

# Analysis of vortex structures in problems of computational fluid dynamics

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**Abstract.** The article discusses methods for identifying vortex structures when analyzing the results of numerical simulations in hydro- and gasdynamics, thermophysics, and in applications. The structures were identified in an authorized visualization system using additional processing of scalar, vector and tensor fields. The technique is illustrated with examples of visualization of simulation results in computational fluid dynamics and astrophysics. Data processing was carried out using topological description methods.

## 1 Vortex flow analysis

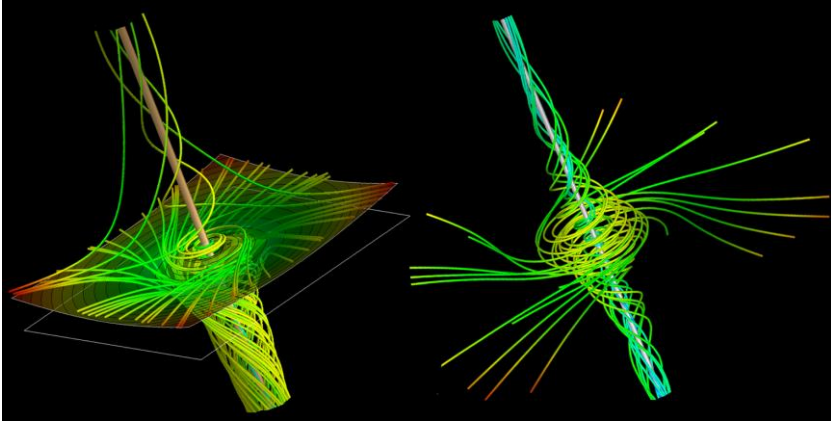
Numerical modeling of transient and unsteady regimes of various vortex flows is a complex task and requires calculations to be accompanied by graphical post-processing and visual analysis of three-dimensional coherent structures, highlighting the characteristic structural features of flows and calculating additional integral characteristics. One of the requirements for visual analysis is the correct illumination of the process of vortex formation, the evolution of vortices, taking into account the peculiarities of the interaction of the fluid and some structural elements in technical devices.

To explain the various effects of vortex formation, increasing attention is paid to topological analysis of flows [1]. In post-processing methods, a significant place is occupied by symbolic techniques of reflecting flows, with graphical display of scalar and vector fields in the form of trajectories of liquid particles, surfaces and variable volumes of gas, etc., replacing experimental flow visualization techniques. This approach allows us to visually show the dynamics of the interaction of vortices generated by various mechanisms of loss of stability of liquid and gas motion, such as Rayleigh-Taylor instability (RTI), Kelvin-Helmholtz instability (KHI) and a number of other factors. The KHI instability, which arises due to a sharp velocity shift in flow layers, is one of the main ones for identifying coherent formations [2]. To graphically represent vortices as vortex tubes, the construction of vector field vorticity filaments in field  $\boldsymbol{\Omega} = \nabla \times \boldsymbol{U}$  is often used. During post-processing, vortex zones are often identified in areas with a positive  $Q$ -criterion value  $Q = (\|\boldsymbol{\Omega}\|^2 - \|\boldsymbol{S}\|^2) / 2$  [3]. For the same purposes, we used the determination of the  $\lambda_2$ -criterion (checking for the value of the second eigenvalue of the tensor  $\boldsymbol{S}^2 + \boldsymbol{\Omega}^2$ , which is

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negative in the vortex core), the use of the Liutex (Rortex) method and other criteria noted in [4]. Proven methods for eddy identification are among the options of the authorized HDVIS visualization system, which has been used for analysis in various problems [5]. In the environment of this system, it is possible to process calculations with interactive analysis of changes in the main variables, the formation of derived values and symbolic data reflecting the features of flows with a graphical representation of the simulated structures. The system was tested using classical examples, in particular to the Burgers vortex formation problem [6], which was used to analyze and compare the effects of shear and stretching/compression of vortices according to the criteria of their topological description. A visualization of the vortex is given in Figure 1.



**Fig. 1.** Burgers vortex streamlines (colored by tangential velocity), and identification of axis using the  $Q$ -criterion.

