probabilities are :

$$P_{x(n-2),x(n-1),x(n)} = P\{X(t_n) = x_n \mid X(t_{n-1}) = x_{n-1}, X(t_{n-2}) = x_{n-2}\} \quad \text{----(1)}$$

So as the model under study consists 4 states, the SOMC Transition Probability Matrix(TPM) (Andre Berchtold*et al.* 2002) [10] can be formulated

The size of the TPM will be m^lx m and the number of transition probabilities to be calculated in each TPM will be m^{l+1}.

2.3 Model Assumptions:

(i) In block replacement decisions, in addition to the functioning state and failure state, two intermediate repairable breakdown states-minor repair and major repair- are introduced which results in four-state Markov chain.

(ii) Obsolescence of the present system, due to the availability of new system with better design and capability is available in market, is assumed as break down/complete failure state.

(iii) Instead of assuming discrete probability distribution from one period to the other period, it is assumed to follow second order discrete-time Markov Chain.

(iv) TPMs for two years or the generators of the Markov chain are assumed. (In real practice these values can be taken from the past experience.)

(v) This model can be applied to a system that consists of N items in four different states and to find out the block replacement period. **In this study, a special reference has been made to the computer and computer based system which is widely used by ICT industry.**

(vi) Repair cost or rectification cost of any item is different for minor repairable state and major repairable state if two intermediary states are allowed. However, it is assumed constant if only one intermediary state is considered.

(vii) The money to be spent is taken as loan for certain items at a given nominal interest rate, r_n .

(viii) Nominal interest rate ‘ r_n ’ assumed to be constant during the life span of the asset

(ix) To address this, a parsimonious model, the Weighted Moving Transition Probabilities (WMTP) method is incorporated that approximates higher order Markov chains. This makes the number of parameters in each TPM to be computed far less. The size of the TPM is m x m only. Each element of the TPM is the probability for the occurrence of a particular event at time t given the probabilities of immediate previous l (= order of the Markov Chain) time periods i.e. at times from (t-l) to (t-1). The effect of each lag is considered by assigning the weights.

Table 3 : Abbreviations/Notations used

S. No.	**Abbreviations /Notations used**
1.	N = Total number of items in the system
2.	C_1 = Individual replacement cost per unit
3.	C_2 = Minor repair cost
4.	C_3 = Major repair cost
5.	C_4 = item cost during Group replacement
6.	r_n = Nominal interest rate
7.	ϕ_t =Inflation rate at the ‘t’th period
8.	r_t = real interest rate= $\frac{(r_n - \varphi_t)}{(1+\varphi_t)}$ from Fisherman’s relation
9.	$\nu = \frac{1}{(1+r_t)}$ =Present Worth Factor
10.	X_0^{I} = proportion of units under working condition initially
11.	X_0^{II} = proportion of units under minor repair initially
12.	X_0^{III} = proportion of units under major repair initially
13.	X_0^{IV} = proportion of units under complete failure initially
14.	X_i^{I}= proportion of units under working condition at the end of i^{th} time period
15.	X_i^{III}= proportion of units under major repair at the end of i^{th} time period,
16.	X_i^{IV}= proportion of units under complete failure at the end of i^{th} time period
17.	P_{ij}= Probability of items switching over from i^{th} state to j^{th} state in a period
18.	δ = weight parameter associated with lag
19.	AC(t) = Weighted average cost per period

As two intermediary states –minor and major repairable states- between functional and complete failure state are considered, the two Transition Probability matrices or the generators of Markov process for a four state system for can be represented as

$$P_{n-1} = \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \\ P_{41} & P_{42} & P_{43} & P_{44} \end{bmatrix}_{n-1} \quad \text{and } P_{n-2} = \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \\ P_{41} & P_{42} & P_{43} & P_{44} \end{bmatrix}_{n-2}$$

where 1,2,3 and 4 represents functional, minor repairable, major repairable and complete failure states.