Using combined techniques for constructing visualization scenes in the system used, it is possible to increase the qualitative enrichment of information about the vortex structure of flows by highlighting their features in zones near shock waves, in areas of jumps and discontinuities, and in other zones important for analysis, with the addition of detailed animation of unsteady flows in the dynamics of process modes.

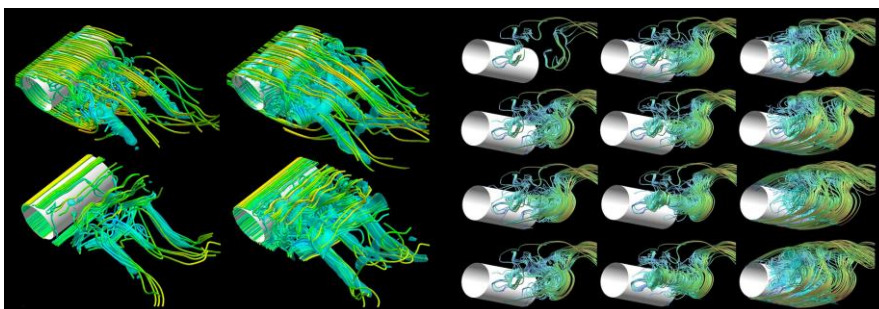
## 2 Numerical solution of hydro- and gasdynamics problems of various scales

To demonstrate vortex shedding, examples of modeling gas flows in devices and environments of different spatial and temporal scales are used. Flow simulations were carried out using Reynolds-averaged Navier–Stokes (RANS) equations. Some problems were solved using in-house program SINF (later SINF/Flag-S) [7]. The code's capabilities have been expanded by using LES, DES and DNS methods. The latter method (Direct Numerical Simulation) was used to study the flow around a heated circular cylinder, a problem discussed in [8]. Another in-house program (beginning representation in article [9]), was intended for the numerical solution of gasdynamics problems using Euler's equations. The code was used to simulate high-speed gas dynamics on a hybrid architecture computer. The program was used to solve a number of problems in computational astrophysics, one of which [10] is discussed from the perspective of experience in visual analysis of results.

## 2.1 Flow around a heated circular cylinder

The purpose of the study [8] was to simulate the vortex structure of a turbulent flow flowing around a heated round cylinder at a Reynolds number of 5900. This case is characterized by a stochastic disturbance of the boundary layer near the surface of the cylinder and a transient flow regime. The transverse size of the calculation area was equal to two diameters of the cylinder. A six-block structured grid containing 218,000 cells, non-coinciding along the internal boundaries of blocks, was used. The uniform velocity conditions were imposed at the inlet of the computational domain, and a constant pressure was specified at the outlet. A constant temperature was set on the surface of the cylinder, higher than at the entrance to the computational domain. Periodicity conditions were specified at the lateral boundaries.

The flow under consideration is characterized by the development of pulsating transverse vortices of high intensity and their variations with large amplitude, both across and downstream. Visualization of the flow structure in the recirculation zone behind the cylinder and in the wake vortex is presented in Figure 2.



**Fig. 2.** Formation of a vortex structure in the recirculation zone behind the cylinder, visualized by streamlines and vortex filaments associated with  $Q$ -criterion, vorticity and pressure isosurfaces.

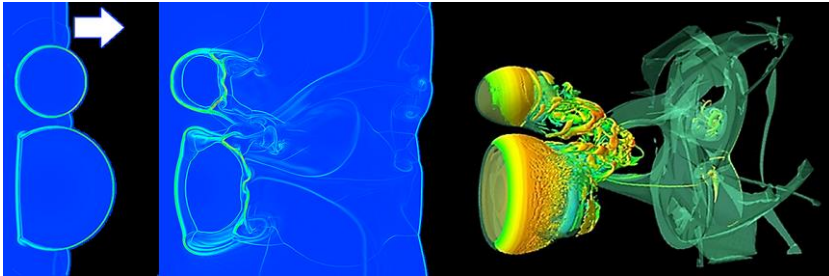
It shows the famous Kármán vortex street (VK instability), resulting from the Kelvin-Helmholtz instability, which generates a sequence of swirling vortices entrained by the flow. At the top and bottom walls of the cylinder, the flow is separated from its surface at the trailing edge due to an unfavorable pressure gradient here. This occurs alternately, which leads to oscillating instability of the vortex flow. A zone of intense recirculation is formed immediately behind the cylinder, where the vortex formed passes into the next vortex, partially repeating the previous shape and changing the direction of rotation. The superposition of areas of low and high velocities and pressure leads to the development of a swirling shear layer that forms the VK structure. The structure of the flow can be seen in the right fragment of Figure 2, which shows the formation of trajectories of spiral vortices passing through points on pressure isosurfaces, with a consistent decrease in its value as the marked particles approach the axis of the twisted spiral.