The Weighted Moving Transition Probabilities-WMTP (for *l* = 2, Second Order Markov process) can be written as:

$$\left(P_{ij}\right)_n = \sum_{g=1}^{2} \delta_g (P_{ij})_g = \delta_{n-1}(P_{ij})_{n-1} + \delta_{n-2}(P_{ij})_{n-2} \quad \text{---(2)}$$

where $\delta_{n-1} + \delta_{n-2} = 1$ and $\delta \geq 0$.

The state probabilities of items in different states can be computed as = (probability of items in different states in initial period) X $\left(TPM\right)_n$

$$\text{i.e. } X_n = X_0 \left(P_{ij}\right)_n \quad \text{--- (3)}$$

Table 5: Probability Parameters of the items in different states in future time periods

S. No.	Description	Period	Parameters
I	Number of individual replacements	1	$\alpha_1 = NX_1^{IV}$
		2	$\alpha_2 = NX_2^{IV} + \alpha_1 X_1^{IV}$
		3	$\alpha_3 = NX_3^{IV} + \alpha_1 X_2^{IV} + \alpha_2 X_1^{IV}$
		4	$\alpha_4 = NX_4^{IV} + \alpha_1 X_3^{IV} + \alpha_2 X_2^{IV} + \alpha_3 X_1^{IV}$
II	Number of minor repairs	1	$\beta_1 = NX_1^{II}$
		2	$\beta_2 = NX_2^{II} + \beta_1 X_1^{II}$
		3	$\beta_3 = NX_3^{II} + \beta_1 X_2^{II} + \beta_2 X_1^{II}$
		4	$\beta_4 = NX_4^{II} + \beta_1 X_3^{II} + \beta_2 X_2^{II} + \beta_3 X_1^{II}$
III	Number of major repairs	1	$\lambda_1 = NX_1^{III}$
		2	$\lambda_2 = NX_2^{III} + \lambda_1 X_1^{III}$
		3	$\lambda_3 = NX_3^{III} + \lambda_1 X_2^{III} + \lambda_2 X_1^{III}$
		4	$\lambda_4 = NX_4^{III} + \lambda_1 X_3^{III} + \lambda_2 X_2^{III} + \lambda_3 X_1^{III}$

Total cost upto ‘n’ time periods:

$$\begin{aligned} TC(n) = C_1\left[\alpha_1 + \alpha_2 \nu + \alpha_3 \nu^2 + \ldots + \alpha_n \nu^{n-1}\right] \\ + C_2\left[\beta_1 + \beta_2 \nu + \beta_3 \nu^2 + \ldots + \beta_n \nu^{n-1}\right] \\ + C_3\left[\lambda_1 + \lambda_2 \nu + \lambda_3 \nu^2 + \ldots + \lambda_n \nu^{n-1}\right] + NC_4 \nu^{n-1} \end{aligned} \quad \text{--- (4)}$$

$$= C_1 \sum (\alpha_n \nu^{n-1}) + C_2 \sum (\beta_n \nu^{n-1}) + C_3 \sum (\lambda_n \nu^{n-1}) + NC_4 \nu^{n-1} \quad \text{---}$$

(5)

where n=1,2,3, …

Weighted Average cost per period=AC(n) = $\dfrac{TC(n)}{\sum \nu^{n-1}}$ --- (6)

3. Results and Discussion:

3.1 Case Study: Influence of forecasted inflation on block replacement decision using second order Markov chain

To understand the impact of the variable maintenance cost and different inflation trends, block replacement strategy is evaluated in the following cases:

a) Variable maintenance cost – low initial maintenance cost with big increments during later periods – in the same range of maintenance cost.
b) Variable maintenance cost - high initial maintenance cost with small increments during later periods – in the same range of maintenance cost.
c) Rapid up-trend in inflation (assumed inflation data)
d) Sluggish up-trend in inflation (assumed inflation data)

Table 6: Parameters involved in the case study

S. No.	**Parameter**	**Cost in Rupees**
1	N	1000
2	C_1	40000
3	C_2	1000
4	C_3	8000
5	C_4	30000/per computer (each item cost under group replacement)
6	r_n	20% = 0.2 (nominal rate of interest)

Real interest rate= $r_t = \dfrac{(r_n - \varphi_t)}{(1+\varphi_t)}$, from Fisherman’s relation

$$X_0 = \left[X_0^{I} \quad X_0^{II} \quad X_0^{III} \quad X_0^{IV}\right] = \left[0.850 \quad 0.085 \quad 0.045 \quad 0.020\right]$$

To generate/estimate state transition probabilities for future periods, Weighted Moving Transition Probability Method (WTPM), a parsimonious model that approximates second order Markov chain is employed.

Generators of Markov process (TPMs) are:

$$P_{n-2} = \begin{bmatrix} 0.8824 & 0.0494 & 0.0388 & 0.0294 \\ 0.6824 & 0.1294 & 0.1059 & 0.0824 \\ 0.1778 & 0.2000 & 0.2222 & 0.4000 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$, \quad \delta_{n-2} = 0.2$$

$$P_{n-1} = \begin{bmatrix} 0.8192 & 0.0577 & 0.0480 & 0.0749 \\ 0.7092 & 0.0716 & 0.0637 & 0.1554 \\ 0.3328 & 0.0791 & 0.0774 & 0.5105 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$, \quad \delta_{n-1} = 0.8$$

The state probabilities of items in different states can be computed as = (probability of items in different states in initial period) X $\left(\mathrm{TPM}\right)_n$

i.e. $X_n = X_0\left(P_{ij}\right)_n$, where n=1,2,3,…

The forecasted inflation data through regression modelis used to calculate the real value of money (or real rate of interest) by Fisherman's relation. This real rate of interest is used to calculate Present Worth Factor (PWF). The calculation of various costs for block replacement decision under the influence of inflation and time value of money using second order Markov chain is shown in Table 7.

As the weighted average annual cost over a period of 4 years is minimum and started increasing from 5th year onwards, it can be inferred that economics can be achieved if the block replacement is done at the end of 4th year i.e. when the inflation with up-trend is considered, the optimal block replacement period (using SOMC also) is advanced to 4th year from 5th year.

Table 7: Computations for block replacement model with the influence of Inflation using SOMC

1	2	3	4	5	6	7
Period (n)	Inflation, ϕ (%)	Real rate of interest, rt= (rn- ϕt) / (1+ ϕt)	PWF, ν = 1/(1+ rt)	Discount factor (νn-1)	Σνn-1	Ind. Replacement cost= αn*C1 * νn-1
1	-18.56	0.473	0.679	1	1	27.96
2	-17.41	0.453	0.688	0.6882	1.6882	34.32
3	-0.49	0.206	0.829	0.6876	2.3759	34.92
4	0.65	0.192	0.839	0.5900	2.9659	33.83

5	17.58	0.021	0.980	0.9217	3.8877	58.33
6	18.72	0.011	0.989	0.9477	4.8355	66.52
7	35.65	-0.115	1.130	2.0865	6.9220	162.25
8	36.8	-0.123	1.140	2.5022	9.4243	215.62

8	9	10 = (7+8+9)	11	12	13 = (11+12)	14 = (13/6)
Minor Repair cost = βn*C2 * νn-1	Major repair cost = λn*C3*νn-1	Maintenance cost(R) (lacs)	ΣR (lacs)	Group replacement cost = N* C4* νn-1	Total cost (TC) (in lacs)	Weighted average annual cost (Rs. in lacs)
0.61	4.15	32.73	32.73	300.00	332.73	332.73
0.43	2.88	37.64	70.37	206.47	276.84	163.98
0.45	3.05	38.43	108.80	206.29	315.10	132.62
0.41	2.74	36.99	145.80	177.01	322.82	108.84
0.69	4.50	63.53	209.33	276.52	485.85	124.97
0.75	4.86	72.14	281.47	284.33	565.81	117.01
1.75	11.24	175.25	456.72	625.96	1082.69	156.41
2.22	14.16	232.01	688.74	750.68	1439.42	152.73

When the influence of predicted inflation and net worth of the money is considered, both ways using FOMC and SOMC resulted in the early replacement of block of computers at the age of 4 years.