The practice of successively refining and identifying the axes of vortex formations has shown that specifying starting points (or intermediate points for tracing) on a given isosurface of vorticity -  $\Omega$ , helicity -  $H$ , or  $Q$  - structures allows you to select areas of swirl as close as possible to the vortex axis, which is most effective for the perception of the vortex turbulization mechanism. The change in the size of the VK vortices in the recirculation zone of the flow can be interpreted as an indicator of the strength of the vortex. Analysis of the fluid movement around the cylinder revealed secondary vortices in which streamlines and vortex filaments twist, connect and break as local pressure changes, causing individual fluid particle lines and vortex filaments to wrap around each other, forming hairpin-shaped vortices of smaller sizes. Calculation of pressure and heat transfer characteristics on the

surface of the cylinder, combined with a detailed study of the flow and experimental data [11], showed good agreement between the DNS modeling, visualizing and measurements.

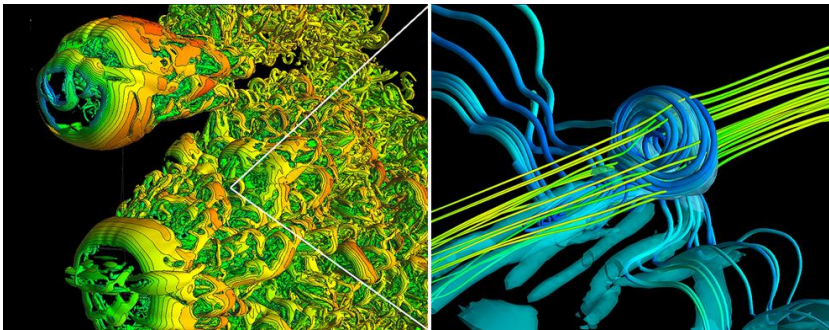
## 2.2 Changes in molecular clouds after collision with SNE shock wave

The second problem, in which methods for identifying vortex structures were used, was modeling the collision of a strong shock wave (SW) arising during a supernova explosion (SNE) with molecular clouds (MC) as it passes through a system of two MCs [10]. The identification of vortex structures was carried out to study the consequences of the influence of emerging KHI and Richtmyer–Meshkov instability (RMI) on the morphology of cloud remnants. The visual analysis was carried out using tensor-invariant markers [12, 13]. The SW/MC collision diagram is shown in Figure 3.



**Fig. 3.** Morphological changes in molecular clouds after a collision with shock wave after SNE.

The formation of transitional coherent structures begins immediately, with the beginning of the wave passing through the boundaries of closely spaced clouds.



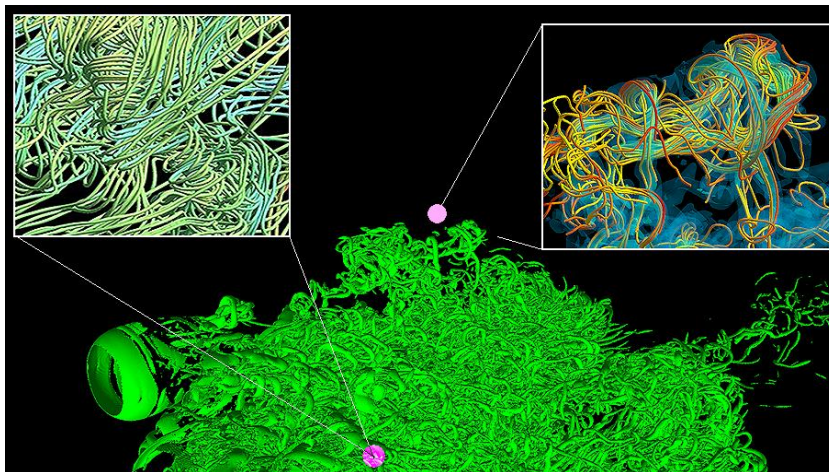
**Fig. 4.**  $Q$ -criterion structures (colored by velocity magnitude) at the boundary of merging clouds and vortices in the form of elongated hairpins with vorticity filaments and streamlines.

When the bow shock wave bends around the clouds, secondary shock waves are formed behind its front, the cloud matter in the impact zone begins to interact with the gas of the SN residual substance behind the shock wave, forming a new cloud formation with closed regions. The substance of new formation has tens and hundreds of times higher density. From the given Figures (3, 4 and 5) it is clear how the primary longitudinal and transverse vortices initiated by KHI are generated, and how the shape of a new gas formation changes. Behind the main shock wave, RM instability arises (in the gas wake from the clouds) and KH instability develops between the outer layers of closely spaced molecular clouds and leading to the formation of folds of more compressed cloud matter, which begin to swirl in the gradient near-surface zone.

Near the boundaries with a sharp change in the density of the cloud matter, vortex rings appear which are deformed and break up into many horseshoe-shaped vortices and

formations elongated in the shape of a hairpin. When analyzing the forming vortex in the cloud wake behind the passing shock wave, it was noticed that it takes a shape close to the shape of a trefoil of fused vortex tubes, which is typical as a manifestation of a secondary KHI with three stretched and skewed vortex pairs [1].

In digital animation (option of the visualization system), the pulsating evolution of the vortex structures of the remnants begin clearly visible, which periodically move away from the upper cloud and merge with the lower cloud, forming packages of secondary budding hairpins and horseshoes vortices (Figures 4 and 5). This is a manifestation of the VK and KH instability already mentioned above.



**Fig. 5.**  $Q$ -criterion structures on the outer boundaries of merged clouds and spiral nodes inside them.

Visual analysis of  $Q$ -structures in the form of hairpins, where the legs, neck and ring head of the hairpin are traditionally distinguished, showed that in a larger number of combinations they are not a continuous formation of vortex filaments but broken structures (Figure 4). The ring head has a  $\Omega$ -shape and in many cases is separated from the quasi-longitudinal branches of the vortex. For the outer regions, the  $Q$  and  $\lambda_2$ - structures partially include vortex filaments (right fragment of the figure). Inside the clouds, vortex nodes are distinguished - spiral forms of vortices containing gas streamlines (left fragment of the figure). It can be noted that in the area of these nodes the density of the cloud remnant matter increases sharply. The identified forms confirm similar observations from [12], with extension to the unlimited areas considered in this work. The revealed morphology of formations in the discussed model is typical for plasma and high-speed gas flows, as a reflection of the decisive influence of KHI on the vortex structure of such flows. The results obtained are in good agreement with the analysis from [13]. The bifurcation of a gas flow is evolutionarily repeated with the non-stationary interaction of residues: various horseshoe-shaped formations, hairpins, individual gas clumps, and twisted filamentary forms during the transformation of vortices from  $\Lambda$ -vortices to multi-level hairpin vortices in the cascade process of turbulization of the MCs substance.

### 3 Conclusions

In the calculations carried out, vortex filament nuclei were identified according to local patterns using tensor-invariant markers. A combined approach to specifying streamlines and vorticity filaments passing through the reference points of different vortex criteria isosurfaces made it possible to improve the perception of vortex structures in gradient

regions of changes in various vector and scalar fields. A proven methodology using HDVIS vortex identification capabilities was used to efficiently graphically analyze vortex structures at multiple scales. The consequences of the influence of the Kelvin–Helmholtz shear hydrodynamic instability on the evolutionary change in the shape of turbolized spots and filaments, as well as on other features of vortex flows of different scale, are revealed.

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