3.1 a: To understand the influence of variable maintenance cost, a pattern with low initial maintenance cost (minor and major repair costs) and big increments at later periods is considered as given in Table 8.

Table 8 : Variable maintenance cost - with low initial maintenance cost and big increments
during the later time periods

Time period (years)	1	2	3	4	5	6	7	8
Minor repair	200	500	700	900	1100	1300	1300	1300

cost (Rs.)								
Major repair cost (Rs.)	2000	5000	8000	11000	13000	13000	13000	13000

Table 9 : Computations for block replacement model with the influence of inflation using SOMC with low initial maintenance cost and big increments during later periods

1	2	3	4	5	6	7	8
Period (n)	Inflation, ϕ (%)	Real rate of interest, $r_t =$ $(r_n - \phi_t)$ / $(1+ \phi_t)$	PWF, $\nu = 1/(1+ r_t)$	Discount factor (ν^{n-1})	$\Sigma\nu^{n-1}$	Ind. Replacement cost= $\alpha_n * C_1 * \nu^{n-1}$	Each Minor repair cost (C_2)
1	-18.54	0.473	0.679	1	1	27.96	0.002
2	-17.41	0.453	0.688	0.6882	1.6882	34.32	0.005
3	-0.49	0.206	0.829	0.6876	2.3759	34.92	0.007
4	0.65	0.192	0.839	0.5900	2.9659	33.83	0.009
5	17.58	0.021	0.980	0.9217	3.8877	58.33	0.011
6	18.72	0.011	0.989	0.9477	4.8355	66.52	0.013
7	35.65	-0.115	1.130	2.0865	6.9220	162.25	0.013
8	36.8	-0.123	1.140	2.5022	9.4243	215.62	.01300

9	10	11	12 = (7+9+11)	13	14	15 = (13+14)	16 = (15/6)
Total Minor Repair cost = $\beta_n * C_2 * \nu^{n-1}$	Each Major repair cost (C_3)	Total Major repair cost = $\lambda_n * C_3 * \nu^{n-1}$	Maintenance cost(R)	ΣR	Group replacement cost = $N * C_4 * \nu^{n-1}$	Total cost (TC)	Weighted average annual cost (Rs. in lacs)
0.12	0.02	1.03	29.12	29.12	300.00	329.12	329.12
0.21	0.05	1.80	36.34	65.46	206.47	271.94	161.07
0.32	0.08	3.05	38.29	103.76	206.29	310.05	130.50
0.37	0.11	3.77	37.98	141.74	177.01	318.76	**107.47**
0.75	0.13	7.31	66.41	208.16	276.52	484.68	124.67
0.97	0.13	7.90	75.40	283.56	284.33	567.90	117.44
2.28	0.13	18.27	182.80	466.37	625.96	1092.34	157.80
2.89	0.13	23.01	241.53	707.90	750.68	1458.58	154.76

3.1 b. Replacement decision with high initial maintenance cost and small increments:

To understand the influence of variable maintenance cost, another pattern with high initial maintenance cost (minor and major repair costs) and small increments at later periods is considered as given in the table 10.

Note: Variable maintenance cost (minor and major repair costs) is taken in the range of fixed maintenance cost.

Table 10: Variable maintenance cost - with high initial maintenance cost and small increments during the later time periods

Time period (years)	1	2	3	4	5	6	7	8
Minor repair cost (Rs.)	1000	1100	1200	1300	1400	1400	1400	1400
Major repair cost (Rs.)	8000	9000	10000	11000	12000	13000	13000	13000

Table 11 shows the computations for block replacement decision *(with the influence of inflation)* using SOMC with high initial maintenance cost and small increments during the later time periods. The data (except the minor and major repair costs) is same as in the case of fixed maintenance cost.

From Table 11, as the average annual cost is minimum over a period of 4 years and started increasing from 5th year onwards, it can be inferred that economics can be achieved if the block replacement is done at the end of 4th year.It is observed that the replacement decision remains same as in the case of fixed maintenance cost (Table 10). It is also to be noted that the increase in inflation resulted in the advancement of replacement period.

Table 11: Computations for block replacement model with the influence of inflation using SOMC with high initial maintenance cost and small increments during later periods

1	2	3	4	5	6	7	8
Period (n)	Inflation, ϕ (%)	Real rate of interest, $r_t = (r_n - \phi_t) / (1+ \phi_t)$	PWF, $\nu = 1/(1+ r_t)$	Discount factor (ν^{n-1})	$\Sigma\nu^{n-1}$	Ind. Replacement cost= $\alpha_n * C_1 * \nu^{n-1}$	Each Minor repair cost (C_2)
1	-18.54	0.473	0.679	1	1	27.96	0.010
2	-17.41	0.453	0.688	0.6882	1.6882	34.32	0.011
3	-0.49	0.206	0.829	0.6876	2.3759	34.92	0.012
4	0.65	0.192	0.839	0.5900	2.9659	33.83	0.013
5	17.58	0.021	0.980	0.9217	3.8877	58.33	0.014
6	18.72	0.011	0.989	0.9477	4.8355	66.52	0.014

7	35.65	-0.115	1.130	2.0865	6.9220	162.25	0.014
8	36.8	-0.123	1.140	2.5022	9.4243	215.62	0.014

9	10	11	12 = (7+9+11)	13	14	15 = (13+14)	16 = (15/6)
Total Minor Repair cost = $\beta_n * C_2 * \nu^{n-1}$	Each Major repair cost (C_3)	Total Major repair cost = $\lambda_n * C_3 * \nu^{n-1}$	Maintenance cost(R) (Rs. in lacs)	ΣR (Rs. in lacs)	Group replacement cost = $N * C_4 * \nu^{n-1}$	Total cost (TC) (Rs. in lacs)	Weighted average annual cost (Rs. in lacs)
0.61	0.08	4.15	32.73	32.73	300.00	332.73	332.73
0.47	0.09	3.24	38.04	70.77	206.47	277.25	164.22
0.55	0.10	3.81	39.28	110.06	206.29	316.35	133.15
0.54	0.11	3.77	38.15	148.21	177.01	325.23	**109.65**
0.96	0.12	6.75	66.05	214.27	276.52	490.79	126.24
10.05	0.13	7.90	75.48	289.75	284.33	574.09	118.72
20.45	0.13	18.27	182.98	472.73	625.96	1098.70	158.72
30.12	0.13	23.01	241.75	714.49	750.68	1465.17	155.46

Table 12 and Fig. 1 furnish the summary of the numerical results obtained (Table 9, 10 and 11), for block replacement decision with high initial maintenance cost and small increments during the later periods, using - first order Markov chain with and without inflation, and Weighted Moving Transition Probability (WMTP) methods.

Table 12: Numerical results for annual average costs with high initial maintenance cost and small increments during later periods

Time period (n years)	Using First Order Markov Chain		Using Second Order Markov chain
	without inflation	with inflation	with inflation
	Annual Avg. cost(lacs)	Weighted Avg. Annual Cost(lacs)	Weighted Avg. Annual Cost(lacs)
1	332.73	332.73	332.73
2	194.00	164.22	164.22
3	155.92	139.7	133.15
4	143.76	123.38	**109.65**
5	142.85	152.73	126.24
6	148.55	159.11	118.72
7	159.04	227.42	158.72
8	173.77	253.77	155.46

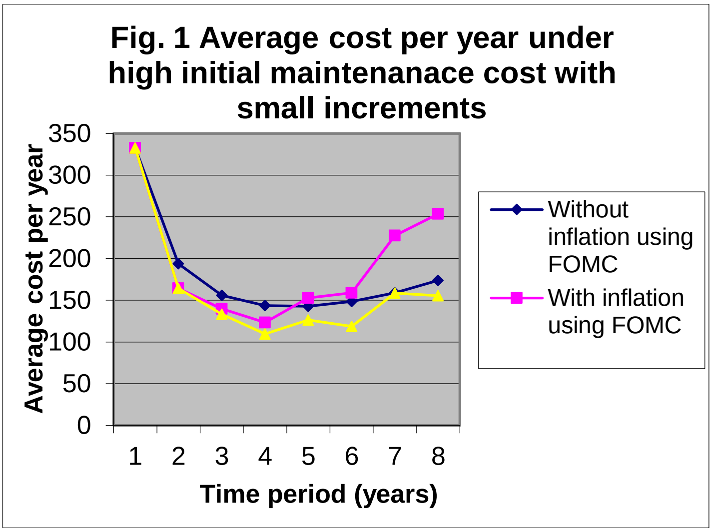


Fig. 1 Average cost per year under high initial maintenanace cost with small increments

When the influence of inflation is considered, FOMC and SOMC calculations resulted in the early replacement of block of computers at the age of 4 years. It can be observed that the optimal replacement decision (block replacement at the end of 4th year) remains same as in the case of fixed maintenance cost

Therefore, when the maintenance cost is varied with different trends – high initial maintenance cost and small increments; and with low initial maintenance cost and big increments; – in the same range of fixed maintenance cost, it is observed that the replacement decision remains the same as in the case of fixed maintenance cost.

However to evaluate behaviour of the developed Block Replacement model under different inflation patterns, two trends – rapid up-trend and sluggish up-trend – in inflation are considered.The data for these rapid and sluggish (inflation) up-trends with respect to the forecasted inflation is shown in Table 13 and plotted in Fig. 2.

Table 13: Inflation trends: Rapid and Sluggish up-trends in inflation with respect to the forecasted inflation.

Time period (year)	Forecasted inflation (%)	Rapid increase in inflation (%)	Sluggish increase in inflation (%)
1	-18.54	-18.54	-18.54
2	-17.41	-5	-15
3	-0.49	5	-12
4	0.65	20	-9
5	17.58	30	-6
6	18.72	40	-3
7	35.65	50	1
8	36.8	60	4

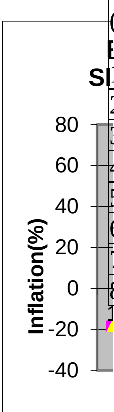


Table 14: Computations for the influence of sluggishup-trend in Inflation on block replacement decision using FOMC

1	2	3	4	5	6	7
Period (n)	Inflation, ϕ (%)	Real rate of interest, $r_t = (r_n - \phi_t) / (1+ \phi_t)$	PWF, $\nu = 1/(1+ r_t)$	Discount factor (ν^{n-1})	$\Sigma\nu^{n-1}$	Ind. Replacement cost= α_n*C_1 * ν^{n-1}
1	-18.54	0.473	0.679	1	1	27.96
2	-15	0.412	0.708	0.7083	1.7083	35.32
3	-12	0.364	0.733	0.5377	2.2461	39.66
4	-9	0.319	0.758	0.4360	2.6822	43.90
5	-6	0.277	0.783	0.3765	3.0587	49.71
6	-3	0.237	0.808	0.3451	3.4038	58.43
7	1	0.188	0.842	0.3555	3.7593	76.18
8	4	0.154	0.867	0.3672	4.1265	98.83

8	9	10 = (7+8+9)	11	12	13 = (11+12)	14 = (13/6)
Minor Repair cost = β_n*C_2 * ν^{n-1}	Major repair cost = λ_n*C_3*ν^{n-1}	Maintenance cost(R) (Rs. in lacs)	ΣR (Rs. in lacs)	Group replacement cost = N* C_4* ν^{n-1}	Total cost (TC) (in lacs)	Weighted average annual cost (Rs. in lacs)
0.61	4.15	32.73	32.73	300.00	332.73	332.73
0.44	2.96	38.73	71.47	212.50	283.97	166.22
0.33	2.25	42.26	113.73	161.33	275.06	122.46
0.27	1.83	46.01	159.75	130.82	290.57	108.33
0.24	1.57	51.53	211.28	112.95	324.23	**106.00**
0.22	1.44	60.10	271.39	103.53	374.92	110.14
0.23	1.48	77.90	349.29	106.65	455.94	121.28
0.24	1.53	100.61	449.90	110.17	560.08	135.72

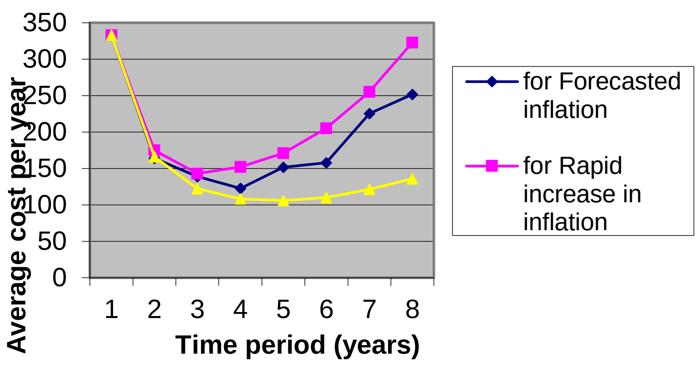
Table 14 and Fig. 2 furnish the summary of the numerical results obtained for the influence of rapid and sluggish up-trends in inflation (with respect to the forecasted inflation) on block replacement decision, using first order Markov chain.

Table 15 and Fig. 3 furnish the summary of the numerical results obtained, for the influence of rapid and sluggish up-trends in inflation (with respect to the forecasted inflation) on block replacement decision, using first order Markov chain. The present results are in line with the previous research results[11].

Table 15: Average cost per year for rapid and sluggish up-trends in inflation

Time period (years)	**Weighted Average annual cost per year (Rs. in lacs)**		
	for Forecasted inflation	**for Rapid increase in inflation**	**for Sluggish increase in inflation**
1	332.73	332.73	332.73
2	163.98	174.99	166.22
3	139.19	**143.07**	122.46
4	**122.62**	152.28	108.33
5	151.59	170.92	**106.00**
6	157.63	205.05	110.14
7	225.52	255.43	121.28
8	251.62	322.92	135.72

Fig. 3 Weighted Average cost per year for different inflation trends



The optimal replacement age for the block items in the case of forecasted inflation is 4 years. *Whereas, in case of rapid up-trend in inflation, the optimal replacement age is advanced to 3rd year and instead, in case of sluggish up-trend in inflation, the optimal replacement age for the block of items is delayed to 5th year.*

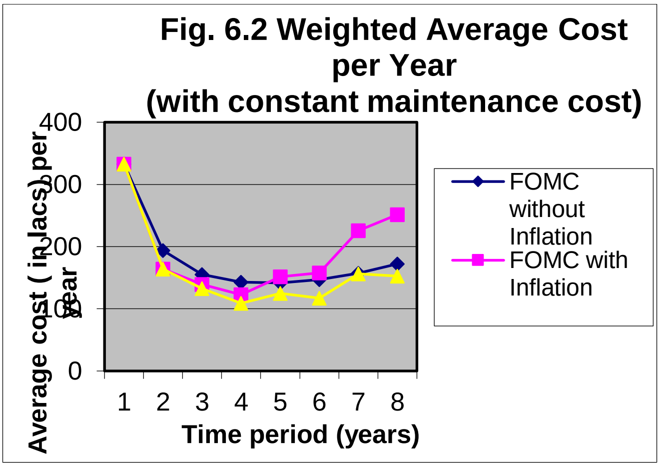
Table 16: Replacement ages with the developed models

Parameter description	**Using Second Order Markov chain**	**Using Second Order Markov chain**
	Without inflation	**with inflation**
With constant maintenance	**4 years**	**8 years**

cost		
Variable maintenance cost: low initial maintenance cost with big increments during later periods	**4 years**	**-**
Variable maintenance cost: High initial maintenance cost with small increments during later periods	**4 years**	**-**

Table 17: Replacement ages with variable trends in inflation

Replacement age in years		
Forecasted inflation	**Rapid increase in inflation**	**Sluggish increase in inflation**
4 years	3 years	5 years



4. Conclusions:

Second order Markov Chain (SOMC) has been applied to calculate the replacement age for the computers as a part of block replacement policy. Replacement decisions for variable maintenance with low initial and high initial costs with big and small increments during later periods is taken as the case study in comparison with rapid increase in inflation and sluggish inflation has been discussed in detail.

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